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# Advanced Electrical Measurements

BY

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## PREFACE

The nine years which have elapsed since the publication of the first edition of Advanced Electrical Measurements have been marked by considerable advance in all fields; especially in the application of electronic methods to electrical measurements and in the development of new and improved networks for alternating current impedance measurements. This progress has made a thorough revision of the text necessary. In the meantime difficulties of a geographical nature and limitations of time have made it impossible for the original senior author to collaborate in the preparation of the new manuscript. It is with extreme regret that I announce his withdrawal from the joint authorship of this book.

Like the earlier publication, this edition has attempted to cover, as well as the standard methods and apparatus peculiar to the electrical laboratory, the application of instruments essentially electrical in character to the measurement of other physical quantities, such as pressures, temperatures and ionic concentrations. An attempt has been made to include sufficient theoretical discussion before the detailed experimental directions so that the good student will be nearly independent of the instructor and so that he will understand the basic principles involved sufficiently well to be able to adapt the standard procedures to unusual circumstances as occasion may arise.

The problems of inclusion and exclusion have offered some difficulty. When a choice between alternative methods or commercial instruments has been available, the author has chosen those with which he has had personal experience and which he has found satisfactory. In general, whenever an improved instrument for the measurement of an electrical quantity is available its historical predecessors have been omitted, but a few of the methods which are partially obsolete, such as the Carey-Foster method for resistance and some of the impedance bridges treated in Chapter V, have been retained because of their peda-

gogical value in preparing the student for more modern measurements.

I wish to express my great indebtedness to the many individuals who have assisted in the preparation of the text. First among these is Professor W. R. Smythe, who has consented to the use of the material of the old edition. So many of my colleagues and graduate students have been of assistance that I cannot list them all, but I want to express my particular thanks to Miss Norma Curtis, who read and offered valuable criticism on the greater part of the manuscript and who, along with Miss Martha Cox and others, has been of great help in the reading of the proofs. Professor William H. Doyle offered valuable suggestions for the chapter on electrochemical measurements. The manufacturers who have supplied the photographs for the illustrations which will be found throughout the book have been very helpful with information and suggestions.

Finally, I wish to express my thanks to the many students and laboratory workers who have located errors in the older edition or have suggested improvements and to say that I should deeply appreciate continued help of this nature.

WALTER C. MICHELS

BRYN MAWR, PENNSYLVANIA
January, 1941

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# CHAPTER 1

## OBSERVATIONAL DATA—GRAPHICAL REPRESENTATION—PRECISION OF MEASUREMENTS

### 1. Observational Data.

In both collegiate and industrial work, the solution of an experimental problem in electrical measurements may be divided, for convenience, into five operations:

1. The choice of the most efficient method, using available apparatus, for obtaining the desired data with the requisite accuracy.
2. The intelligent manipulation of the apparatus.
3. The recording of the data thus obtained in a form that will remain permanently legible and unambiguous.
4. The estimation of the accuracy of the results.
5. The preparation of a report presenting the results to the interested persons, who may be non-technical.

The first two operations will occupy the later chapters of this book and the last three will be treated briefly in the next few pages. It is hoped that the treatment of these will cover ordinary cases and serve as an introduction to more complete works on the subject. A problem is not completely solved until all the above operations have been completed.

### 2. Original Data.

The practice of jotting down a few data on loose sheets of paper and trusting to memory to fill in the gaps is certain, eventually, to produce mistakes. The record should be sufficiently legible and complete to be intelligible at any future time, so that not only the author, but any technically qualified person can prepare a report from it. Hence original data should be kept in a bound notebook and should contain the following elements:

1. A carefully drawn wiring diagram of the apparatus used, showing all possible current paths and emphasizing any original or unusual features.

2. Some means of identifying every piece of apparatus which can affect the accuracy of the results. This precautionary measure may enable an experimenter to revise his data instead of repeating them, if, for example, a certain meter is later found to have been in error. The usual means of identification is by the use of the manufacturer's name and serial number, although company numbers are sometimes more convenient.

3. A neat tabulation of all readings taken. At the head of each column should be given the quantity measured, the units used, and a symbolic reference to the wiring diagram. *The units are important.* For example, "current in milliamperes" is intelligible to anyone while "current" may be clear only to the experimenter himself.

4. All formulas used with a sample, at least, of each calculation involved. Any approximations introduced should be noted and the way in which each reading enters into the final result should be apparent.

5. The final result, in numerical or graphical form, with an estimate of its accuracy.

6. A statement of any unusual features which may affect the accuracy or the interpretation of the results. It may be well to include an extended discussion here, particularly if unexpected or apparently illogical results are obtained. The data sheet is the place to show whether such results are real or spurious.

The exact form of the data sheet depends on the type of test performed and the laboratory custom, but should include, in some manner, the elements listed above.

### 3. The Final Report.

No exact rules can be given here for the preparation of the final report since each laboratory has its own form. In general, however, the final report contains all the elements of a data sheet except, in some cases, the detailed readings. In addition, it usually includes a discussion of the interpretation which the experimenter puts on the result and often some account of the theory leading to that interpretation. In writing the report, always keep in mind the technical qualifications of those for whom it is intended.

**4. Graphical Representation.**

One very efficient method of presenting results is by the use of graphs. It is concise and clear and can be made to bring out relationships which are not at all evident in the original tabulation.

Before plotting results, it is necessary to select for coordinates suitable functions of the measured quantities. For many types of measurements, this selection is standardized and should be adhered to. The choice of the scale must meet two requirements; namely, that the curve fill the entire sheet as nearly as possible and that it be easily read. The latter restriction practically demands that one division of the coordinate paper correspond to decimal multiples of one, two or five units of the plotted quantities. When possible, the accuracy with which the coordinates can be read should be at least as great as that with which the quantities have been determined. Coordinate axes should be distinctly labeled and the relationship between plotted and measured quantities indicated somewhere on the sheet.

Unless overcongestion results, it is usually preferable to show each plotted point on the final graph. If there are physical reasons for expecting a smooth curve and one can be found which does not deviate excessively from the observed points, such an average curve may be drawn and the deviations of the points from the curve will picture the consistency of the results. There is usually no theoretical reason to expect a smooth calibration curve for an instrument such as an ammeter, and a broken line, connecting the observed points, is the preferred approximation.

The simpler the form of the approximation curve, the easier it is to select a good one. For this reason, as indicated above, one frequently chooses as coordinates simple functions of the measured quantities rather than the quantities themselves. For example, we may desire to determine the leakage rate of a condenser. The direct measurements give voltage as a function of time. However, since the logarithm of this voltage is a linear function of the time, we can obtain a straight line by plotting $\log V$ against $t$ on ordinary paper,—or $V$ against $t$ on semi-logarithmic paper. It is now very easy to select a good straight line approximation which shows at a glance the law of the decay of the potential. Such a graph is shown in Figure 1.

Extrapolation should be used with caution but it is obviously

much easier to extrapolate accurately a straight line than a curve. For example, in Figure 1, the first reading was not taken until thirty seconds after the start of the time count. If desired, the initial voltage can easily be obtained by extrapolation and is found to be 0.95 volts.

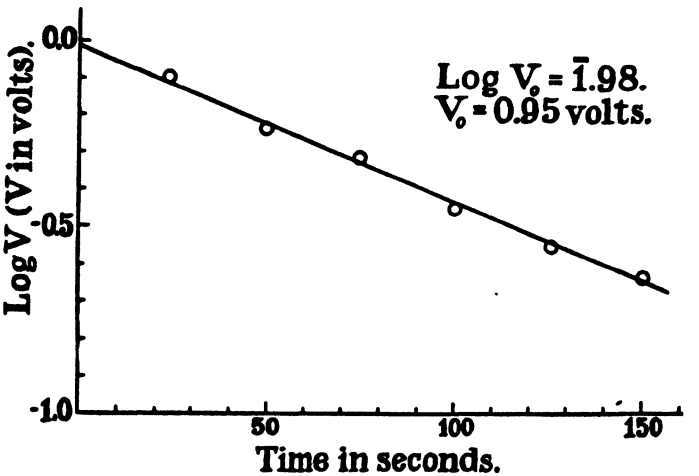


FIG. 1. Use of logarithmic scale.

**5. Precision of Measurements.**

The term "physical measurement" implies the more or less exact determination of numbers representing physical magnitudes. These numbers have little value unless their accuracy is known. This does not mean that high accuracy is always desirable, for, in general, the greater the precision, the more expensive and tedious the method. The best plan is to use the simplest method which will give the *necessary* accuracy. Inaccuracies may be due to determinate errors which can be discovered and eliminated and to indeterminate errors which usually can be estimated.

**6. Determinate Errors.**

Under this head come many instrumental, personal and "analytical" errors. *Instrumental errors* include such items as the inaccuracies of scale divisions of meters, deviations of standards from the designated values and effects of temperature on

resistance. They may often be determined by calibrating all instruments against laboratory standards, which, in turn, have been compared with primary standards by the Bureau of Standards in Washington, and by carefully controlling and correcting for such factors as temperature and atmospheric pressure. Some types of instrumental errors are extremely elusive. For example, every joint between two dissimilar metals is a possible source of a thermal E.M.F., which it is often impossible to observe. Whenever such errors may be appreciable, it is well to repeat the experiment, using other apparatus and, if feasible, a different method. In certain measurements, we will point out special precautions which must be taken to avoid these "hidden" errors.

*Personal errors* are those due to psychological or physical tendencies of the observer. For example, one person may consistently read a meter slightly high and another slightly low. Such errors may be eliminated by using some automatic system of recording or by having several persons repeat the observations. Their results may be combined as shown later. *"Analytical" errors* are those due to a misunderstanding of the apparatus or its manipulation. Such an error, for example, may occur in using a watt hour meter, for if the current coil be connected ahead of the potential coil the energy dissipated in the latter will be recorded, while if the potential coil be connected ahead the power in the current coil is included. In either case, a correction is necessary and can be made by a competent observer.

Thus, results by one observer using a single set of apparatus may be inaccurate, though consistent in themselves. Only when results are repeated with different apparatus and methods by several observers who thoroughly understand them does consistency become a sure test.

**7. Indeterminate Errors.**

After all known determinate errors have been found and corrected for, there will still remain certain other errors. Calibrations and standards may be in error and the magnitude of such *residual errors* is usually specified by the manufacturers or the standardizing laboratory. Other sources of error whose existence is unsuspected may still be present, but obviously cannot be estimated. However, there is one type of error which always

enters and whose magnitude may be estimated from examining the consistency of the results. These are known as *accidental errors* and are subject to the mathematical laws of probability. Fluctuations in room temperature, in the reaction time of an observer, or in the voltage of a power source are possible origins of such errors.

Certain rules governing the occurrence of accidental errors have been found:

1. Small errors are much more frequently found than large.
2. Very large errors are extremely unlikely.
3. Positive and negative errors are equally probable.

The last statement points the way to the elimination of these errors, for it indicates that if we take the arithmetic mean of a large number of readings the error in the final result should tend to average out.

It is desirable, after this method has been applied to a given set of readings, to be able to estimate the accidental error in the result and also, conversely, to be able to determine the number of readings necessary to give a result of specified accuracy. For this purpose, the *mathematical theory of least squares* has been developed.

The conclusions of this theory will be accepted here without proof. They are only valid when the measured values are distributed about the true value in a perfectly random fashion. One of them is that the most probable deviation, $D$, of the arithmetic mean, $M$, of $n$ measurements, from the true value is $1/\sqrt{n}$ times as great as the numerical average, $\bar{d}$, without regard to sign, of the deviations of the observed values from the mean. If we let $m_1, m_2, m_3, \ldots m_n$ be the observed values and $d_1, d_2, d_3, \ldots d_n$ the absolute values of their deviations from the mean, then this statement is represented by the formula

$$D = \bar{d}/\sqrt{n},$$

where

$$\bar{d} = (d_1 + d_2 + \cdots + d_k + \cdots + d_n)/n,$$

$$d_k = |m_k - M|, \qquad k = 1, 2, \cdots, n,$$

and

$$M = (m_1 + m_2 + \cdots + m_k + \cdots + m_n)/n.$$

The number $D$ is called the *deviation measure* of the mean.

The theory of least squares shows us that this deviation measure may be combined with the known residual errors, $r_1, r_2, \cdots r_n$, to give the precision measure, $\delta$, by means of the formula

$$\delta = \sqrt{D^2 + r_1^2 + r_2^2 + \cdots + r_n^2}.$$

This should not be confused with the " measure of precision," $h$, used in the theory of probability. (Art. 10.) The type of addition used in this expression we shall hereafter call *geometrical addition*. Another term often used in physics is *probable error*. This is a number so chosen that there are equal chances that the actual error exceeds or falls short of it. This number is $0.8453\bar{d}$ for each observation and $0.8453\bar{d}/\sqrt{n-1}$ for the mean.

The precision measure, as defined above, is the *absolute precision measure*. In addition to this quantity, the *fractional precision measure*, $\delta/M$, and the *percentage precision measure*, $100\,\delta/M$, are sometimes used.

Since the reliability of the result increases only with the square root of the number of observations, the labor involved grows much faster than the accuracy obtained. Hence the usual practice is to obtain the desired accuracy by taking more observations only when the additional number required is small, say five or ten, and, if higher accuracy is required, to improve the apparatus itself.

**8. Significant Figures.**

One of the most useful methods of keeping the accuracy of the work in mind during an experiment and of indicating the accuracy later is the proper use of significant figures. A *significant figure* is a digit which is believed to be nearer the actual value than any other. Zeros are significant if any other digits precede them in the number, otherwise not. Thus 0.00143 has three significant figures while 126700 has six and indicates that the value lies between 126699 and 126701. If it is desired to show that a number lies between 126600 and 126800, it may be written $1267_{00}$ or as a power of ten, thus: $1.267 \times 10^5$.

In many cases there is some uncertainty as to how many figures are significant, and the standard practice is to retain only one or two doubtful figures in the measurements. The advantage

of retaining two instead of one is that cumulative errors introduced will not affect the first doubtful figure. In the final average, a good rule is to retain those digits corresponding to the first two digits of the precision measure. This often means the rejection of superfluous figures, which simplifies the computation without loss of accuracy. If the value of the rejected figures is greater than half a unit in the last place retained, one should increase the last digit retained by one and if it is less than half one should leave this digit unchanged. If it equals half a unit, one should increase the digit by one only half the time, for example when the last retained digit is odd.

It is important to know whether any residual errors may be neglected without affecting the first significant figure of the precision measure. In the expression for the precision measure

$$\delta = \sqrt{D^2 + r_1^2 + r_2^2 + \cdots + r_n^2}$$

let

$$r_1^2 + r_2^2 + \cdots + r_i^2 = a^2 \quad \text{and} \quad D^2 + r_{i+1}^2 + \cdots + r_n^2 = b^2.$$

We wish to find the maximum value that the residuals represented by $a$ can have without affecting this figure, which, in the worst case, may be nine. Differentiating,

$$\frac{\partial \delta}{\partial a} = \frac{a}{\sqrt{a^2 + b^2}} = \frac{a}{\delta}, \text{ if } a \text{ is small.}$$

In this case, the change in $\delta$ is limited to 10 per cent, so that

$$\frac{\partial \delta}{\delta} = \frac{a \partial a}{\delta^2} = \frac{\partial (a^2)}{2\delta^2} \leqq 0.1.$$

If we drop $a^2$ entirely, $\partial(a^2) = a^2$ and this becomes

$$a^2 = 0.2\delta^2 \text{ or } a = 0.447\delta.$$

Hence a group of residual errors may be neglected only when

$$\sqrt{r_1^2 + r_2^2 + \cdots + r_i^2} \leqq 0.447\delta.$$

For convenience this limit is usually taken as $\delta/3$ and in good experimental work an attempt is always made to keep the residuals below this limit.

**9. Combination of Observations.**

In precise work, sets of observations are often repeated by different observers, with different apparatus or under different conditions, and it is desired to combine these sets to obtain a final value of the quantity measured. In such cases, the reliability of the different sets will not be the same, so some rule must be devised for their combination. Each set is given a weight inversely proportional to the square of its precision measure. Thus, letting $p_i = 1/\delta_i^2$, we have for the final weighted mean

$$M = \frac{p_1M_1 + p_2M_2 + \cdots + p_nM_n}{p_1 + p_2 + \cdots + p_n}.$$

**10. Rejection of Observations.**

When one observation departs widely from the others, it should be scrutinized closely. The data should be checked carefully for obvious blunders such as errors in copying or mistakes in arithmetic. If a reason for the mistake is found and cannot be corrected for, that reading should be rejected. If all work appears correct, the observer must use his own judgment or apply a test based on the theory of probability.

Assuming that the deviations of our measurements from the true value are perfectly random, it can be shown that, if $ydx$ is the probability that a deviation lies between $x$ and $x + dx$, then

$$y = \frac{h}{\sqrt{\pi}} e^{-h^2x^2},$$

where the true value is taken as the origin and $h$ is a constant related to the mean deviation by the relation

$$h = 1/d\sqrt{\pi}.$$

The relative probability of a deviation equal to the mean deviation and one $n$ times as great is therefore

$$e^{-1/\pi}/e^{-n^2/\pi} = e^{(n^2-1)/\pi}.$$

If we take $n = 4$, the value of the probability becomes 1/118, which is an almost negligible chance. Therefore, if the deviation of an observation from the mean is more than four times the average deviation, it may safely be discarded.

**11. Indirect Measures.**

Thus far, we have discussed only direct measures: that is, quantities determined by reading a single instrument. By far the greater part of electrical measurements, however, are indirect measures, obtained by combining the readings of several instruments using theoretical relations. Thus, we may measure directly the power consumed in a direct current circuit, or we may determine it indirectly as current times voltage, or as current squared times resistance, or as voltage squared divided by resistance.

The accuracy of such an indirect measure depends on the accuracy of the constituent measures and upon the way in which they enter the calculations. In general, the error in each component tends to increase the error in the result. Let $Q$ be the quantity obtained indirectly, $M_1M_2 \cdots M_n$ the components entering into it, $\delta_1, \delta_2, \cdots \delta_n$ the corresponding precision measures, and $\Delta$ the precision measure of $Q$. The dependence of $\Delta$ on the $\delta_k$'s is required. Let $\Delta_k$ be the change produced in $Q$ by a change, $\delta_k$, in the single component $M_k$. Then

$$Q = F(M_k) \quad \text{and} \quad Q + \Delta_k = F(M_k + \delta_k),$$

so that

$$\Delta_k = F(M_k + \delta_k) - F(M_k).$$

If $\delta_k$ is small, we may write this

$$\Delta_k = F'(M_k) \cdot \delta_k = \frac{\partial Q}{\partial M_k} \cdot \delta_k.$$

It can also be shown that

$$\Delta = \sqrt{\Delta_1^2 + \Delta_2^2 + \cdots + \Delta_n^2}.$$

It is instructive to apply these results to a few algebraic operations commonly used in obtaining indirect measures. First take the case

$$Q = M_1 \pm M_2 \pm \cdots \pm M_n.$$

Here $\partial Q/\partial M_k = \pm 1$; hence $\Delta_k = \pm\delta_k$ or $\Delta_k^2 = \delta_k^2$ and we have

$$\Delta = \sqrt{\delta_1^2 + \delta_2^2 + \cdots + \delta_n^2}.$$

This establishes

Rule I. *In processes of addition or subtraction, the absolute precision measure of the result is obtained by geometric addition of the absolute precision measures of the components.* It should be noted that in the case of subtraction the resultant quantity is smaller, but the precision measure is larger than for either component. If the subtracted quantities are nearly equal, this may completely destroy the accuracy of the result, so that this operation should be avoided when possible.

Consider the case

$$Q = M_1^a M_2^b \cdots M_k^q \cdots M_n^s,$$

where $a, b, \cdots z$ are any real numbers, positive or negative, integers or fractions.

$$\Delta_k = \frac{\partial Q}{\partial M_k}\delta_k = M_1^a M_2^b \cdots M_{k-1}^p q M_k^{q-1} M_{k+1}^r \cdots M_n^s \delta_k.$$

$$\Delta = \sqrt{a^2 M_1^{2(a-1)} M_2^{2b} \cdots M_n^{2s}\delta_1^2 + \cdots + q^2 M_1^{2a} \cdots M_{k-1}^{2p} M_k^{2(q-1)} M_{k+1}^{2r}\delta_k^2 + \cdots}.$$

$$\frac{\Delta}{Q} = \frac{\Delta}{M_1^a M_2^b \cdots M_n^s} = \sqrt{a^2\frac{\delta_1^2}{M_1^2} + b^2\frac{\delta_2^2}{M_2^2} + \cdots + z^2\frac{\delta_n^2}{M_n^2}}.$$

This establishes two rules:

Rule II. *In processes of multiplication and division, the fractional precision measure of the result is obtained by adding geometrically the fractional precision measure of the components.*

Rule III. *In raising to powers or extracting roots, the fractional precision measure of the result is obtained by multiplying the fractional precision measure of the component by its exponent.*

Finally in logarithmic computation, we have:

$$Q = \log M, \quad \Delta = \frac{\partial(\log M)}{\partial M}\delta = \frac{\delta}{M}.$$

This shows that the absolute precision measure of the logarithm of a quantity equals the fractional precision measure of the quantity. A glance at a logarithm table will show that a variation in the $n$th place of the mantissa corresponds very roughly to a variation of the $n$th significant figure of the number itself. This leads to:

Rule IV. *In all logarithmic computations, retain as many decimal places in the mantissa of the logarithm as there are significant figures in the quantity involved.*

As a numerical example of the methods developed above, we shall analyze the following data taken to determine the power consumption in two D.C. circuits connected in parallel. An ammeter with a full scale reading of 1.50 amperes, guaranteed to ¼ per cent of full scale deflection, was included in each circuit and a 150 volt voltmeter carrying the same guarantee was connected across the two circuits. We desire to compute the precision measure of the power value computed from these data:

| Potential (Volts) | | Current (Amperes) | | | |
|---|---|---|---|---|---|
| | $d_v$ | Circuit 1 | $d_1$ | Circuit 2 | $d_2$ |
| 110.3 | 0.12 | 1.32 | 0.010 | 0.51 | 0.004 |
| 110.8 | 0.38 | 1.36 | 0.030 | 0.51 | 0.004 |
| 110.1 | 0.32 | 1.30 | 0.030 | 0.48 | 0.026 |
| 110.0 | 0.42 | 1.30 | 0.030 | 0.50 | 0.006 |
| 110.9 | 0.48 | 1.37 | 0.040 | 0.53 | 0.014 |
| Mean 110.42 | 0.34 | 1.330 | 0.028 | 0.506 | 0.013 |

We shall use the notation of this chapter throughout and shall use the subscripts $v$, 1, 2, and $I$ to indicate voltage, circuit 1, circuit 2 and the combined current respectively.

Dividing the mean deviation measures by the square root of five, we obtain:

$$D_v = 0.15 \text{ volts} \quad D_1 = 0.012 \text{ amps.} \quad D_2 = 0.0058 \text{ amps.}$$

The residual errors are:

$$r_v = 0.0025 \times 150 = 0.38 \text{ volts}$$
$$r_1 = r_2 = 0.0025 \times 1.50 = 0.0038 \text{ amps.}$$

By the criteria of Art. 8, the residual error may be neglected for circuit 1, but not for circuit 2 or the voltage. Combining

geometrically:

$$\delta_v = 0.51 \text{ volts} \quad \delta_1 = 0.012 \text{ amps.} \quad \delta_2 = 0.0069 \text{ amps.}$$

$$\Delta_I = \sqrt{(0.012)^2 + (0.0069)^2} = 0.014 \text{ amps.}$$

$$\frac{\Delta_p}{P} = \sqrt{\left(\frac{0.51}{110}\right)^2 + \left(\frac{0.014}{1.8}\right)^2} = 0.0091.$$

The power may therefore be determined with a precision of 0.91 per cent. For this accuracy, a slide rule is sufficiently precise.

$$P = V \cdot I = 110.4 \times (1.330 + 0.506) = 202.6\ (\pm 1.8) \text{ watts.}$$

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# CHAPTER 2

## MEASUREMENT OF RESISTANCE

**12. Resistance Units and Standards.**

In the construction of an electrical and magnetic system of units it is necessary that, in addition to the fundamental units of length, mass and time, the dimensions of some fourth unit be arbitrarily chosen. This choice has been made in many ways in the past and from each selection a complete set of units has grown. There is a tendency at present to use for all practical purposes a so called MKSC system of units in which the basic quantities are the meter, the kilogram, the second and the coulomb. In such a system, which we shall use principally in this text, the unit of resistance is the *ohm*, and it is defined as *the resistance across which a steady potential of one volt is required to maintain a current of one ampere.**

For standardization purposes it is more convenient to have a unit of resistance defined in terms of a fixed standard, which is designed to duplicate the above definition. The present legal standard in the United States is the *International ohm*, established by the International Conference at London in 1908. Its definition is:

*The International ohm is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross sectional area and of length 106.300 cm.* Subsequent measurements have shown that this definition is not quite identical with the determination from Ohm's law, and it is probable that a new legal standard, of resistance equal to 0.99949 of the present ohm will be established in the near future. It was intended, in fact, that the new International ohm should come into use in 1941, but the impossibility of collaboration between the various national standardizing laboratories, due to the war, delayed its adoption.

* See Arts. 21 and 30 for definitions of the ampere and volt.

The primary standard defined above is, of course, not well adapted to laboratory practice. It is usually replaced by secondary standards such as those shown in Fig. 2. These are commonly built in 0.001, 0.01, 0.1, 1.0 and 10 ohm sizes and are compared with primary standards by the national standardizing laboratories. Even they are used only for quite accurate work,

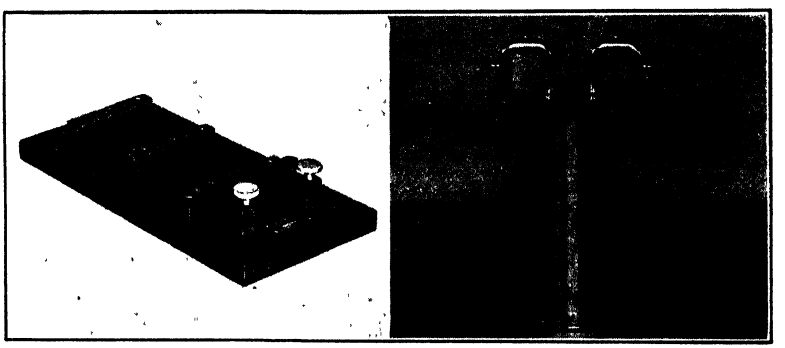

FIG. 2. Resistance standards. (Courtesy of Leeds and Northrup Co.)

and, for ordinary resistance measurements, portable and variable standards of resistance, built up of wire-wound coils and controlled by dials or plugs, are used.

**13. Elementary Methods of Measurement—The Wheatstone Bridge.** The three most common methods of measuring resistance are the volt-ammeter method, in which the resistance is computed from Ohm's law, the substitution method, in which the current through the unknown resistance and that through a standard resistance of the same order of magnitude, impelled by the same electromotive force, are compared, and the Wheatstone bridge.

The latter consists of a network of six conductors connected as shown in Fig. 3. The bridge is balanced when no current flows through the galvanometer. The potentials at $C$ and $D$ are then equal, so that

$$i_1R_1 = i_2R_3, \quad i_1R_2 = i_2R_4$$

and

$$R_1/R_2 = R_3/R_4. \tag{1}$$

Thus, if the value of one resistance and the ratio of the values of

two others are known, it is possible to determine the value of a fourth resistance by this method. Several devices for varying the ratio of the two resistances $R_3$ and $R_4$ are used. The conductor $ACB$ may be made of a single piece of uniform resistance wire with a sliding contact at $C$, so that the resistances of the two sections are proportional to their lengths. For more accurate work, various arrangements, by which resistance coils can be transferred from $R_3$ to $R_4$ by shifting plugs or turning dials, are made.

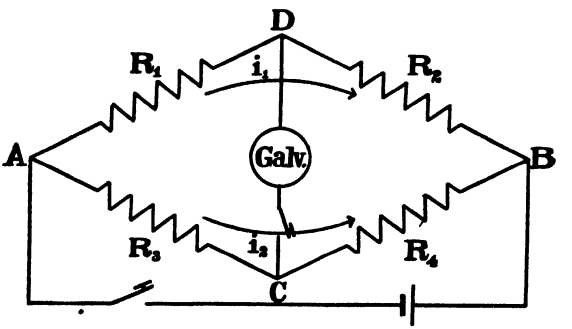


FIG. 3. The Wheatstone bridge network.

While Eq. (1) is the balance condition for any network of the type of that shown in Fig. 3, the sensitivity of the bridge to a slight unbalance, which determines the accuracy with which an unknown resistance can be determined, depends upon the relative values of the resistances of the four arms and that of the galvanometer, as well as upon the sensitivity * of the latter. If we let the resistance of the galvanometer be $R_g$, the potential difference across the bridge be $V$, the galvanometer current (reckoned positive from $D$ to $C$) be $i_g$, and the currents through $R_1$, $R_2$, $R_3$ and $R_4$ be $i_1$, $i_2$, $i_3$ and $i_4$, respectively (reckoned positive from left to right in Fig. 3), then the various currents may be determined from the equations:

$$
\begin{aligned}
i_1 &= i_2 + i_g, \\
i_3 &= i_4 - i_g, \\
R_1 i_1 + R_2 i_2 &= V, \\
R_3 i_3 + R_4 i_4 &= V, \quad \text{and} \\
R_1 i_1 + R_g i_g - R_3 i_3 &= 0.
\end{aligned}
$$

* See Chap. III for a discussion of this quantity.

Eliminating four of the currents from these equations gives:

$$i_g = \frac{(R_2R_3 - R_4R_1)V}{R_1R_2R_3+R_2R_3R_4+R_3R_4R_1+R_4R_1R_2+R_g(R_1+R_2)(R_3+R_4)}, \quad (2)$$

from which it is possible to calculate the current which will flow in the galvanometer for a slight unbalance of the bridge, say for $R_1 = (R_2R_3/R_4)(1 + k)$, where $k$ is a small fractional variation in $R_1$. Whenever possible, the galvanometer should be so chosen that it gives a deflection of about one division for the current which results from these equations when $k$ is taken as the maximum allowable fractional uncertainty in the measurement. If a less sensitive galvanometer is used the requisite accuracy will not be obtainable, and if a much more sensitive one is used time will be wasted in achieving the balance. In the same connection, it is important that the resistances of each of the four resistance arms and of the galvanometer be kept to the same order of magnitude. It will be obvious from Fig. 3, without formal proof, that a very low value of resistance in any arm or in the galvanometer circuit will divert a large portion of the current from the adjoining conductors, and will thereby decrease the effectiveness of those conductors in contributing to the unbalance. In the usual commercial bridge, if $R_1$ is the unknown, $R_2$ consists of a decade box of the dial or plug type and $R_3$ and $R_4$ are arranged to be varied simultaneously so that their ratio at any setting is a multiple or submultiple of ten. With such a bridge a number of variations are possible for measuring a given resistance and, when extreme accuracy is needed, it will often be found advisable to calculate the sensitivity from Eq. (2) for the different possible arrangements, taking into account the galvanometers which are available and the fact that $R_2$ can be varied only in finite steps.

Considerations such as those outlined in the last paragraph have limited the Wheatstone bridge to the measurement of medium values of resistance, say from 1 to 100,000 ohms. Below the lower limit extraneous influences, such as the resistance of contacts, become appreciable and affect the accuracy considerably, while above the upper limit it becomes difficult to obtain galvanometers of high enough resistance to act as sensitive de-

tectors. Recently, however, the vacuum tube voltmeter * has begun to come into use to replace the galvanometer for high resistance bridge measurements, since this instrument has a resistance of $10^8$ ohms or more. For low resistances, below 1 ohm, the volt-ammeter method may be advantageously employed, and with a good millivoltmeter or a potentiometer resistances of 0.001 or even 0.0001 ohm may be read with considerable accuracy. Attempts to extend the method further, however, usually necessitate heavy currents with disturbing heating effects.

Since the two methods mentioned above cover the lower range, the substitution method is usually limited to high values of resistances. The currents drawn in this range do not seriously affect the potential difference of the source, and may be read with a galvanometer. The upper limit of this method may be extended to about $10^{10}$ ohms if good standards are available in this region, or if a good Ayrton shunt is used to cut down the galvanometer deflection when the unknown resistance is replaced by a considerably lower resistance standard. Commercial apparatus for facilitating substitution measurements, such as the Leeds and Northrup No. 5602 guarded insulation test set, will be found to be convenient when an appreciable amount of high resistance testing is necessary.

**14. The Kelvin Double Bridge.**

The lower limit of the Wheatstone bridge, as was noted above, is determined by the fact that the contact resistances between conductors become appreciable in comparison with the resistances of the conductors themselves. The Kelvin double bridge, which replaces the six conductor network by one of nine conductors, is designed to overcome this difficulty, and is undoubtedly the most elegant method yet devised for measuring low resistances.

The bridge network is shown in Fig. 4. The elimination of contact resistances is brought about by the fact that the principal current path is from the battery, $B$, through the unknown resistance, $X$, the connector, $A$, and the standard resistance, $S$, while the balancing is accomplished in the comparatively high resistance potential network formed by the other five conductors. The current flowing in these conductors is small, and contact

* See Chap. VII.

resistances have very little effect. Let the value of the unknown resistance be $X$, of the standard, $S$, and of the other conductors, $a$, $b$, $c$, and $d$ as shown. Let the currents flowing be $I_1$, $I_2$ and $I$ as marked. The balance is obtained in two steps. First, the

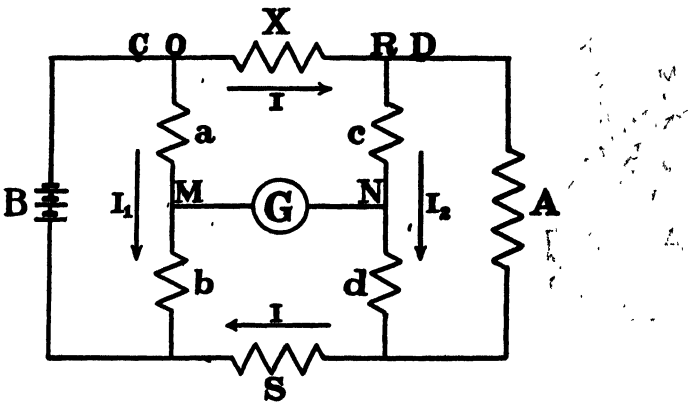


FIG. 4. The Kelvin double bridge network.

values $a$, $b$, $c$ and $d$ are adjusted to give an approximate balance, then $S$ is varied until the galvanometer shows no deflection. The bridge is so constructed that any changes in the potential circuit leave unchanged the proportion:

$$\frac{a}{b} = \frac{c}{d}. \tag{1}$$

The balance condition implies that the points $M$ and $N$ are at the same potential, hence:

$$I_1 a = IX + I_2 c \tag{2}$$

and

$$I_1 b = IS + I_2 d. \tag{3}$$

Solving for $IX$ and $IS$:

$$IX = a\left(I_1 - \frac{c}{a} I_2\right),$$

$$IS = b\left(I_1 - \frac{d}{b} I_2\right).$$

But, from Eq. (1), the factors in parentheses are identical, therefore:

$$\frac{X}{S} = \frac{a}{b} = \frac{c}{d}. \tag{4}$$

A self contained Kelvin double bridge,* manufactured by the Leeds and Northrup Company, is shown in Fig. 5 and its wiring

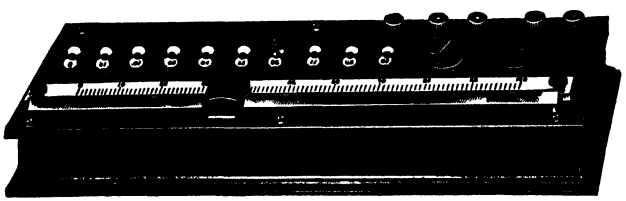

FIG. 5. The Kelvin double bridge. (Courtesy of Leeds and Northrup Co.)

diagram set up for laboratory use is shown in Fig. 6, which is lettered to correspond with Fig. 4. The sample to be measured, usually in the form of a rod, is fastened in the clamps *C* and *D* and contact is made to the network through potential contacts *O* and *R*. The standard resistance is divided into ten parts, nine 0.01 ohm coils and a sliding contact resistance of 0.01 ohm. A scale, divided into 100 units, is placed along this resistance, so that by reading to one-tenth of these units, the standard may be adjusted to 0.00001 ohm. The two ratio dials must always be set alike in order that Eq. (1) may be satisfied. As will be seen from the values of the resistance steps on these dials, the ratios *a*/*b* and *c*/*d* may assume the values 0.1, 0.2, 0.5, 1.0, 2.0, 5.0, and 10.0, so that any unknown resistance between .01 and 1.0 ohms can be measured with about the same accuracy.

For samples not in the usual form, an auxiliary pair of potential contacts, *P*–*P*, are provided. The maximum current capacity of the bridge is 8 amperes, and 3 to 5 amperes will be found, in general, satisfactory. For an indicating device, a galvanometer of 100 to 600 megohms sensitivity † should be used.

* One modification of the Kelvin double bridge, the Hoopes conductivity bridge, is designed to measure with greater rapidity and accuracy very low resistances.

† See Chap. III for explanation.

The ease of working is greatly increased by using the more sensitive galvanometers with an Ayrton shunt * to cut down the deflections during the rough adjustment.

## EXPERIMENT 1

### MEASUREMENT OF SPECIFIC RESISTANCE

Set up the apparatus as shown in Fig. 6. Three or four samples of copper, aluminum, steel and brass will be measured. Determine the diameter of the specimen with micrometer calipers

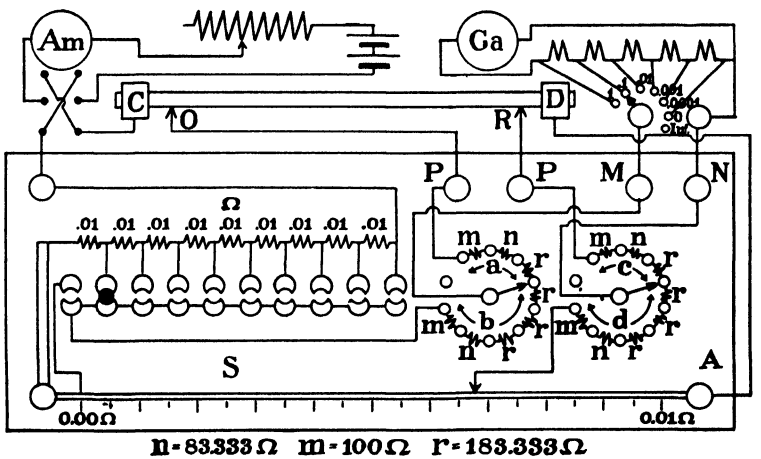


FIG. 6. Kelvin double bridge with battery, rheostat and Ayrton shunt.

in at least five places along the rod, taking two determinations, at right angles to each other, at each place. Just before the current and potential clamps are attached, *lightly* polish the rod with fine sandpaper or emery cloth at the points at which the potential contacts are to be made. After clamping, measure the length of the sample between the knife edges of the potential contacts, *O* and *R*, with a good scale. Make two determinations on each rod, one for a nine inch length and one for an eighteen inch length.

* See Chap. III for explanation.

First determine the approximate resistance by setting the ratio dials for a 1–1 ratio and balancing, keeping the Ayrton shunt on the 0.01 setting. If a balance is obtained find the ratio of this value of $X$ to the maximum possible value of $S$ (0.1 ohm) and set the ratio dials at their nearest smaller value for the final determination of $X$. If no balance can be obtained the resistance is above 0.1 ohm and the higher ratios must be used. Having decided upon the proper ratio, adjust the standard resistance as closely as possible, then set the shunt on the 1.0 point and obtain a final balance. Check this balance by keeping the galvanometer key closed and opening the current circuit. There may be a momentary kick of the galvanometer, but it should return to the same rest position. This test will locate any contact or thermal potentials in the galvanometer circuit. Now reverse the current by means of the switch $T$ and again determine the balance. Any accidental E.M.F. in the network itself will thus be eliminated, as it will aid the current in one case and oppose it in the other.

The resistance of a conductor is given by the formula:

$$R = \frac{\rho l}{A}$$

where $R$ is the resistance in ohms, $\rho$ the *resistivity* in ohm-centimeters, $l$ the length in centimeters, and $A$ the cross-sectional area in square centimeters. Therefore:

$$\rho = \frac{AR}{l}.$$

Reduce all your readings for a single sample to one centimeter length, average these and compute the resistivity.* Compare your result with that given by standard tables.

**15. The Carey-Foster Method.**

Another means which was devised for the measurement of low resistances by the elimination of contact resistances is the Carey-

* It may be advantageous to have the student do a smaller number of samples than suggested, and devote the balance of his time to computing the precision of his results. The residual error of the bridge may be taken as 1/10 per cent of the full range.

Foster method. It is equally well adapted, however, to medium values of resistance, and finds its principal application in the comparison of two resistances of nearly the same value, such as two laboratory standards which are to be interchecked. By interchanging the standard resistance and the unknown during the measurement, this method provides a simple way of eliminating all influences of extraneous resistances and E.M.F.'s. The network used is shown in Fig. 7. The large letters indicate the known resistances of the arms and the small $r$'s are the contact resistances in these arms. The portion $M$–$N$ is a slide wire of length $l$ and of resistance $\rho$ per unit of length. Consider the

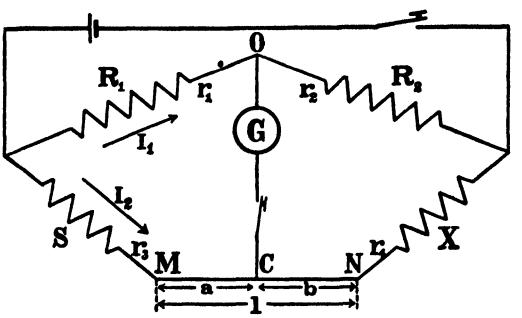


FIG. 7. Network for the Carey-Foster method.

bridge to be balanced (i.e. no current flowing through the galvanometer) with the contact $C$ at a distance $a_1$ from $M$ and $b_1$ from $N$. Then, since $O$ and $C$ are at the same potential:

$$(R_1 + r_1)I_1 = (S + r_3 + a_1\rho)I_2$$

and

$$(R_2 + r_2)I_1 = (X + r_4 + b_1\rho)I_2.$$

Dividing:

$$\frac{R_2 + r_2}{R_1 + r_1} = \frac{X + r_4 + b_1\rho}{S + r_3 + a_1\rho}. \qquad (1)$$

If the positions of $S$ and $X$ are interchanged, a new balance will be obtained, with the contact at distances $a_2$ and $b_2$ from the ends of the wire,—then:

$$\frac{R_2 + r_2}{R_1 + r_1} = \frac{S + r_4 + b_2\rho}{X + r_3 + a_2\rho}. \qquad (2)$$

Combining Eqs. (1) and (2) and adding unity to both sides of the new equation:

$$\frac{X + r_4 + S + r_3 + (a_1 + b_1)\rho}{S + r_3 + a_1\rho} = \frac{S + r_4 + X + r_3 + (a_2 + b_2)\rho}{X + r_3 + a_2\rho}.$$

But

$$a_1 + b_1 = l = a_2 + b_2;$$

therefore, the numerators of the two fractions are identically equal and the denominators must also be equal:

$$S + a_1\rho = X + a_2\rho,$$

$$X = S + (a_1 - a_2)\rho. \tag{3}$$

Since only the difference between the two resistances is measured, the fractional error in the total resistance is very small. This is particularly true when $a_1 - a_2$ is a large quantity so that it may be determined with considerable accuracy. This may be accomplished, even though the difference of the resistances is exceedingly small, by shunting the slide wire with a wire of known resistance. Let this resistance be $r$; then the total resistance of the slide wire is:

$$R = \frac{r\rho l}{r + \rho l}$$

and the equation of the bridge becomes:

$$X = S + (a_1 - a_2)\frac{r\rho}{r + \rho l}. \tag{4}$$

The quantity $a_1 - a_2$ may be made as large as desired by taking $r$ sufficiently small. Commercial bridges are usually made with two or three shunts for this purpose.

The second factor in the difference, the resistivity, must also be known or determined with a considerable degree of accuracy. One method of determining this is to balance two known resistances against each other exactly as above, and to compute $\rho$. A much more elegant method, however, is that which follows. The bridge is set up as shown in Fig. 8, where $S$ is an accurately known standard, $S_1$ is any resistance, not necessarily accurately

known, but near enough to the value of $S$ to give a balance near the center of the slide wire, and $C$ is a third resistance, fairly accurately known, of about 100 times the value of $S$, and arranged

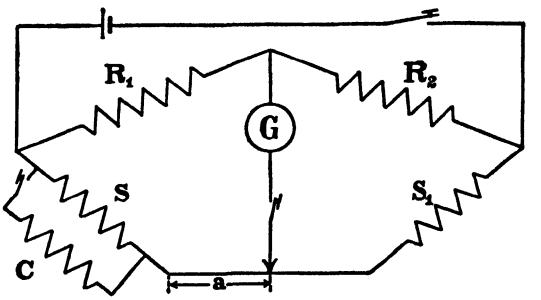


FIG. 8. Calibration of slide wire.

to be shunted across the latter. If the resistance of $S_1$ is measured in the ordinary way, direct and interchanged:

$$S_1 = S + (a_1 - a_2)\rho. \tag{5}$$

If $C$ is now shunted across $S$, the new standard resistance becomes:

$$S' = \frac{CS}{C + S}.$$

And, if a new balance is obtained:

$$S_1 = \frac{CS}{C + S} + (a_1' - a_2')\rho. \tag{6}$$

Combining Eqs. (5) and (6):

$$\rho = \frac{S^2}{(C + S)(a_1' + a_2 - a_2' - a_1)}. \tag{7}$$

**16. Calibration of the Slide Wire.**

The method of determining the resistance, as described above, gives, of course, only the average resistance per unit length over the portion considered. For extremely accurate work, not this average, but the true resistance between the points $a_1$ and $a_2$ is needed. The following method for calibrating the slide wire

along its length, or testing its uniformity, is essentially the same process as would be used in calibrating a scale. In such a case, the obvious procedure would be to apply the same standard of length to successive parts of the scale and to see if the scale indicates the same length at all points.

Consider the set up of Fig. 8, except that the fixed resistance $R_1$ is replaced by a variable resistance and $C$ is permanently shunted across $S$. We shall indicate the combination, $S$ and $C$, by $S'$. The values of $R_1$ and $R_2$, or even their ratio, do not affect

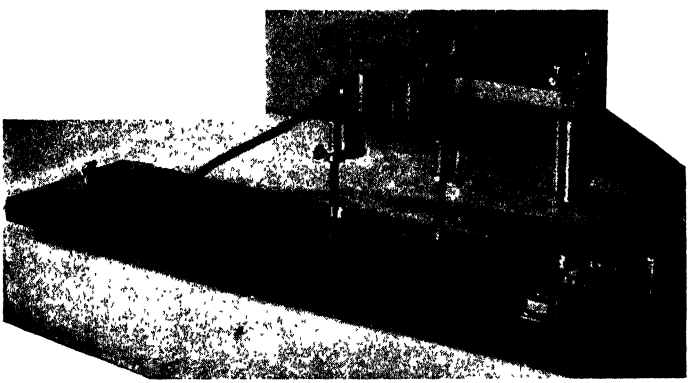

FIG. 9. Carey-Foster coil interchanger and slide wire. (Courtesy of Leeds and Northrup Co.)

the quantity $a_1 - a_2$, as Eq. (3) shows, but do affect the individual quantities $a_1$ and $a_2$, as shown by Eqs. (1) and (2). Set the slider near the right end of the wire, say at $a_0$ cm. from the left end, and bring the bridge to a balance by adjusting $R_1$. When this has been accomplished, interchange $S'$ and $S_1$ and balance again by moving the slider to some new position $a_1$ cm. from the left. Then:

$$S_1 - S' = (a_0 - a_1)\rho_1 \quad \text{by Eq. (3), Art. 15.}$$

Now restore the resistances to their original position, leaving the slider in place, adjust $R_1$ to some new value, $R_1'$, for a new balance, interchange and balance by means of the slider. This will give:

$$S_1 - S' = (a_1 - a_2)\rho_2.$$

Repeat this procedure until the slider has been moved all the way across the slide wire. Now, since $S_1 - S'$ has remained constant in value:

$$(a_0 - a_1)\rho_1 = (a_1 - a_2)\rho_2 = (a_2 - a_3)\rho_3 = \cdots.$$

If $\rho$ were constant for all parts of the wire, then we should have:

$$(a_0 - a_1) = (a_1 - a_2) = (a_2 - a_3) = \cdots.$$

In general, however, this condition will not be satisfied. It is possible to correct for the variations in resistance either by using the corrected value obtained by solving for $\rho_1/\rho_2$, etc. in the above equation or by using for the calculation a corrected length which just compensates for the variations in $\rho$. Actually, this procedure is the one to be preferred.

Denote the quantities $a_0 - a_1$, etc. by $l_1$, $l_2$, $l_3$, $\cdots$. Since they should all be equal if $\rho$ were constant, their variation from the average is a measure of the variation of $\rho$ from the average. Let their mean value be $l_m$. Take the point $a_0$ as a base point. Then the second reading should have been $a_0 - l_m$ instead of $a_1$. Consequently, a correction:

$$c_1 = l_1 - l_m \tag{1}$$

should be applied to $a_1$. Similarly, the correction for the second point is:

$$c_2 = l_1 + l_2 - 2l_m, \text{ etc.} \tag{2}$$

Since the correction is only for relative variations, the total distance is taken as correct and the last correction, as well as the first, is zero.

While the Carey-Foster method is not used as frequently now as it was in the past it is typical of a number of methods in which precision is obtained by interchanging a standard and unknown resistance. This method of calibration, moreover, is not limited to the Carey-Foster bridge or to slide wires. It may be applied to the ordinary Wheatstone bridge, for example, by mounting the slide wire between two of the arms and then arranging to change periodically the resistance of one arm by placing a fixed resistor in shunt with it. Similarly, any continuously variable standard,

such as a movable plate condenser, which can be measured in a bridge circuit can be calibrated over its range by essentially the same method.

## EXPERIMENT 2

### CALIBRATION OF A CAREY-FOSTER SLIDE WIRE

Set up the apparatus as shown in Fig. 8. Dial boxes or plug boxes may be used for $R_1$ and $R_2$. For $S$ use a very accurately known laboratory standard. Also use laboratory standards for $S_1$ and $C$, although these do not need to be so accurately known. The nominal value of $S_1$ should be the same as that of $S$, that of $C$ about one hundred times as great.

First calibrate the wire by the method described in Art. 16, taking the initial point at the extreme right hand end of the wire. Plot a curve of correction * in cm. against the scale reading in cm.

Having tested the uniformity, determine the average value of the resistance per unit length by the method of Art. 15. In Eq. (7), use the corrected values for the $a$'s obtained by adding the corrections given by your curve.

## EXPERIMENT 3

### STANDARDIZATION OF LABORATORY STANDARD RESISTANCES

Set up the apparatus as in Fig. 7. A series of certified standard resistances should be available and a number of laboratory standards of the same nominal values are to be checked against these. For each determination keep the values of the resistances in the four arms approximately equal, as this results in maximum sensitivity. Use the corrected lengths, as found in Exp. 2, in the calculations. Calculate the *per cent error* of each of the resistances. Also calculate the *correction*, in ohms, to be applied to each.

* By common convention, a *correction* is defined as the correct reading minus the observed, and is, therefore, always added to the observed reading. An *error* has the same magnitude, but the opposite sign.

**17. Measurement by the Voltmeter Method.**

When a constant source of potential is available, moderately high resistances may be measured with a voltmeter alone. This is really a variation of the substitution method, the voltmeter acting in the double capacity of galvanometer and standard resistance. Consider the circuit of Fig. 10. The voltmeter is first connected directly across the line, reading $V_0$, then in series with the resistance, reading $V_1$. If the line voltage is constant, the currents in the two cases are:

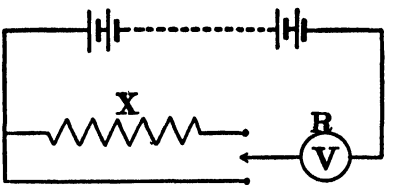


FIG. 10. Resistance measurement by the voltmeter method.

$$I_0 = \frac{V_0}{R} \quad \text{and} \quad I_1 = \frac{V_0}{X+R} = \frac{V_1}{R};$$

therefore:

$$X = \frac{V_0 - V_1}{V_1} \cdot R.$$

This method may be applied with considerable accuracy to resistances varying from one-tenth to ten times the voltmeter resistance. This resistance is usually specified by the manufacturer and, for a good 150 volt meter, is about 15,000 ohms. The method may therefore be used to advantage for the measurement of such resistances as the lower resistance radio grid leaks. If the voltmeter resistance is not given, it may be obtained by the use of a good standard resistance of the same magnitude, by the method given above, solving for $R$.

**18. High Resistance Leaks.**

For certain purposes, particularly for the measurement of very minute currents,* it is desirable to have high resistances of known and constant value. One convenient form of such resistance is an ink line ruled on paper. A long uniform line is ruled with ordinary India ink. The ends of the strip are then pressed with copper clamps to which lead-in wires have been

* See Chap. IV.

soldered. These extend two or three centimeters from the end to insure good contact with the ink. The whole is then boiled in paraffin for a few minutes to drive off any water in the paper and then put into a glass tube, either straight or U shaped. This tube is filled with melted paraffin beyond the ends of the paper and the ends are closed with sealing wax. Such resistances have remained almost constant for several years. It requires experience to make such a resistance of a specified magnitude, but the maker may be guided somewhat by the fact that one made with an ink line 0.5 mm. wide and 45 mm. long between clamps gave a resistance of $5(10)^7$ ohms.

Two common types of commercial high resistance elements are available for this range. The first is formed by the deposition of a very thin metallic coating on a good insulating material. The entire resistor is then coated with a protective layer of insulator to protect it against mechanical abrasion or atmospheric disturbances. *Metallized resistors* of this type are manufactured by the International Resistor Company in values up to $5 \times 10^7$ ohms. For still higher values quite satisfactory stable units are supplied by the S. S. White Dental Manufacturing Co. in the form of *molded resistors*, composed of a vulcanized mixture of conducting material and binder. These are manufactured in the range from $10^3$ ohms to $10^{12}$ ohms. When used on direct currents they may show polarization effects, but they are among the most satisfactory forms of resistances for high ranges.

**19. Resistance Measurement by Leakage.**

For such resistances as those described in the last article, the most satisfactory method of measurement is by the rate of leakage from a condenser. Consider a condenser of capacitance * $C$, shunted by a resistance of value $R$. The condenser, having been charged, acts as a source of electricity, and the current which flows in the circuit at any instant will be:

$$i = \frac{V}{R} = \frac{Q}{CR}, \tag{1}$$

since $V = Q/C$ where $Q$ is the charge on, and $V$ the potential

* See Chap. V.

difference across, the condenser. But, by definition:

$$i = -\frac{dQ}{dt}.$$

Note that the negative sign is taken, since the current is essentially positive while $Q$ is decreasing. Substituting:

$$-\frac{dQ}{dt} = \frac{Q}{CR}, \tag{2}$$

$$-\frac{dQ}{Q} = \frac{dt}{CR}.$$

If we integrate from the time zero to the time $t$, and the charge, initially $Q_0$, has been reduced to $Q_1$ at this time:

$$\log_e \frac{Q_0}{Q_1} = \frac{t}{CR},$$

$$R = \frac{t}{C \log_e \frac{Q_0}{Q_1}} \tag{3}$$

where $R$ is the resistance in ohms, $t$ the time in seconds, and $C$ the capacitance in farads.

## EXPERIMENT 4

## MEASUREMENT OF A HIGH RESISTANCE BY LEAKAGE

Set up the apparatus as shown in Fig. 11. $C$ is a condenser of about one microfarad capacitance, $R$ is the resistance to be measured and $G$ is a ballistic galvanometer, the deflection of which, when a condenser is discharged through it, is proportional to the charge on the condenser.*

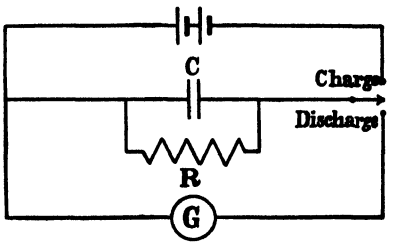


FIG. 11. Resistance measurement by leakage with a ballistic galvanometer.

* See Chap. V.

The charge and discharge key should be a rapid action key actuated by a spring, so that the time lag between pressing the key and discharge is negligible.

If the resistance is roughly known, calculate from Eq. (3) the approximate time required for the charge to leak to 0.9, 0.8, 0.7, 0.6, and 0.5 times its initial value. If it is not known, a preliminary run may be made with a sixty second lag.

Charge the condenser, then immediately discharge it through the galvanometer, noting the deflection. As this reading is the most important single reading, repeat four or five times. Now charge again and time with a stop watch from the instant that the condenser is insulated. At the end of your first calculated time, discharge and again read the deflection. Charge and repeat for the second time, etc. Changing Eq. (3) to logarithms to the base 10, and replacing the $Q$'s by the proportional deflections:

$$R = \frac{t}{2.303C \log \frac{d_0}{d_t}}$$

or

$$\log d_t = \log d_0 - \frac{1}{2.303CR} t. \qquad (4)$$

Plot the logarithms of the successive deflections against the time elapsing between insulation and discharge. Plot an average straight line through the points, passing it exactly through the initial point, which was most accurately determined. Calculate the slope of this curve and equate it to $-\frac{1}{2.303CR}$ and solve for $R$.

This would be the resistance sought if the resistance across the condenser itself were infinite. To measure this resistance, disconnect $R$ and repeat the experiment, using as the longest time the same time as was used to discharge the condenser to half value. In this way determine the condenser resistance, $R_c$. Then the true resistance, $R_1$, is given by:

$$R_1 = \frac{R_c R}{R_c - R}.$$

The same measurement may be made using an electrometer as described in Chapter IV.

**20. Other Methods.**

A number of commercial instruments for measuring high resistances with extreme rapidity are on the market. Among these may be mentioned the Ohmmeter and the Megger, both of which read ohms directly on their scales. As these are merely special applications of standard methods, the reader is referred to the literature of the manufacturing companies for their construction and use.

In this chapter we have discussed only those methods of measuring resistance which can be carried out with the help of only resistance standards and galvanometers. The various potential measuring devices,—the potentiometer, the electrometer and the vacuum tube amplifier,—furnish tools for the measurement of resistance and will be considered in this connection in later chapters.

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# CHAPTER 3

## MEASUREMENT OF CURRENT

**21. Units of Current Measurement.**

Because of the nature of the electric current the only useful definition of a standard of this quantity must be an operational one. The present legal standard, the *International ampere*, is based on the electrolytic properties of a current * and is defined as *the current which, when passed through a solution of nitrate of silver in water, in accordance with certain definite specifications, deposits silver at the rate of 0.00111800 grams per second.* This definition was intended to coincide with that of the *absolute ampere*, which is one tenth of the *absolute electromagnetic unit.* This quantity, in turn, is defined as *the current which, when flowing in a conductor of one centimeter length, bent in an arc of a circle of one centimeter radius, produces a magnetic field of one oersted* † *at the center.* It has been found that there is a small discrepancy between the two definitions, and it is probable that a new legal standard will be adopted at the same time that the International ohm is changed. According to the best present measurements, the new International ampere will be 1.00005 times the old.

**22. The Tangent Galvanometer.**

The earliest electro-magnetic instrument for the measurement of current, and, at the same time, an instrument which is based directly on the definition of the absolute ampere, is the tangent galvanometer. This consists of a circular coil of wire of $n$ turns and of radius $r$ placed at right angles to the horizontal component of the earth's magnetic field. At the center is a magnetic needle of dimensions small compared with those of the coil. If a current $I$ flows in the coil, since the magnetic field is inversely proportional to the radius and directly proportional to the length of the con-

* See Chap. XIII.

† See Art. 56 for a definition of the oersted.

ductor, the field strength (amp. turns/meter) at the center is:

$$H' = \frac{nI}{2r}.$$

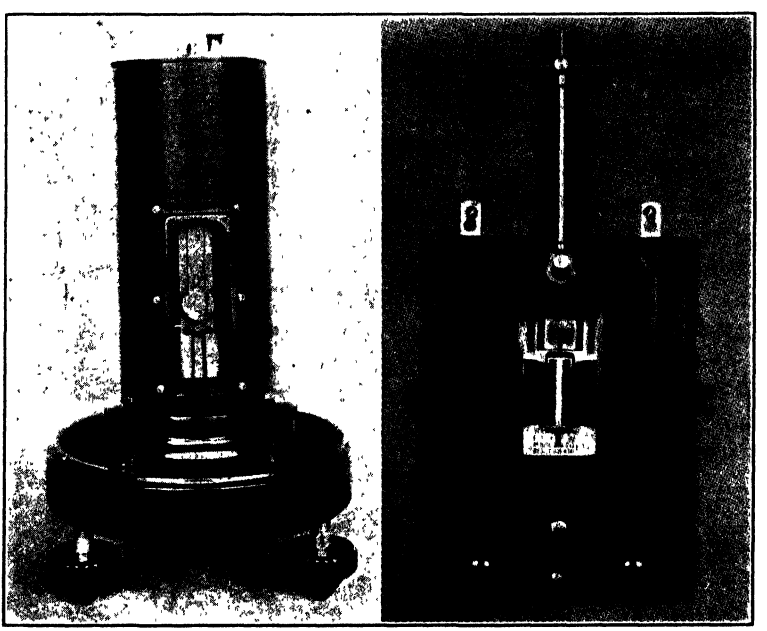
If the strength of the earth's field at the point is $H$, then the angle which the resultant field makes with that of the earth is defined by:

$$\tan \theta = \frac{H'}{H} = \frac{nI}{2Hr},$$

and, since the needle will take the direction of the resultant field:

$$I = \frac{2Hr \tan \theta}{n}$$

where $I$ is the current in amperes and $\theta$ is the deviation of the needle from its position of rest.



a b

FIG. 12. Types of the D'Arsonval galvanometer. (Courtesy of Leeds and Northrup Co.)

**23. The D'Arsonval Galvanometer.**

The tangent galvanometer has three disadvantages: first, its use requires a precise determination of the horizontal component of the earth's field at the point of measurement; second, the current is proportional to the tangent of the angle of deflection rather than the angle itself; third, the instrument is very sensitive to external disturbances; all of these are overcome by the D'Arsonval galvanometer. In this instrument, illustrated in Fig. 12 and shown schematically in Fig. 13, the current-carrying coil is the rotating part, and the permanent magnet is fixed. A rotating coil of fine wire, $C$, is hung by a thin conducting suspension which also serves as a lead for the current. The other lead usually consists of a very loose spiral of fine wire attached to the bottom of the coil, and not stiff enough to produce appreciable torque upon it. The coil rotates about a cylindrical core of soft iron, $A$, in the narrow air gap between this core and the poles, $N$–$S$, of a strong permanent magnet. In this way, the field is kept nearly radial to the axis of rotation over a large range.

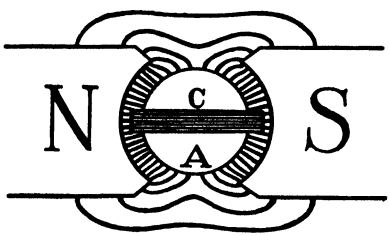



FIG. 13. Arrangement of theD'Arsonval galvanometer.

Suppose that the flux density in the air gap is $B$ webers/meter$^2$, that the vertical sides of coil are $l$ meters long and the horizontal sides $w$ meters, and that it has $N$ turns and carries a current of $I$ amperes. A negligible force acts on the top and bottom wires of the coil, since they are outside the air gap, but the force on one vertical wire, in newtons, is

$$F = BIl. \tag{1}$$

Since the field is radial, this force is always tangential, and the torque, in newton meters, produced by it is

$$T' = BIlw/2.$$

The total torque produced by both sides of all turns is

$$T = 2NT' = BIlwN. \tag{2}$$

If, for convenience, we let $lwN = A$, i.e. the number of turns times the area of the coil, then

$$T = BIA. \tag{3}$$

This torque causes a rotation of the coil which meets a resisting torque in the suspension. By Hooke's law this torque, for small deflections, is

$$T_r = k_1\theta$$

where $k_1$ is a constant and $\theta$ is the angle through which the coil has rotated. When the coil has come to rest, the two torques will just balance, therefore

$$k_1\theta = BIA \tag{3A}$$

or

$$I = \frac{k_1\theta}{BA} = K\theta, \text{ where } K = k_1/BA. \tag{4}$$

It will be noticed that this is a linear law of deflection and that the constant $K$ depends only on the characteristics of the galvanometer itself.

**24. Figure of Merit, Sensitivity.**

The constant $K$ is a measure of the sensitivity of the instrument, but before we can approach the more convenient and customary expressions for the sensitivity, we must consider the galvanometer as actually used in the laboratory. Since the entire moving system must be kept very light, and carefully balanced, a beam of light is used as a pointer. This beam falls on a small mirror which is rigidly mounted on the suspension just above the coil and, after reflection, is focused, either by a lens or by having the mirror itself concave, on a scale at some distance from the instrument, or on the cross hairs of a telescope eyepiece. Such an arrangement, with a lamp as the

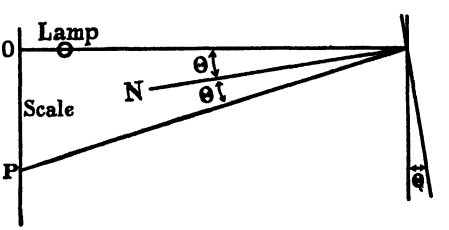



FIG. 14. Optical system for galvanometer.

source of the light and a scale for reading the position of the reflected beam, is shown in Fig. 14. When the mirror is undeflected, the beam is reflected normally to meet the scale at $O$. The illustration shows the mirror deflected through an angle $\theta$. The normal is then at $N$ and the beam meets it at the angle $\theta$. Since the angle of incidence is equal to the angle of reflection, the latter angle is also $\theta$ and the total deviation of the beam is $2\theta$ or twice the deflection of the mirror. Now, if the scale is curved in an arc of a circle whose center lies on the axis of rotation of the coil, or if the distance $OP$ is sufficiently small, so that $\tan 2\theta = 2\theta$ to the desired degree of accuracy:

$$OP = 2D\theta \tag{1}$$

where $D$ is the distance from the mirror to the scale and $OP$ is the *deflection*.

The most common measure of sensitivity is the *figure of merit* which is defined as the current, in amperes, required to produce a deflection of one millimeter with the scale at a distance of one meter. Combining Eq. (1) with Eq. (4) of Art. 23, the figure of merit is:

$$K_1 = K\frac{d}{2D}$$

where $d$ is the deflection in mm., and by definition:

$$K_1 = \frac{1}{2000}K. \tag{2}$$

The smaller the figure of merit, the greater is the sensitivity of the galvanometer. Other ways of expressing the sensitivity are:

1. The *microvolt sensitivity* is defined as the deflection in millimeters at one meter when a potential difference of one millionth of a volt is maintained across the terminals of the galvanometer. If the resistance of the galvanometer is $R$ ohms, then the microvolt sensitivity, by Ohm's law is:

$$K_2 = \frac{2000 \times 10^{-6}}{KR} = \frac{2}{KR}(10)^{-3}. \tag{3}$$

2. The *megohm sensitivity* is the resistance necessary in the galvanometer circuit to reduce the deflection to one millimeter at one meter when a potential difference of one volt is impressed on the circuit. Since the resistance of the galvanometer is always negligible compared with this figure, the megohm sensitivity is:

$$M = \frac{2000}{K} = \frac{1}{K_1}. \tag{4}$$

## EXPERIMENT 5

### DETERMINATION OF GALVANOMETER SENSITIVITY

Since the currents measured by a galvanometer are quite small, it will be necessary to use an arrangement such as is shown in Fig. 15. Then, by Kirchoff's laws:

$$(I - i)r_1 + Ir_2 = V$$

where $V$ is the voltage read by the voltmeter and

$$(I - i)r_1 = iR.$$

Eliminating $I$ between these equations:

$$i = \frac{Vr_1}{(R + r_1)(r_1 + r_2) - r_1^2}. \tag{5}$$

Set the zero reading of the meter near the center of the scale and determine the values of $r_1$ and $r_2$ necessary to deflect the image

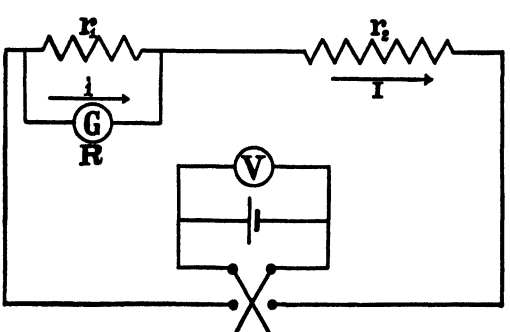



FIG. 15. Sensitivity of a galvanometer.

about 15–20 centimeters. Then, leaving $r_2$ unchanged, use values of $r_1$ equal to 10 per cent, 20 per cent, etc. of this value. For each setting of $r_1$, close the reversing switch and read the position of the image when it has come to rest, then reverse the switch and again read the position. This eliminates errors in the zero point. Call the distance between these two positions $d_1$. Then, by definition: $i = K_1 d$ at a distance of one meter, and with a distance of $D$ millimeters between the mirror and scale, and with the double deflections used:

$$d = d_1 \times \frac{1000}{D} \times \frac{1}{2}$$

so that:

$$K_1 = \frac{Di}{500 d_1}. \tag{6}$$

Compute the current through the galvanometer for each value of resistance and plot the values of $i$ against $d_1$ using an average straight line through the ten points obtained. Determine the average value of $i/d_1$ from this curve and compute the figure of merit by Eq. (6). Also calculate the microvolt and megohm sensitivities from Eqs. (3) and (4).

In some cases, particularly when high sensitivity galvanometers are being calibrated, the circuit of Fig. 15 may result in heavy overdamping. With the same instruments the method as outlined also becomes difficult because the extreme sensitivity necessitates either that $r_1$ be very small or that $r_2$ be inconveniently large. Both of these difficulties can be removed, if they arise, by inserting a resistance $r$, of the order of the critical damping resistance of the galvanometer,* in series with the instrument, so that the series resistances $R$ and $r$ are in parallel with $r_1$. In this case Eq. (5) is replaced by:

$$i = \frac{V r_1}{(R + r + r_1)(r_1 + r_2) - r_1^2} \tag{7}$$

and with this change in the calculations the experiment may be carried out as above.

* See Art. 25.

**25. Critical Damping.**

The moving system of a D'Arsonval galvanometer is essentially a torsional pendulum, and its equation of motion is:

$$M\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + k_1\theta = 0 \tag{1}$$

where $\theta$ is the angular deviation from the rest position, $M$ is the moment of inertia of the moving system, and $D$ is a *damping constant.* That is, the motion is opposed by a torque, due to air friction, shearing friction in the suspension and eddy currents, which is assumed proportional to the velocity. To solve this equation, put $\theta = xe^{\lambda t}$, then:

$$M\frac{d^2x}{dt^2} + (2M\lambda + D)\frac{dx}{dt} + (M\lambda^2 + D\lambda + k_1)x = 0. \tag{2}$$

The second factor of this equation may be eliminated by setting the arbitrary constant, $\lambda$, equal to $-D/2M$. The equation then reduces to the more usual form:

$$\frac{d^2x}{dt^2} = \left(\frac{D^2}{4M^2} - \frac{k_1}{M}\right)x. \tag{3}$$

This equation has three distinct solutions, depending on the value of the coefficient of $x$. If this quantity is negative, the solution is:

$$\theta = a\cos(\mu_1 t + b)e^{-(D/2M)t} \tag{4}$$

where

$$\mu_1^2 = \left(\frac{k_1}{M} - \frac{D^2}{4M^2}\right).$$

This is the equation of a damped oscillatory motion; that is, of a periodic motion with exponentially decreasing amplitude. On the other hand, if the coefficient is positive:

$$\theta = (ae^{\mu_2 t} - be^{-\mu_2 t})e^{-(D/2M)t}, \tag{5}$$

where

$$\mu_2^2 = \left(\frac{D^2}{4M^2} - \frac{k_1}{M}\right),$$

which gives an *aperiodic motion;* that is, if the system be started

from rest with some displacement, $\theta_1$, $\theta$ will decrease to zero asymptotically.

In practice, it is most convenient to have an aperiodic motion, but it is also desirable to have the system approach its rest position in the minimum possible time. This is accomplished by the transition case between periodic and aperiodic motion, i.e. when $\mu = 0$. This equation of motion then becomes:

$$\theta = (at + b)e^{-(D/2M)t} \tag{6}$$

in which, after a short time, the exponential factor becomes the only one of importance. To bring about this condition, we must have

$$\frac{k_1}{M} - \frac{D^2}{4M^2} = 0$$

or

$$D = 2\sqrt{Mk_1}. \tag{7}$$

In general, $D$ does not attain this value, and unless it be artificially increased by some means, the galvanometer will have a periodic motion.

Consider a galvanometer connected in parallel with an external resistance, $R_d$. Due to the motion of the coil, an E.M.F. is set up according to Lenz's law and is equal, in the notation of the last article, to:

$$V = BlwN\frac{d\theta}{dt} = BA\frac{d\theta}{dt}. \tag{8}$$

This potential difference will produce a current:

$$I = \frac{V}{R_d + R} = \frac{BA}{R_d + R}\frac{d\theta}{dt}$$

which, in turn, sets up a resisting torque according to Eq. (2) of Art. 23:

$$T = BAI = \frac{B^2A^2}{R_d + R}\frac{d\theta}{dt} = \frac{100k_1^2}{K^2(R_d + R)}\frac{d\theta}{dt}. \tag{9}$$

But this is a resisting torque which is proportional to the angular velocity, exactly as was that of Eq. (1). Consequently, if the other damping forces account for $100f$ per cent of the necessary

value of $D$, we may put:

$$(1 - f)D = \frac{100k_1^2}{K^2(R_d + R)}.$$

Solving for $R_d$ and combining with Eq. (7) and Eq. (4) of Art. 23:

$$R_d = \frac{100k_1^2}{2K^2(1 - f)\sqrt{Mk_1}} - R = \frac{B^2A^2}{2(1 - f)\sqrt{Mk_1}} - R. \quad (10)$$

This value of external resistance, just sufficiently low to reduce the motion of the moving system to an aperiodic motion, is known as the *critical damping resistance.* It is usually specified by the manufacturer, and for rapid work the galvanometer should be shunted by a sufficiently low resistance to reduce the total resistance of the external circuit nearly to this value. When the resistance is greater, the galvanometer is *under-damped*, when the resistance is less, the galvanometer is *over-damped.* The insertion of a damping resistance, of course, shunts part of the current from the galvanometer, and this decreases the effective sensitivity. If the external resistance is lower than the initial damping resistance, a series resistance may be added, and this decreases the voltage sensitivity. The best condition of operation, usually, is with the galvanometer just sufficiently underdamped so that the beam moves past its rest point by a distance equal to the estimated error of a single reading. Under these conditions no errors of more than negligible importance are introduced, and the speed of operation is greater than with any other arrangement.

**26. The Ayrton Shunt.**

This device, which is used to decrease the sensitivity of galvanometers, is shown in Fig. 16. Its purpose is to shunt a certain portion of the current from the galvanometer. For example, let the galvanometer resistance be $r$, the current in the line $I$, and let the shunt be set on the 0.01 tap, as shown. Then the current in the galvanometer, $i$, is determined by the relation:

$$\frac{(I - i)R}{100} = i\left(\frac{99R}{100} + r\right),$$

$$i = \frac{I}{100} \cdot \frac{R}{R + r}. \tag{1}$$

On the other hand, suppose that the shunt is on the 1.0 tap, then:

$$i = I \frac{R}{R + r}. \tag{2}$$

The *relative* values of the currents in the galvanometer and line in the two cases are, therefore, independent of the resistances of the

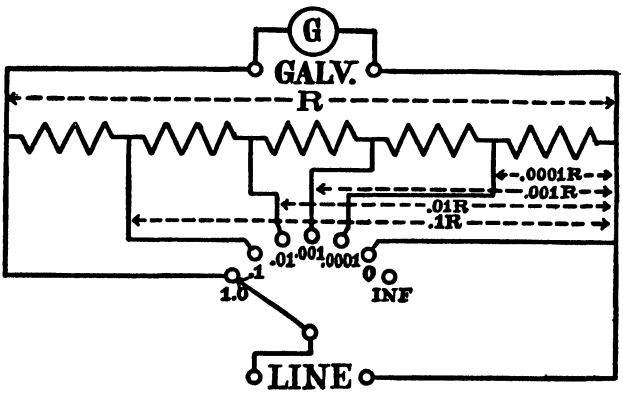



FIG. 16. The Ayrton shunt.

shunt and the galvanometer. Unless the resistance of the circuit, exclusive of the shunt, is high compared with that of the shunt, however, the value of $I$ will itself change as the shunt is moved from point to point, and a correction must be made for this change. The value of $R$ is usually so chosen that it approximates the critical damping resistance of the galvanometer. It should be sufficiently above this value so that the resultant resistance, when it is connected to the external circuit, is approximately the critical damping value. When a galvanometer is to be used with a shunt, it is best to calibrate it with the shunt connected, so that the ratio $\frac{R}{R + r}$ is included in the constant. If this is impossible, the correction may be calculated.

**27. Remarks on the Choice of a Galvanometer.**

The uses of D'Arsonval galvanometers may be divided, in general, into two classifications:

1. Deflection applications, in which the galvanometer is used directly to measure a current by placing it in series with the circuit, and

2. Null applications, in which changes in some network are made until the galvanometer, properly connected, reads zero. The Wheatstone bridge, discussed in Chap. II, and the potentiometer, to be discussed in Chap. IV, are examples of null methods.

For either type of application, considerable experimental difficulty may be avoided by intelligent choice of the galvanometer to be used. The principal characteristics which enter into this choice are the sensitivity, the resistance, the critical damping resistance and the period of the instrument. The sensitivity should be at least equal to, but not very much greater than, that required to measure the desired quantity with the requisite accuracy. Use of a galvanometer which is more sensitive than is needed results only in greater difficulty of adjustment and decreased stability. Few general remarks can be made about the best resistance to be used, as this depends on the particular circuit. Usually when the galvanometer is used in series with a circuit, its resistance should be low compared with the circuit resistance in order that the instrument shall not seriously increase the resistance of the circuit, whereas, when the galvanometer is placed in parallel with a resistance (e.g. for a potential measurement), its resistance, plus any resistance in series with it, should be high compared with the resistance across which the potential drop is measured. Finally, in many networks (cf. Art. 13), the best resistance is uniquely determined. In regard to the period, for most measurements it is desirable that this be kept low, in order that the time consumed in waiting for readings shall be a minimum. For the same reason, it is desirable that the galvanometer be almost critically damped, as explained in Art. 25, and it is well if such a galvanometer can be so chosen that the circuit itself will critically damp it. If the galvanometer's critical damping resistance is lower than the resistance in parallel with

it, it will be necessary to shunt it with an added resistance, thereby decreasing the effective current sensitivity, while if it is higher a series resistance must be added, decreasing the voltage sensitivity.

The requirements outlined in the last paragraph are often mutually contradictory and some sort of a compromise solution must be reached. In choosing the proper galvanometer for a given circuit we generally have two choices to make, the first involving the general type of instrument and the second involving the resistance and period of the instrument within that type. The most commonly used D'Arsonval galvanometers may be grouped into general types, in increasing order of sensitivity, as:

(Typical value of period, figure of merit and critical damping resistance given for each type, assuming 100 ohm coil.)

1. Pointer type meters, characterized by low sensitivity, rugged construction and extreme stability. (3 sec., $10^{-6}$ amp./mm., 400 ohms)
2. Portable reflecting meters, with short, fairly rugged suspensions. (3 sec., $5 \times 10^{-8}$ amp./mm., 1600 ohms)
3. Wall galvanometers, with longer phosphor bronze or gold suspensions, medium sensitivity and fairly easy adjustment, such as are shown in Fig. 12b. (12 sec., $5 \times 10^{-9}$ amp./mm., 1000 ohms)
4. Galvanometers of the type of Fig. 12a, of somewhat greater sensitivity and less stability than wall galvanometers, due to the use of a lighter moving system, smaller air gaps in the magnetic circuit and a lighter suspension. (5 sec., $2.5 \times 10^{-9}$ amp./mm., 1000 ohms)
5. High sensitivity galvanometers, in which the general design indicated under (4) has been carried as far as possible, with resulting extreme sensitivity and required nicety of adjustment. (5 sec., $1.5 \times 10^{-9}$ amp./mm., 700 ohms)

The characteristics given for each type above are rough averages, obtained by considering a number of galvanometers of different manufacturers and of slight differences of design, but they represent what can be expected in general. For convenience they have all been referred to instruments having a coil

resistance of 100 ohms. By the use of the equations already deduced for the galvanometer, however, we can calculate the approximate characteristics of each type for different resistances. Within a given type the characteristics are determined by the size of the suspension which, in turn, is fixed by the weight of the moving system and the desired period. Let us suppose that a given type of instrument has been chosen and that the period is fixed. Then the weight of the coil and the torsional constant, $k_1$, are predetermined, but a choice of the size of wire and the number of turns can be made, subject to the condition:

$$Na = C_1 \tag{1}$$

where $a$ is the cross sectional area of the wire. The resistance, figure of merit, microvolt sensitivity and critical damping resistance then become

$$R = C_2N/a, \tag{2}$$

$$K_1 = (d/2D)(10k_1/BNwl) = C_3/N, \tag{3}$$

$$K_2 = 2(10)^{-8}BNwl/10k_1R = C_4a, \tag{4}$$

and

$$R_d = \frac{B^2l^2w^2N^2}{2(1-f)\sqrt{Mk_1}} - R = C_5N^2 - R. \tag{5}$$

If we eliminate $a$ and $N$ from the last three of these equations by the use of Eqs. (1) and (2), we find three relations between the observable constants of the instrument:

$$K_1 = C_i/\sqrt{R}, \tag{6}$$

$$K_2 = C_v/\sqrt{R}, \tag{7}$$

and

$$R_d = C_dR. \tag{8}$$

These equations show that for extreme current sensitivity we should use a high resistance galvanometer (often as high as several thousand ohms), while for extreme voltage sensitivity we should use a low resistance instrument (often about ten ohms), provided that the circuit can be so arranged that Eq. (8) can be approximately satisfied. The use of Eqs. (6)–(8) with the general characteristics given above for 100 ohm coils will in general

indicate whether a standard galvanometer of a given type will be satisfactory for a particular task. If these equations indicate that such a choice is not possible there remain several possible solutions:

1. A galvanometer of longer period than those indicated above may be used.
2. A special instrument may be designed by the manufacturer, who should be given full data on the circuit with which it is to be used. When too great a departure from standard characteristics is not required this is frequently possible.
3. Vacuum tube amplifiers, which will be discussed in Chap. VI, may be used in conjunction with a galvanometer.

**28. Portable Direct Current Ammeters.**

The D'Arsonval principle is used in the great majority of direct current ammeters. In place of the galvanometer type suspension, the rotating system is mounted in jeweled bearings and the restoring torque is supplied by spiral springs, which also act as current leads. The light beam is replaced by a light, rigid pointer which reads over a graduated scale. The coils themselves, obviously, cannot carry the heavy currents which they are called upon to measure, hence they are usually paralleled by heavy metal shunts, of low temperature coefficient. The usual good grade of meter gives a full scale deflection for about 0.01 ampere, and the balance of the current must be carried by the shunt. Since shunts can be built of any capacity, there is no practical upper limit to the currents that can be measured with D'Arsonval type instruments. The lower limit is set by the sensitivity with which a portable galvanometer may be built. This type of instrument has been built to give full scale deflection for one-half microampere.

In addition to the D'Arsonval type of ammeter, electrodynamometer and hotwire meters, which will be described in detail in Chap. VII, are often used. Moving vane type instruments, which are operated by an iron vane which tends to place itself parallel to the field, are more commonly used for switch board meters than for portable use.

**29. Standardization of Ammeters.**

Due to the difficulty of setting up a current standard, the standardization of ammeters is seldom done directly, and its discussion must be postponed to later chapters. The usual method for laboratory work is to measure carefully the voltage drop across a standard resistance and to compute the current from Ohm's law. This method will be considered in the next chapter, after potential measuring instruments have been discussed. For primary standardization, the electrolytic definition of the International ampere is used, and this will be considered in Chap. XIII.

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# CHAPTER 4

## MEASUREMENT OF POTENTIAL DIFFERENCE

### 30. Units and Standards.

Potential difference is a measure of energy per unit charge and its fundamental definition in the MKSC system is *the steady potential difference which exists across a conductor in which a current of one ampere is flowing and the power dissipated is one watt.* The present legal standard, the *International volt,* was set up by the London Conference as *the electrical pressure which, when steadily applied to a conductor whose resistance is one ohm, will produce a current of one International ampere.* The conference also recommended that the Weston Normal standard cell be used as a convenient means of measuring potential differences and that its E.M.F. be provisionally taken as 1.0184 Int. volts at 20° C. More recent investigation by the Bureau of Standards has shown that the E.M.F. of a carefully made Weston Normal cell is 1.01830 ($\pm$0.00010) Int. volts at 20° C. The MKSC volt defined above is equal to 1.00046 ($\pm$0.00005) Int. volts, and it is probable that the new legal standard will be defined on this basis.

While the definitions given above determine exactly a difference of potential, they do not specify the potential itself and any point may be chosen arbitrarily as being at zero potential. It is often convenient to choose infinity as this zero, and the *potential* of a point is defined as the work required to bring a positive charge of one coulomb up to the point from infinity. But the arbitrary quality of potential must always be kept in mind.

### 31. The Weston Standard Cell.

The construction of this cell, developed by Edward Weston, in 1892, is shown in Fig. 17. The following types of this cell are used:

*A.* The Weston Normal cell, referred to in the preceding article, has crystals of cadmium sulphate left in contact with the

neutral solution so that it is saturated at all temperatures. The voltage of such cells varies appreciably with temperature according to the equation

$$E_t = 1.0183\{1 - 0.0000406(t - 20)\},$$

so that their use requires precise temperature control, and they are rarely found in the commercial laboratory. In many of these cells the voltage falls off appreciably with age.

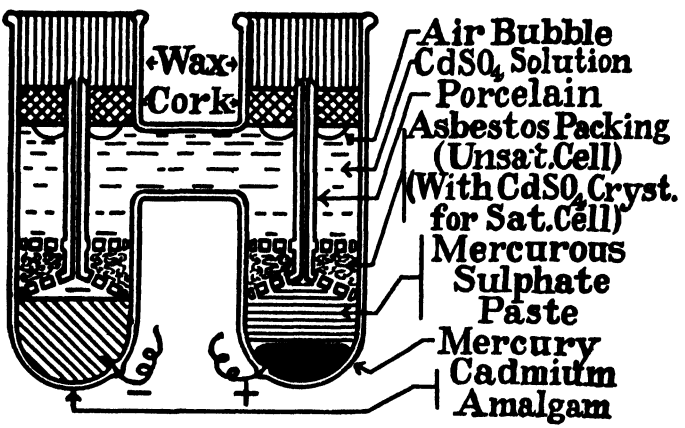


FIG. 17. The Weston standard cell.

*B.* The same cell, but with the electrolyte slightly acid, has a lower voltage which depends on the acidity but shows no preceptible change with age.

*C.* The unsaturated neutral type has a solution which is saturated at 4° C. and a voltage which does not vary much with temperature in the usual range. This type is difficult to reproduce accurately so that each cell is usually accompanied by a certificate stating its exact voltage, which should lie between 1.0180 and 1.0190 volts. This cell often shows an aging effect.

*D.* The unsaturated acid cell is also difficult to reproduce accurately and the voltage of each cell is specified by the maker.

In this country cells of type *C* are made by the Weston Electrical Instrument Company and cells of type *A* and type *D* by the Eppley Laboratories, Inc. It is assumed throughout this book that unsaturated cells, which require no temperature correction, are used.

Certain precautions must always be used in handling standard cells:

1. The cell should not be exposed to temperatures below 4° C. or above 40° C.
2. Abrupt changes of temperature should be avoided.
3. All parts of the cell should be at the same temperature.
4. Current in excess of 0.0001 ampere should never pass through the cell.
5. The cell should be checked against certified standards once a year or when it is suspected that abuse may have affected its electromotive force.

A cell slightly injured by neglecting the above precautions will often recuperate after standing idle for a few months.

**32. Portable Direct Current Voltmeters.**

The D'Arsonval voltmeter is used for most D.C. voltage measurements as well as for current measurements. The actual instrument is exactly similar to the D'Arsonval ammeter, the sole difference being that an external series resistance is inserted in the circuit in place of the low resistance shunt. The value of this resistance may be computed easily from the known sensitivity and, for many commercial meters, is about 100–1000 ohms per volt for full scale deflection. As a rule, one or more resistance coils are contained in the instrument case, but for high voltages external series resistances or *multipliers*, calibrated with the particular instrument involved, are usually used.

Any instrument which will indicate small currents can be used with a resistance to measure voltage. Therefore, there are voltmeter types to correspond to all the ammeter types mentioned in Chap. III. In addition, there are two types of instruments designed particularly for voltage measurements, the electrostatic voltmeter and the vacuum tube voltmeter, which,

since they are used chiefly with alternating currents, will be discussed in Chap. VII.

**33. The Potentiometer.**

The principle of the potentiometer, which is one of the most widely used instruments in precise electrical measurements, may be understood by reference to Fig. 18.

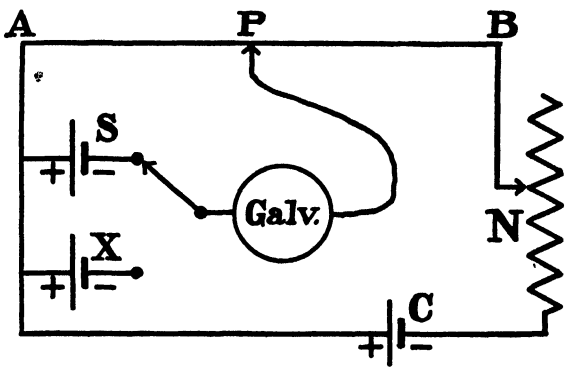


FIG. 18. The slide wire potentiometer.

A slide wire, $AB$, having a resistance $r$ per unit length, is connected in series with a cell $C$ and variable resistance $N$. The latter is adjusted so that, when a current $I$ flows through the wire, the voltage drop across $AB$ exceeds the voltage of the standard cell. If the circuit be closed through the standard cell, $S$, and the galvanometer, $G$, the latter will, in general, be deflected. By moving the contact, $P$, along the wire, a point of zero deflection can be found. In this position, the potential drop across $AP$ must equal the E.M.F. of the standard cell. If $l_1$ is the distance $AP$, and $V_s$ the voltage of the standard cell, we have:

$$V_s = Irl_1. \tag{1}$$

By throwing the switch, the unknown E.M.F., $X$, is substituted for the standard cell and a new balance obtained when $AP$ equals $l_2$. Then:

$$V_x = Irl_2. \tag{2}$$

Since $N$ is unchanged and no current flows through the galva-

nometer at balance, $I$ is also unchanged, and we may divide (1) by (2) to obtain:

$$\frac{V_s}{V_x} = \frac{l_1}{l_2}. \tag{3}$$

This permits us to compute $V_x$. It may be that, due to a variation in the voltage of the battery $C$ or other causes, the current $I$ has actually changed during the measurement. To check this, a second balance with the standard cell is made as soon as that with the unknown is completed. If this position is not the same as in the initial balance, the whole measurement must be rejected and repeated.

It may be well to consider here some of the advantages of the potentiometer system of measurement:

1. The unknown voltage is compared directly against a secondary standard.
2. At the balance condition no current is drawn, so that contact resistances and resistances of leads have no affect on the result.
3. At all times, only a minute current is drawn from the standard cell.

**34. The Direct Reading Potentiometer.**

The simple slide wire potentiometer described above has the disadvantage that the results require computation. This may be overcome easily by a proper adjustment of the current. Suppose that the wire is divided into arbitrary units of length. Now $l_1$ (measured in these units) depends upon $V_s$, $r$ and $I$. The first two of these quantities are fixed, but the third may be varied by means of the rheostat $N$. If the contact is set at 1.0183 divisions from the left end, with the standard cell in the circuit, the current may be adjusted to give a balance. The balance with the unknown is then carried out in the usual way, and:

$$V_x = l_2 \frac{V_s}{l_1} = l_2 \frac{1.0183}{1.0183} = l_2. \tag{1}$$

The potentiometer is now said to be *direct reading*.

When a direct reading potentiometer is designed for convenient practical use, the simple circuit of Fig. 18, while remaining identical in principle, becomes considerably more involved by the addition of various resistance coils and switching arrangements. The details of these arrangements vary widely among the various types of potentiometers, but, before discussing these individually, we shall discuss some of the principal requirements of the circuits. In all potentiometers there are two partially independent current paths, the *battery circuit*, including $C$, $A$–$B$ and $N$ in Fig. 18, in which a steady current flows and the *galvanometer circuit*, including $S$ or $X$, the galvanometer, the sliding contact and the resistance $P$–$A$ in this simple instrument. It will be noticed from Eq. (1) that the resistance of the battery circuit does not enter the reading of the instrument, except as it determines the magnitude of the battery current, hence a considerable latitude is allowed in choosing this constant. It is determined within limits by the particular design used, however, since too low a value of potentiometer resistance results in a heavy drain on the battery, the necessity of heavy wire in the resistors because of the heat dissipation, and of very low resistance contacts in the control resistances of the battery circuit, while too high a value results in low and variable sensitivity and variable damping of the galvanometer (since the portion $A$–$P$ of the battery circuit is also in series with the galvanometer) as well as decreased stability due to the fact that high resistances are more apt to change with time than moderate or low resistances. Practice in compromising between these difficulties varies with different manufacturers as will be seen in the detailed description, and this should be taken into account when a potentiometer is being chosen for a particular task. To test the adaptability of a particular potentiometer to a given measurement it is in general necessary to consider the entire circuit, and to calculate the deflection of the galvanometer when the quantity being measured is changed by the minimum allowable uncertainty. The galvanometer itself must be chosen so as to be approximately critically damped in the potentiometer circuit and in accord with the considerations of Art. 27.

Any switches or moving contacts in the battery circuit should

have very low and constant resistances, in order that the battery current should not be affected seriously by them. In the galvanometer circuit low resistance is not necessary, but absolute freedom from parasitic electromotive forces is essential if the instrument is to be used for precise measurements. For those parts of the instrument which are common to the two circuits both of these requirements must be met and therefore sliding contacts in this part of the circuit must either be avoided or built with extreme care. Various devices for avoiding difficulties in this part of the circuit will be found in the commercial instru-

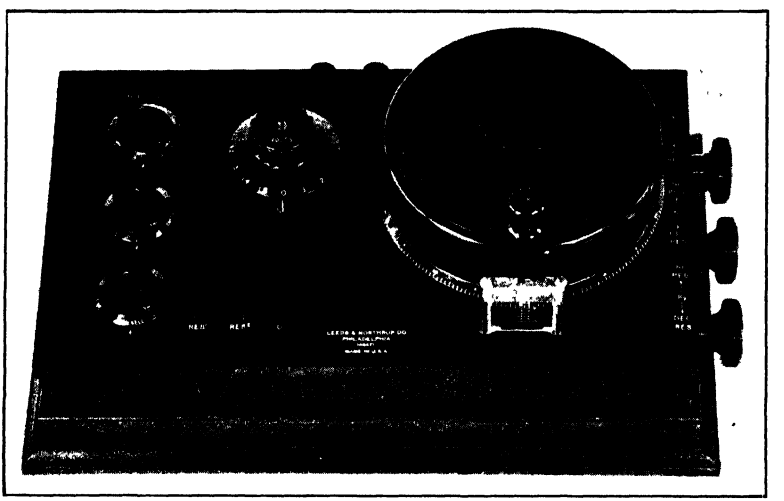

FIG. 19. The Leeds and Northrup Type K-2 potentiometer. (Courtesy of Leeds and Northrup Co.)

ments, and any potentiometer which is being considered for accurate measurements should be carefully inspected and checked for the presence of variable resistance contacts and thermal E.M.F.'s by checking the constancy and the reproducibility of various settings, including the zero of the instrument.

**35. The Leeds and Northrup Type K Potentiometers.**

One instrument, brought out in 1930 by the Leeds and Northrup Company and known as the Type *K*2, is shown in

Fig. 19 and the essential features of the wiring are shown in Fig. 20. The slide wire, $AB$, of Fig. 18 is replaced by the following resistances in series.

1. A set of fifteen five ohm resistance coils in series across each of which the potential drop is 0.1 volt when the 1.6 volt range is used. Between each pair of coils is a dial contact.

2. A helical slide wire of eleven turns between the ends of which a shunt is adjusted so that the potential drop across the slide wire is exactly 0.11 volt when the 1.6 volt range is used.

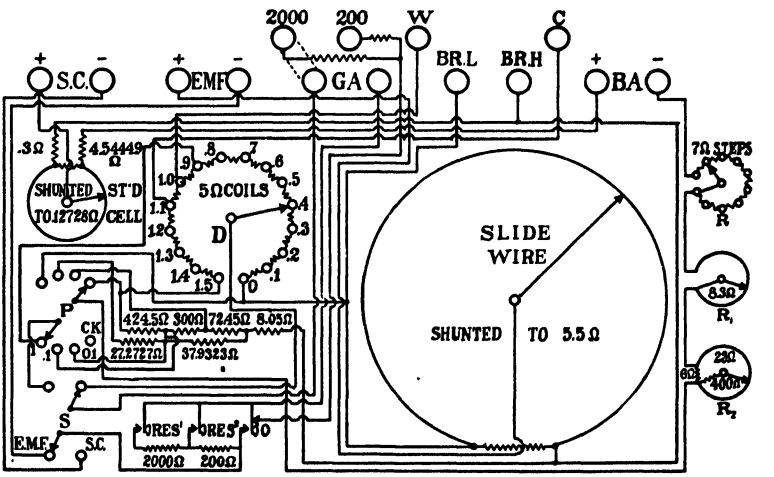


FIG. 20. Arrangement of the Leeds and Northrup Type K-2 potentiometer.

The position of the contact on this helical wire is read from two scales. A vertical glass scale gives the number of the turn and a graduated drum, which has 100 main divisions, gives the fraction of a turn. The setting may be estimated to one tenth division which corresponds to 0.00001 volt.

3. An auxiliary coil and dial resistance slide wire upon which the standard cell voltage may be set without disturbing the other contacts.

The E.M.F. to be measured is applied to the section between the dial contact and the main slide wire contact while the standard cell E.M.F. is applied to the section between the ninth dial coil

and the standard cell dial contact. A dial form of double pole double throw switch permits a rapid transfer from one to the other. Three dial resistances permit coarse and fine adjustment of the battery current. Three galvanometer keys are provided, two for the preliminary adjustment, putting resistances of 2000 or 200 ohms in series with the instrument, and one without resistance for the final setting. In the case of the first two it is possible to insert a link so that the keys automatically shunt the galvanometer with either 2000 or 200 ohms for damping.

The normal range of the instrument is 1.6 volts and is obtained when the dial switch, $P$, is set at 1. The sum of the resistances of the slide wire and dial is 80.5 ohms of which, in this case, 50.5 ohms is included in the standard cell circuit. Shunted around this is 805 ohms so that the total resistance contributed to the battery circuit is 73.1818 ohms and 10/11 of the battery current flows through the slide wire.

When the switch $P$ is set at 0.1 the resistance of the dial and slide wire plus the series resistance is 805 ohms of which 505 is included in the standard cell circuit. In parallel with this is now 80.5 ohms so that the resistance contributed to the battery circuit is still 73.1818 ohms but the current through the slide wire is now 1/11 of the battery current and hence the range is from .000001 to 0.16 volt.

When the switch $P$ is set at 0.01 the resistance of the dial, slide wire and series coils is 877.45 ohms and 8.05 ohms is in parallel so that the current through the slide wire is 1/110 of the battery current. To keep the latter constant 65.205 ohms is placed in series with the whole so that the contribution to the battery circuit is still 73.1818 ohms. The standard cell circuit still includes all but 27.2727 ohms of this. The range is now from. 0000001 to 0.016 volt.

As will be seen from the above the adjustment of the battery current against the standard cell can be made with the switch $P$ set for any range, by turning the switch $S$ to S.C.

Special provision is made for checking the resistances in this instrument. If the cover of the dial switch $D$ be removed it will be found that the posts on which the contacts slide are drilled and reamed to fit tapered plugs with flexible leads supplied with the instrument. Thus, after disconnecting all other leads, each coil,

and finally, using the terminals $BR \cdot L$ and $BR \cdot H$, the slide wire itself, can be connected into a Wheatstone Bridge and measured. Because of the resistance in parallel each coil should give about 4.9718 ohms and the slide wire 1.1 times as much, but this is not as important as that all coils give the same value. The standard cell slide wire can be checked in the same way by connecting to the posts $C$ and $Ba+$ and should give the same value. To check the uniformity of the slide wire the turns are compared as follows. Leave only the battery and galvanometer leads connected and short circuit the E.M.F. binding posts as well as the $BR \cdot H$ and $C$ posts. Turn switch $P$ to the check position and switch $S$ to the E.M.F. position and the arrangement becomes that of Fig. 21 which is an ordinary Wheatstone Bridge. Thus each turn of the slide wire may be compared with the corresponding coil of the dial resistance. As in any slide wire instrument, rapid sliding of the contact may heat it sufficiently to produce a transient thermal E.M.F. of several microvolts. This disappears quickly and does not interfere with the final balance.

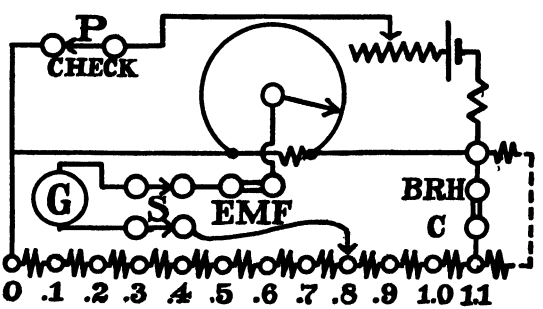


FIG. 21. Calibration of a potentiometer slide wire.

The properties of the Leeds and Northrup Type $K2$ potentiometer may be summed up by saying that it is a low resistance instrument with ranges of 1.6, 0.16 and 0.016 volts and operates on a battery current of 0.022 amperes. Larger voltages can be measured by using a volt box.

**36. The Wolff Potentiometers.**

A potentiometer in which only fixed resistance coils are used is manufactured by Otto Wolff and is shown schematically in

Fig. 22. There are five large dials containing eighteen 1000 ohm, ten 100 ohm, twenty 10 ohm, twenty 1 ohm and twenty 0.1 ohm coils respectively, as well as a small dial for the standard cell containing ten 1 ohm coils. The total resistance of the auxiliary cell circuit between *A* and *B* is 20,000 ohms, and the operating current is 0.0001 ampere. By tracing out this circuit, it will be seen that it includes all three upper sets of coils and that portion of each of the lower sets of coils lying to the left of the two contact points which move together. Hence this resistance, analogous to *AB* in Fig. 18, remains constant regardless of the dial positions. Setting the upper left hand switch at "*X*" and tracing the unknown potential circuit in the same way, we find it includes that portion of the 1000, 100 and of the *lower halves* of the 10, 1, and 0.1 sets of coils lying to the left of the contact points. Thus, this resistance, analogous to *AP* in Fig. 18, can be increased or

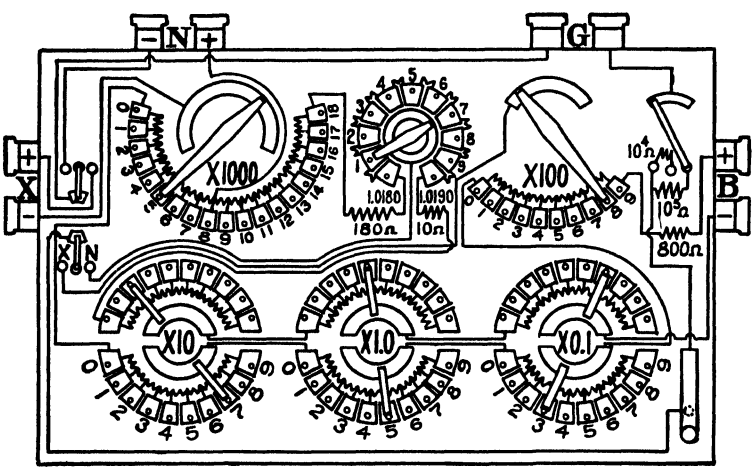


FIG. 22. Arrangement of the Wolff potentiometer.

decreased in steps of 1000, 100, 10, 1 or 0.1 ohms by turning the corresponding dials from one contact to the next. When the switch is set at "*N*," the standard cell circuit includes those coils lying between the fixed connection, *D*, on the 1000 dial and the contact point on the standard cell dial. The resistance of this

circuit can be adjusted by means of the latter from 10,180 ohms to 10,190 ohms in ten steps. The instrument can thus be set to read correctly with any standard cell having an E.M.F. between 1.0180 and 1.0190 volts. The battery current is adjusted by the external rheostat. The switch in the upper right hand corner permits resistances of 10,000 or 100,000 ohms to be inserted in series with the galvanometer to protect it during the preliminary balancing. Tapered holes with plugs to fit are provided for testing the coils just as in the Type *K*2 instrument.

**37. The Rubicon Type B Potentiometer.**

The two potentiometers thus far described represent extremes of design, each of which possesses particular advantages. The Leeds and Northrup instruments satisfy the requirements of Art. 34 by extreme simplicity of design and by avoiding all moving contacts in that part of the instrument which is shared by the battery and the galvanometer circuits. On the other hand, operation of the sliding contact over the eleven turn slide wire slows down the operation slightly and introduces some slide wire wear. The Wolff instruments, on the other hand, have many contacts in the common part of the circuit, but are extremely reliable due to the care exercised in building these contacts and the relatively high resistance of the instrument, which is sometimes a disadvantage. The precautions necessitated by the contacts make the Wolff instruments considerably more expensive than slide wire types. Attempts have been made to build cheaper potentiometers on the Wolff principle, but these have often resulted in very unsatisfactory operation. One potentiometer which succeeds in compromising between the two types, however, is manufactured by the Rubicon Company, and its wiring is shown in Fig. 23, while the instrument is shown in Fig. 24.

In this instrument there are two sets of one ohm coils to supply the 0.01 volt steps, with dials *II* and *II'* moving together, as in the Wolff instrument. In order that very expensive construction need not be used for these contacts the circuit is so arranged that they carry only a small part of the battery current. There are always nine one ohm coils between the contacts *II* and *II'*, and the slide wire *III*, which is shunted to an effective re-

sistance of 11 ohms, together with the 70 ohms series resistance *c*, is in parallel with these coils. Thus only one tenth of the battery current flows through the contacts and the slide wire, minimizing the effect of contact resistances. The slide wire *III'* is a dummy wire which is in the galvanometer circuit only. It is mounted close and parallel to *III*, and the contacts on the two wires move together. When the contact of a slide wire is being moved

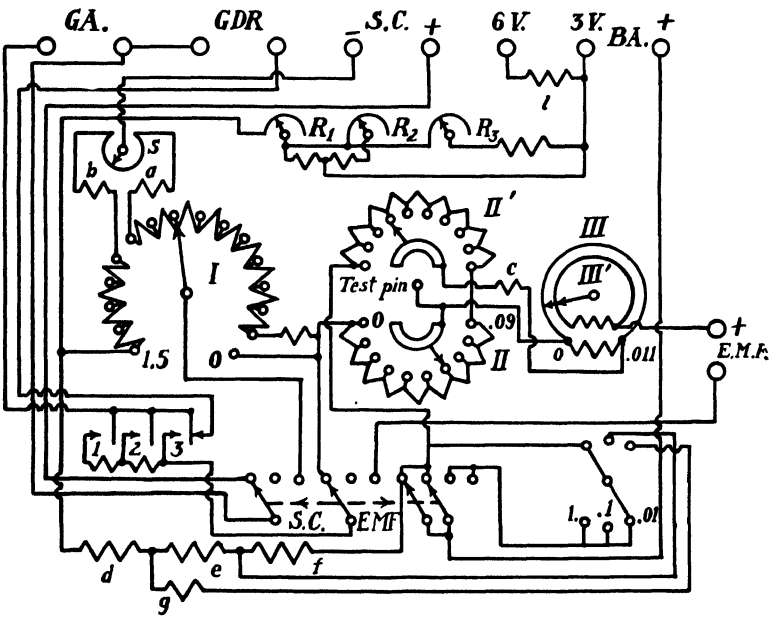


FIG. 23. Arrangement of the Rubicon Type B potentiometer.

rapidly there is some chance of thermal E.M.F.'s being set up, and this arrangement is intended to cancel partially this effect by setting up equal and opposite disturbances. In the tenth volt dial the coil between the *1.0* and the *1.1* volt studs is replaced by the resistances *a*, *b*, and *s* to enable testing against the standard cell without disturbing the setting of the other dials. Three keys are provided in the galvanometer circuit, putting resistances of 10,000 ohms, 1,000 ohms or zero in series with the galvanometer. When key 3 is released it connects the galvanometer across

the terminals *GDR*, across which a properly chosen external resistance may be inserted to damp critically the galvanometer on its return to zero. The resistances $R_1$, $R_2$, $R_3$, and $l$ allow adjustment of the battery current, which should be supplied by two dry cells or a six volt storage battery, the latter giving somewhat more satisfactory operation.

The normal range of the instrument is 1.6 volts, but ranges of 0.160 and 0.016 volts are provided. The circuit which accomplishes this is similar to, although not identical with, that used in the Type *K* instruments, and will be understood by reference

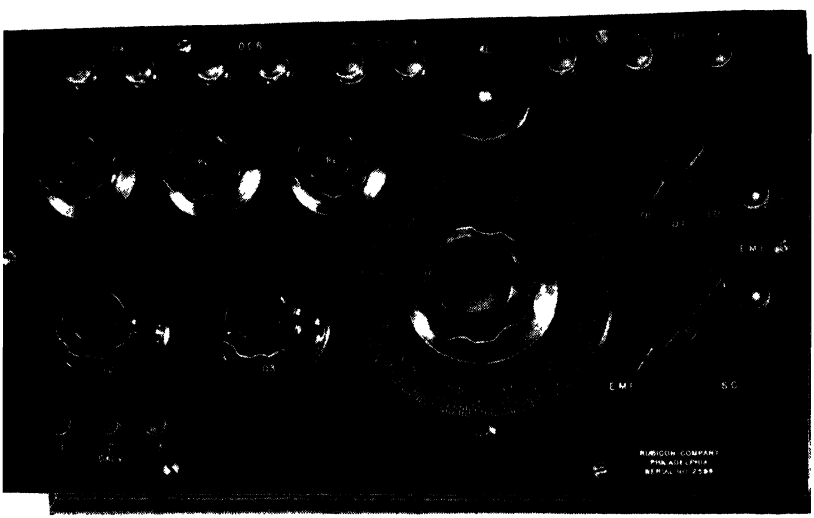

FIG. 24. The Rubicon Type B potentiometer. (Courtesy of Rubicon Co.)

to the diagram. (The coils *d*, *e*, *f* and *g* have resistances of 16.710, 150.39, 1503.9 and 135.35 ohms, ± 0.01%, respectively.) One interesting feature of the instrument is that throwing the E.M.F.-S.C. switch to the S.C. position automatically returns the instrument to the high range, so that there is no danger of short circuiting the standard cell when working on lower ranges.

The Model B instrument does not lend itself to easy internal testing, as plug holes are not provided. If it becomes necessary to test the resistances the instrument must be removed from its case and low resistance clamps attached to the stud ends.

## EXPERIMENT 6

## INTERCHECKING OF STANDARD CELLS*

Connect up the potentiometer, being careful that all polarities are as marked on the instrument. Connect the standard cell, the voltage of which is known, across the standard cell terminals and the cell which is to be checked across the terminals marked "unknown." An accurate thermometer should be mounted beside each cell, which should not have been subjected to appreciable temperature changes for several hours. Throw the double-pole switch to the "standard cell" position, set the standard cell dial to the certified value for the cell, corrected for temperature if necessary, and close the galvanometer circuit, having the highest resistance in series. Reduce the deflection nearly to zero by adjusting the current controlling rheostat, then repeat with the low resistance in series with the galvanometer, and finally, with all the resistance removed, bringing the galvanometer exactly to zero. Now throw the switch to the "unknown" position. Leave the control rheostat untouched and balance with the dials (and the slide wire, if the instrument has one) using the high resistance and low resistance successively in series as before. As soon as the balance is obtained, return to the "standard cell" position and, without changing any adjustments, test the balance. If the deflection is still zero, the reading may be taken, but if it is not, it is necessary to see by how much the current has departed from the correct value. To do this, change the setting of the dial by an amount equal to the uncertainty in the standard cell voltage and again try the balance. If the deflection is larger than before, make the same change in the opposite direction and test again. If the deflection reduces to zero or changes sign for either of these settings, the error introduced by the change of the current is no greater than that which must be expected in the final result, and the reading may be taken. *In any case, return the dial to the correct value and get a new balance by adjusting the control rheostat.* If none of the

* If the student is not familiar with the instrument, he should first make several determinations of the voltage of a dry cell against the poorer standard cell. The procedure will be as outlined, except that the two cells will not be interchanged.

changes give a balance, reject the measurement and start over. This procedure should be repeated three or four times for checking purposes.

Now interchange the "standard" and the "unknown" cells, set the "standard cell" dial to the value obtained for the unknown and adjust the current as before. The instrument should now read the certified voltage of the standard cell when balanced. Also check this reading several times. If the potentiometer circuit contains disturbing electromotive forces, this reading will not be the certified value. In that case, compute the percentage change necessary to bring the observed value to the correct value and apply one-half of this change *in the opposite direction* to the value obtained in the first part of the experiment. This is equivalent to averaging the two values obtained, and gives the correct value for the unknown cell, since any extraneous E.M.F.'s are additive in one case and subtractive in the other.

In your report, specify the accuracy of the result and the temperature at which it was obtained.

## EXPERIMENT 7

### CHECKING OF A POTENTIOMETER

Set up a Wheatstone bridge capable of reading the highest resistance coil in the potentiometer to five significant figures. In the unknown arm, insert flexible leads with the testing plugs for the potentiometer attached. Measure individually the resistance of each coil of the instrument. If the instrument has a slide wire, measure its total resistance and then connect it as shown in Fig. 21 and check the sections of the wire against the coils.

Since it is the relative values of the resistances which are desired, each coil will be checked against the value which it would have if the total resistance were as measured, but the individual resistances were correctly apportioned. To obtain this, divide the sum of the resistances of all the coils, considering the slide wire as one coil, by the voltage which all these coils in series represent with the normal current flowing. This will give an average number of ohms per volt, and by multiplying this by the voltage which any given coil represents (i.e. 0.1, 0.01 etc.) the correct value for any coil can be found. This gives a

connected to the "unknown" terminals of the potentiometer. Be careful of the polarities.

With the circuit open, check the zero reading of the instrument and adjust it if it is not correct. Then close the circuit and adjust the potentiometer rheostat until the meter reads exactly one-fifth of the full scale deflection. Determine the voltage across the terminals of the potentiometer exactly as in Exp. 6, adjusting the

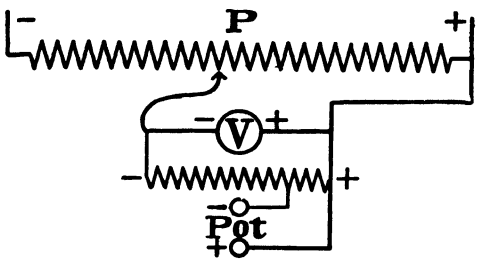


FIG. 26. Calibration of a voltmeter.

current against the standard cell first and taking care to include the resistances in the galvanometer circuit for the rough adjustments. The percentage variation which is now permissible in the potentiometer current is about one-tenth of the precision with which the meter may be read (usually one-tenth of the smallest division). The voltage read by the potentiometer, multiplied by the ratio of the volt box, gives the true voltage across the meter. This also should be determined to one more significant figure than the last one read by the meter.

Now adjust the meter to read 2/5, 3/5, 4/5 and 1 times full scale deflection, determining the true voltage for each reading. Repeat the procedure starting at full scale and decreasing the voltage, so as to compensate for friction in the meter. Average the two results obtained for each reading and calculate the correction for each point by subtracting the observed voltage from the true. Plot a curve of correction in volts against reading in volts. In the correction, retain only as many significant figures as correspond to the accuracy with which the meter may be read.

**40. The Electrometer.**

An electrometer is an instrument for determining small potential differences by measuring the displacements of charged

conductors under electrostatic forces. Such instruments draw no current, and the more sensitive ones require a relatively high and constant auxiliary potential. The type of set up used depends upon the nature of the potential to be measured. If this is a low resistance source, such as a battery, no special precautions are needed. If, however, the potential of a small insulated electrode is required, as frequently happens, great care is necessary. The vital lead must be supported by the best insulators, such as amber, sealing wax or quartz. The surface of these supports must be dry, clean and uncharged. Since quartz is somewhat hygroscopic, it should be cleaned with some non-hygroscopic solvent such as xylol. An insulating surface should never be touched by the fingers after cleaning, and the air in contact with them should be dried with $P_2O_5$, $CaCl_2$, $H_2SO_4$, or some alkali metal. Care should be taken to exclude the ions formed when these agents absorb water.

Since the potential of the system will be changed by the movement of any electrical charges in the neighborhood, it is important that it be shielded from such influences by surrounding it with conductors connected to earth by soldered connections. This soldering should be done with some chemically inert flux, such as rosin, but if this is not feasible, the joint should be thoroughly cleaned of all traces of salts, otherwise a voltaic cell may be formed at this point. The switch making contact to the system must be free from contact electromotive forces. Amalgamated copper points dipping into mercury cups in paraffin blocks suffice for rough work, but the platinum contacts are better. This switch must be electrically shielded and well insulated at all times. Sliding parts should be avoided as frictional charges may result.

**41. The String Electrometer.**

The simplest sensitive instrument is the string electrometer which consists of a fine fibre of sputtered quartz or Wollaston * wire stretched at right angles to the electric field maintained between two parallel electrodes by means of a battery. When the potential of the filament is changed, it is displaced towards the electrode of opposite polarity. This displacement is observed through a microscope or projected on a scale and is roughly pro-

* Fine platinum wire drawn in silver. The silver coat is later removed by acid etching.

portional to the applied potential. The sensitivity of the instrument is regulated by the tension on the fibre and the strength and configuration of the field. In order that the tension of the fibre be unaffected by temperature, the supporting frame must have nearly the same coefficient of expansion as the fibre. The device for adjusting the tension is equipped with stops which are set when the fibre is mounted, so that it cannot be tightened to the breaking point. The distance of the electrodes from the filament is adjustable for a final adjustment, but in some models it is more convenient to make this adjustment by changing the electrical ground contact as indicated below.

The scale of a string electrometer is rarely linear, so that, for accurate work, it is necessary to calibrate it, by connecting it in parallel with a good voltmeter across a variable source of potential. If the adjustments are changed, a new calibration is necessary. Fig. 27 shows the set up for measuring an unknown

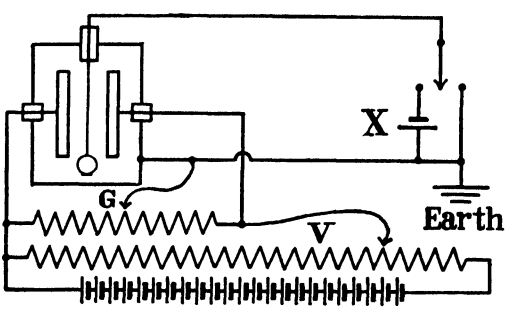


FIG. 27. String electrometer circuit.

E.M.F., $X$. The electrode voltage can be adjusted by means of the contact $V$ which, together with the fibre tension, determines largely the sensitivity. By a further manipulation of the position of the electrodes and adjustment of the earth contact, $G$, various forms of calibration curves can be obtained.

Many types of construction are used, but nearly all of them use amber insulation for the filament. In the Cambridge and Paul type, a very fine adjustment of the tension is possible by means of a micrometer screw on the long end of an invar lever mounted on the invar frame. The microscope scale has fifty divisions and may be read to a tenth of a division. A sensitivity

of .01 volt per division is obtainable with 45 volts between the plates. The capacitance is about 2 mmf. The Stebbins type, made by the Pyrolectric Instrument Company, uses quartz supports for temperature compensation. In the Wulf type made by E. Leybold, Cologne, and the Lutz-Edelmann type made by Edelmann and Son, Munich, the filament is in the plane of two knife edge electrodes and its lower end is fastened to a thin quartz bow. In the latter instrument, both electrodes as well as the fibre are insulated with amber, so it is possible to use several other connections beside that already mentioned. For example, the fibre may be charged to a high potential and the unknown potential applied across the knife edges. This instrument has a sensitivity range from .001 to 100 volts per division. The capacitance lies between 1.5 and 7.0 mmf.

The advantage of string electrometers over other forms lies in their compactness, portability, small capacitance, short period, and wide sensitivity range. For most types, accessory apparatus for photographic recording can be obtained.

**42. The Quadrant Electrometer.**

The quadrant electrometer is probably more widely used than any other type. The important part of this instrument consists

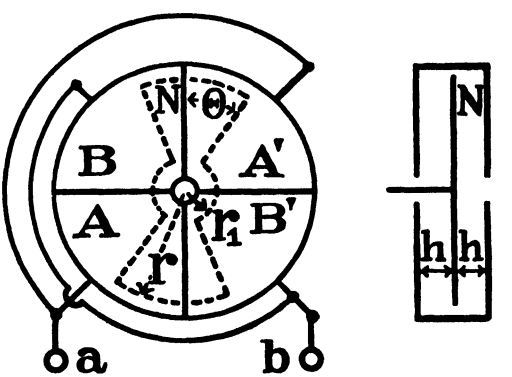


FIG. 28. The quadrant electrometer.

of a cylindrical metal box, whose height is small compared with its radius, divided into four identical, separately insulated quadrants, *A*, *B*, *A'*, *B'*. Diagonally opposite pairs are connected

by wires joined to the terminals $a$ and $b$ as shown in Fig. 28. A light thin needle, $N$, of the form indicated by the dotted line, is suspended horizontally in the box by a conducting fibre. Let the total area of the needle be $S$, and the area in the quadrants $AA'$ be $S_A$, then the area in $BB'$ will be $S-S_A$. Let the potentials be $V_A$, $V_B$ and $V_N$. The total energy of the system will be that of the condensers formed by the needle and $AA'$ and the needle and $BB'$. Including both sides of the needle, this will be approximately:

$$E = \frac{1}{2}CV^2 = \frac{S_A(V_A - V_N)^2}{4\pi h} + \frac{(S - S_A)(V_B - V_N)^2}{4\pi h}. \quad (1)$$

That part of the torque, $T$, on the needle due to the electrical forces will be $\partial E/\partial\theta$ and will be opposed by the torsional resistance of the fibre $\tau(\theta - \theta_0)$. The resultant torque is therefore:

$$\begin{aligned} T &= \frac{\partial E}{\partial \theta} - \tau(\theta - \theta_0) \\ &= \frac{(V_A - V_N)^2 - (V_B - V_N)^2}{4\pi h}\frac{\partial S_A}{\partial \theta} - \tau(\theta - \theta_0). \end{aligned} \quad (2)$$

But $S_A = r^2\theta + r_1^2(\pi/2 - \theta)$ so that $\partial S_A/\partial\theta = r^2 - r_1^2$. In equilibrium $T = 0$ so that we have

$$(\theta - \theta_0) = \frac{r^2 - r_1^2}{4\pi h\tau}(V_A - V_B)(V_A + V_B - 2V_N). \quad (3)$$

The deflection, $D$, is proportional to $(\theta - \theta_0)$ and collecting all the constants into the value $K$, we obtain the usual form

$$D = K(V_A - V_B)(V_N - (V_A + V_B)/2). \quad (4)$$

In most cases, the potential to be measured is $V_A - V_B$ where $V_A$ and $V_B$ are extremely small compared with $V_N$ so that this formula may be written, with far greater accuracy than the measurements can be made,

$$D = KV_N(V_A - V_B). \quad (5)$$

In order that the above analysis hold, it is necessary that when all quadrants are grounded, the position of the needle be the same whether it be uncharged or charged. These two positions, known as the mechanical and electrical zeros, can usually be brought

together by raising and lowering or rotating the needle or by leveling the instrument. Additional adjustments are provided by some makers. Due to approximations in the theory and manufacture of the instrument, the scale is not quite linear and must be calibrated. The instrument described is called the Dolezalek type and has a capacitance of about 80 mmf. and a maximum sensitivity of about 2000 mm. per volt on a scale one meter distant.

Another type of quadrant electrometer is known as the Compton type. In this instrument, advantage is taken of a slight dissymmetry between the needle and quadrants, one of which can be moved up and down independently. In this way, it is possible to cause the electrostatic deflecting force to increase as the deflection increases in such a way that the restoring torque of the suspension is nearly neutralized. By this means, a maximum sensitivity of about 50,000 divisions per volt has been obtained, with a capacitance of from 12 to 15 mmf. Great care is necessary in setting up this instrument, which is extremely sensitive to changes in level and other disturbances.

**43. The Hoffman Electrometer.**

The Hoffman electrometer has probably the greatest workable sensitivity of any. In this instrument, binants are used instead of quadrants and the needle is unsymmetrical as shown in Fig. 29. The voltage to be measured is applied to the needle and a fixed voltage put across the binants. The needle system is made of very thin platinum alloy sheet with a light mirror about 2 mm. in diameter and suspended by a fine Wollaston wire. The inside of the binant box is platinum plated to avoid any contact E.M.F. A heavy copper shield, swinging on pivots, closes about the moving system. This insures uniform temperature and makes the volume of gas in contact with the system small, thus minimiz-

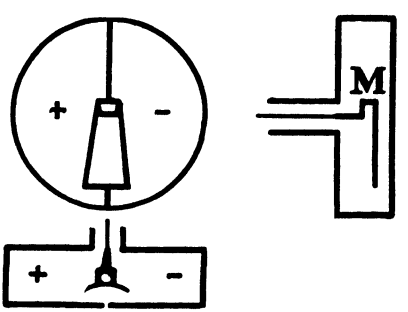


FIG. 29. The Hoffman electrometer.

ing the effect of the gas ions produced by the radioactivity of the walls and penetrating radiation. This effect is further reduced and rendered negligible by evacuating the instrument to a pressure of from 2 to 7 mm. of mercury. The moving system is made to have the proper damping at about these pressures. The frame is compensated, so that no relative movement of the needle and the binants is possible due to temperature changes. The filament clamp is supported by a thin flat amber plate, mounted in a heavy brass cylinder which can be rotated or moved up and down through a vacuum tight, leather packing joint by means of external screws. Embedded in the walls of this cylinder is a small electromagnet and armature which operates a grounding contact of thin platinum wire in the vacuum.

The electrical and mechanical zeros may be brought together either by adjusting the ground between the binants or by charging the "induction ring." The latter is a thin brass cylinder concentric with the filament clamp which when charged induces a charge on the suspended system. Due to the downward concavity of the needle, the system approaches instability as the binant voltage is increased. This point of "infinite" sensitivity frequently occurs in the neighborhood of 13 volts, but depends on the individual system and the distance of the needle above the binants. When using this instrument to measure very small quantities, it is necessary that the leads be completely shielded and evacuated to a few millimeters pressure. The instrument is quite insensitive to level and is not disturbed by vibration as badly as a sensitive galvanometer. It is claimed that with special care currents as low as $10^{-19}$ ampere have been measured, but without special precautions $10^{-17}$ ampere is about what may be expected. A special switch is provided by which the voltage may be thrown on the binants and induction ring just after the ground is broken, and thrown off just after the ground is made. The capacitance is about 2 mmf. and the period thirty seconds.

## EXPERIMENT 9

### CALIBRATION OF QUADRANT OR STRING ELECTROMETER

Set up the apparatus as shown in Fig. 30. Now bring the mechanical and electrical zeros (see Art. 42) together by leveling

the instrument and raising, lowering or rotating the needle until, with all quadrants grounded, charging or grounding the needle does not change the scale reading by more than a millimeter. Calibrate the instrument at needle or electrode voltages of 45

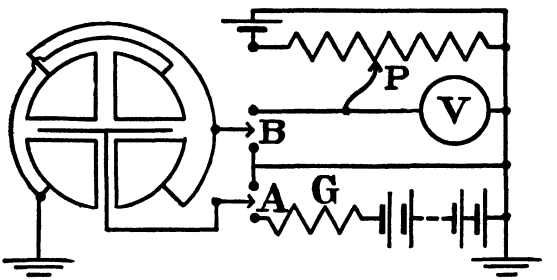


FIG. 30. Calibration of a quadrant electrometer.

and 22.5 volts. To do this, keep switch *A* in charge position, being sure to have a high protective resistance, *G* (a grid leak will do), in series with the needle battery, and apply potential to the quadrants or string through the potentiometer *P*. Go from zero to one volt by one-tenth volt steps for each needle potential. Plot curves of deflection against potential.

## EXPERIMENT 10

### MEASUREMENT OF HIGH RESISTANCE

A convenient form of high resistance for this experiment is that described in Art. 18. Measure several of these by finding the rate of leakage when they are connected across a good 1 mf.

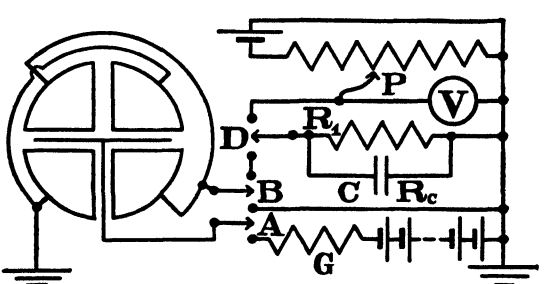


FIG. 31. Measurement of high resistance with a quadrant electrometer.

condenser, $C$. Connect as shown in Fig. 31. Using the switch $D$, charge the condenser to a voltage sufficient to give nearly a full scale deflection on the electrometer, then, by reversing $D$ connect the condenser, with $R_1$ in parallel, to the quadrants (or fibre, if a string electrometer is used). Take about ten readings at regular intervals on the scale until the voltage has dropped to between three-fourths and one-half of its initial value. These deflections, corrected from the calibration curve, are proportional to the charges just as in Art. 19, and the results are plotted in the same way. Compute $R$ and then remove the resistance and determine $R_c$ if necessary. Compute $R_1$ as in Art. 19.

If, when the results are plotted, it is found that a straight line cannot be made to fit the points within the accuracy of the observations, it is probable that the product $RC$ is too small, making the rate of leak too high for the period of the instrument. If this curvature is small, a smooth curve may be drawn through the points and values of the slope taken for different sections of the curve. When these values are plotted against the time they should approach the true value asymptotically and it can be estimated. The trouble can also be remedied by using a larger condenser. This error cannot occur if a string electrometer, with a period of about 0.02 seconds, is used, since this is faster than the reaction time of the observer.

**44. Potentiometers for Current Measurement.**

Since instruments for the measurement of potential difference have been developed to such a high degree, they are usually used for the measurement of current in the standardizing laboratory. For this purpose, the current to be measured is passed through a very accurately determined resistance, and some voltage measuring device is used to measure the potential drop across this resistance. The current is then computed from Ohm's law. An ammeter, for example, may be considered as a milli-voltmeter reading the drop across a shunt. For standardization of D.C. ammeters, a potentiometer across a standard resistance of the type described in Art. 12 is almost universally used. Another application is in the measurement of very small currents, for which an electrometer is connected across a very high resistance leak.

## EXPERIMENT 11

### CALIBRATION OF A D.C. AMMETER

Set up the apparatus as shown in Fig. 32, where $R_1$ is a rheostat of proper value to limit the current to less than one-fifth of the full scale deflection of the ammeter, $A$, and $R_2$ is a rheostat of about ten times the resistance of $R_1$. The latter resistance is

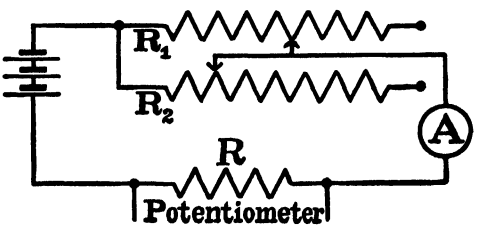


FIG. 32. Calibration of a D.C. ammeter.

used for a fine adjustment of the current. Connect the lines marked "Pot" to the unknown terminals of the potentiometer. $R$ is a standard resistance of such value that the full scale deflection of the meter will give a voltage drop as close as possible to the maximum read by the potentiometer.

Check and correct the zero of the meter. Build up the current until the meter reads 1/5 of full scale deflection. Holding this constant, determine the voltage drop, observing all the precautions of Exps. 6 and 8. Read the potentiometer to one more significant figure than the meter can be read. Repeat this procedure for 2/5, 3/5, 4/5 and 1 times full scale current, then for the same values with decreasing current.

Calculate the true current for each ammeter reading, average the values ascending and descending, and plot a curve of correction in amperes against reading in amperes.

## EXPERIMENT 12

### COMPARISON OF STANDARD RESISTANCES WITH THE POTENTIOMETER

By a slight variant of the method used in the last experiment, the potentiometer may be used for the accurate comparison of

two resistances which are not too far different in value from one another.

Set up the apparatus as shown in Fig. 33, using for $S$ an accurately known standard not differing from the unknown $R$ by a factor of more than 100 to 1000. The battery used for the current supply should have sufficient capacity so that the potential difference across its terminals will remain constant to the percentage accuracy desired in the comparison, and all contacts must be free from resistance variations or thermal electromotive forces. For this reason a double pole, double throw mercury contact switch is best used for the connection to the potentiometer. Balance the potentiometer across $S$, then switch it to $R$ and balance, and finally return to $S$ for a check on the constancy of the current. Discard the readings unless the two potential differences across $S$ agree within the desired percentage accuracy.

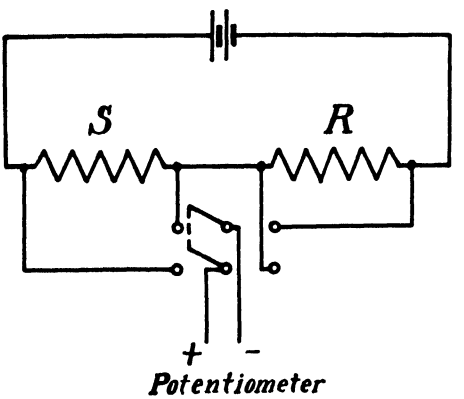


FIG. 33. Comparison of resistances with the potentiometer.

If $S$ and $R$ are of the same order of magnitude, both readings can be taken on the 1.0 range of the instrument, with sufficient battery potential used in the external circuit to give a drop of one volt or slightly more across the larger resistance. If they differ by an appreciable factor (say more than five), accuracy may be gained by measuring the drop across the larger on the 1.0 range and across the smaller on the 0.1 or the 0.01 range if these are available.

Since the same current flows through the two resistances, their magnitudes are directly proportional to the measured potential drops. Calculate the percentage correction to be applied to the measured resistance.

**45. The Brooks Deflection Potentiometer.**

In the use of any of the ordinary potentiometers, a great deal of time is consumed in bringing the galvanometer exactly to zero deflection. Where extreme accuracy is not required, the idea of balancing the potentiometer approximately, and then using a milli-voltmeter to detect the amount of the departure from balance, suggests itself.

The difficulty with this in the ordinary arrangement is that the galvanometer circuit, to be used as a voltmeter, has a resistance which depends on the position of the slide wire contact and the adjustment of the battery rheostat.

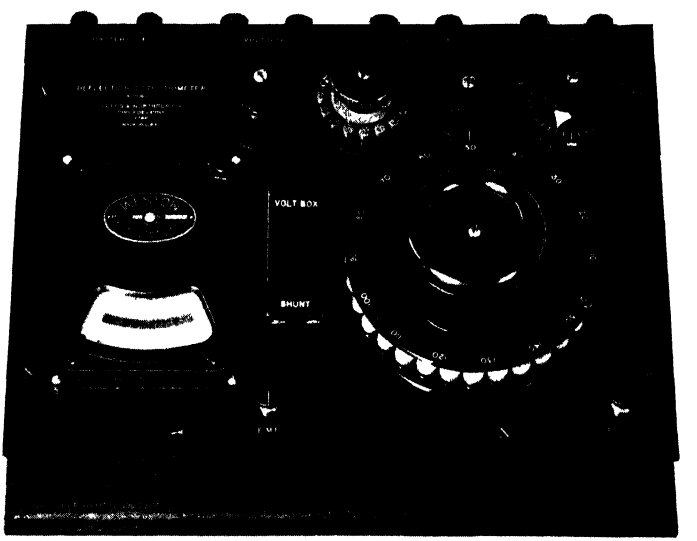


FIG. 34. The Brooks deflection potentiometer. (Courtesy of Leeds and Northrup Co.)

This difficulty is overcome in the Brooks Deflection Potentiometer. The Model 7 instrument, manufactured by the Leeds and Northrup Company, is shown in Fig. 34 and its actual wiring is given in Fig. 35B. The operation will be better understood by considering Fig. 35A, which is perfectly analogous to the simple potentiometer diagram, Fig. 18, Art. 33. If this circuit

is solved for the current through the galvanometer we find that

$$i_g = \frac{X - ER_n/(R_n + R)}{R_g + r_n + R_nR/(R_n + R)}.$$

The numerator is the difference between the voltage to be measured, $X$, and the voltage, $ER_n(R_n + R)$, indicated by the position of the contact, $P$. It will be seen that this difference is always proportional to the galvanometer current, provided the denominator is constant for all settings of $P$ and the battery rheostat. $R_g$, which includes the galvanometer resistance and

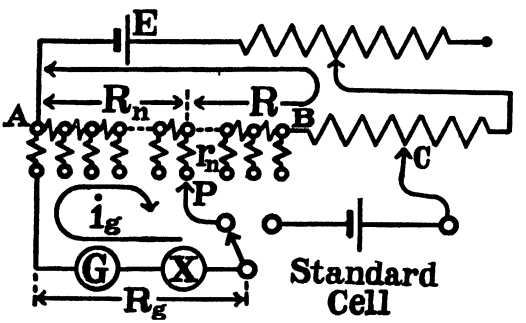


FIG. 35A. The deflection potentiometer principle.

the resistance of the unknown source, is kept constant by the special construction of voltage and shunt boxes mentioned later. $R$, which has a minimum value of about 150 ohms, is always much larger than $R_n$, which has a maximum value of about 6 ohms. We may therefore make the following expansion:

$$\begin{aligned} R_nR/(R_n + R) &= R_n(1 + R_n/R)^{-1} \\ &= R_n(1 - R_n/R + R_n^2/R^2 - \cdots). \end{aligned}$$

We may certainly neglect the square term and write:

$$i_g = \Delta X/(R_g + r_n + R_n - R_n^2/R).$$

We now choose the $r_n$'s, so that $r_n + R_n - R_n^2/R$ is independent of the position of the contact provided $R$ is constant. But $R$ may have to be adjusted between 180 and 150 ohms to take care of different battery voltages. The denominator has roughly the

value 10 ohms and the worst variation due to adjustments of the battery rheostat will be $R_n^2(1/R_0 - 1/R) = 0.04$ ohms. Assuming the instrument to be adjusted for the mean value, the galvanometer reading may thus be in error by $\pm 0.2$ per cent, or one part in 500. Since the maximum deflection of the galvanometer is 30 divisions and tenths of a division may be estimated, it cannot be read to better than one part in 300 at best, so that

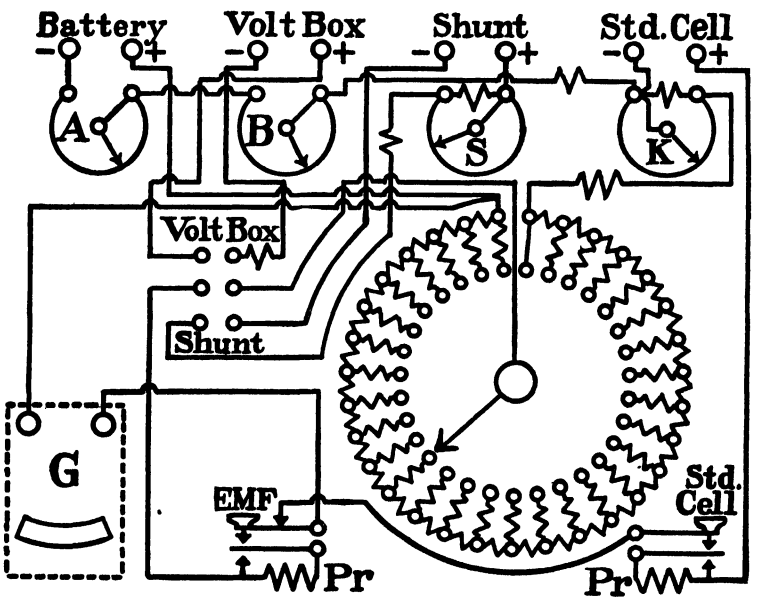


FIG. 35*B*. The Brooks deflection potentiometer.

in this instrument errors due to variations in the galvanometer circuit resistance have been reduced to negligible proportions.

The range of the instrument is 0.150 volts, which is covered by thirty steps on the dial. The galvanometer has a scale of three millivolts on each side of the balance which may be estimated to 0.01 millivolts. The dial resistances are accurate to 0.02 per cent and the galvanometer, in proper adjustment, has an error of less than 0.04 millivolts at full deflection. Examining Fig. 35 *B*, we see that rough and fine adjustments by the dials *A* and *B* are

provided for the auxiliary battery. The standard cell dial can be set for any value between 1.017 and 1.019 volts.

In order to meet the requirements of constant resistance in the unknown circuit, a special volt box is provided for voltage measurements and a special dial adjustment, $S$, compensates for the resistance of shunt and leads in current measurements. To facilitate simultaneous measurement of current and voltage, separate terminals are brought out for each and a double throw switch shifts from one to the other. The keys for closing the standard cell and unknown circuits are electrically interlocked, so that the two circuits cannot be closed at the same time. Each key connects a high protective resistance, $P_r$, in series with the galvanometer when pushed part way down and connects it directly when pushed clear down.

## EXPERIMENT 13

### CALIBRATION OF A WATT-METER

Set up the apparatus as shown in Fig. 36. The circuits for control of voltage and current are exactly similar to those used in Exps. 8 and 11, except that the reversing switches, $S$, have been included. $V$ is a volt box * designed for the deflection potentiometer and $R$ is the standard resistance.* The object of using separate supplies for the current and potential coils is to decrease the energy which must be dissipated in the resistances.

Having connected the apparatus, set the standard cell dial to the assigned value and adjust the current to the proper value with the dials $A$ and $B$, being careful to press the key only to the first contact position until a close balance is obtained. Bring the galvanometer exactly to zero. Check this balance before each reading.

Set the potential within one volt of 110 volts, and hold it within that limit throughout the measurement. Adjust the current until the wattmeter reads 1/5 of full scale. Throw the double-throw switch to the volt box position and, pressing

* Where the accuracy obtainable with a potentiometer is not required, the volt box and standard resistance may be replaced by a calibrated voltmeter and ammeter, which are known as laboratory standard instruments.

the E.M.F. key half way, turn the main dial until you locate the position nearest to the balance. Then press the key all the way and read the dial and galvanometer. (Be careful to use the scale which has its middle figure the same as the last figure of the dial reading.) Throw the switch to the shunt position and measure the drop across the standard resistance in the same way.

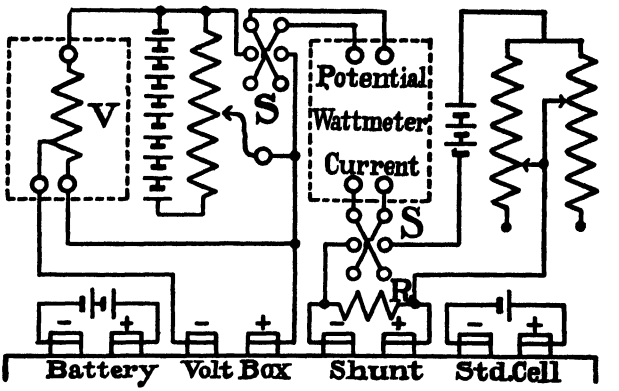


FIG. 36. Calibration of a wattmeter.

Since a watt-meter consists of a rotating coil, carrying a current proportional to the voltage, and moving in a magnetic field supplied by coils carrying the current, it is affected by the earth's magnetic field, or by any stray fields, unless it is carefully shielded. To avoid such effects, reverse both switches, *S*. This reverses the field in the instrument, but leaves the direction of deflection unchanged. Repeat the measurements under these conditions, and average the values thus obtained.

Repeat for 2/5, 3/5, 4/5 and 1 times full scale deflection. Plot a curve of correction in watts against reading in watts at constant potential of 110 volts.

If a meter is to be used on a constant current circuit, its calibration at this current is necessary. Set the current at the rated current of the meter and vary the readings by changing the voltage. Plot a correction curve at constant current of the specified value.

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# CHAPTER 5

## MEASUREMENT OF QUANTITY OF ELECTRICITY

**46. Units.**

The unit of quantity of electricity is the *coulomb* which is defined as *that quantity which flows in one second through any surface of a conductor across which a current of one ampere is flowing.* Consequently the ratio of the present International coulomb to the probable new legal standard of quantity is the same as in the case of the current standards.

**47. The Ballistic Galvanometer.**

Quantities of electricity may be measured by electrolytic, caloric or magnetic effects. The most commonly used instrument, which depends upon the last named, is the *ballistic galvanometer.* This instrument is identical with the ordinary D'Arsonval current galvanometer with the exception that the current carrying coil is made with a much larger moment of inertia. This is accomplished without appreciably increasing the weight of the moving system by using a wider coil and enables the instrument to measure an impulse, rather than a steady deflecting torque.

Consider the galvanometer of Art. 23, and assume that a quantity $Q$, such as the discharge from a condenser, has been passed through the coil, and that the total time of discharge is small compared with the period of the instrument. Then the torque is (See Eq. (3), Art. 23):

$$T = BAI$$

and since, by definition:

$$Q = \int I dt,$$

the torque-impulse:

$$\int T dt = BA \int I dt = BAQ. \qquad (1)$$

But the torque produces an angular acceleration, $d^2\theta/dt^2$, defined

by the equation (see Art. 25):

$$T = M\frac{d^2\theta}{dt^2}, \tag{2}$$

so that

$$\int T dt = BAQ = \int M\frac{d^2\theta}{dt^2}dt = M\frac{d\theta}{dt} + C_1.$$

The angular velocity, $d\theta/dt$, is obviously zero if no discharge has been passed; hence the constant $C_1$ has the value zero, and:

$$\frac{d\theta}{dt} = \frac{BA}{M}Q. \tag{3}$$

The coil, therefore, has a kinetic energy of rotation:

$$\text{K.E.} = \frac{1}{2}M\left(\frac{d\theta}{dt}\right)^2 = \frac{B^2A^2Q^2}{2M}. \tag{4}$$

If there is no friction or damping, the coil will swing to a position in which the entire energy has been transferred to potential energy in the suspension. But this energy is:

$$\text{P.E.} = \int T d\theta$$

where $T = k_1\theta$, and therefore:

$$\text{P.E.} = \int k_1\theta d\theta = \frac{k_1\theta^2}{2}. \tag{5}$$

Equating the two energies:

$$\frac{B^2A^2Q^2}{2M} = \frac{k_1\theta^2}{2},$$

$$Q = \sqrt{Mk_1}\frac{\theta}{BA} = \sqrt{\frac{M}{k_1}}\cdot\frac{k_1}{BA}\theta = K_b'\theta. \tag{6}$$

That is, the deflection of the ballistic galvanometer is directly proportional to the quantity of electricity passed through the coil, provided this quantity is passed in a very short time compared with the period of free swing of the instrument. The quantities here used to define the ballistic constant, $K_b'$, are not

easily measured, however, and a more convenient expression may be obtained by combining Eq. (6) with Eq. (4) of Art. 23 and with the well-known relation for the total period, $P$, of an undamped torsional pendulum:

$$P = 2\pi\sqrt{\frac{M}{k_1}}. \tag{7}$$

It is then found that:

$$Q = \frac{P}{2\pi} \cdot K\theta. \tag{8}$$

Since the ballistic galvanometer is usually used with a lamp or telescope and scale, like the current galvanometer, the constant of the galvanometer, as used practically, is, from Eq. (2), Art. 24:

$$K_b = \frac{K_b'}{2000} = \frac{P}{2\pi} K_1. \tag{9}$$

The constant of a ballistic galvanometer is, therefore, directly proportional to the period and also to the figure of merit of the instrument when it is used to indicate currents.

**48. Damping.**

In the above discussion, it was assumed that all of the energy of the initial impulse was consumed in setting the galvanometer in harmonic motion. In practice, of course, this is not the case, and energy is dissipated in friction and the electrical circuit. In Art. 25, Eq. (4), it was shown that the motion of such a damped galvanometer, when it is swinging freely, is:

$$\theta = a \cos (\mu t + b) e^{-(D/2M)t}$$

where

$$\mu^2 = \frac{k_1}{M} - \frac{D^2}{4M^2}.$$

The damping, therefore, has two effects:

1. The period is slightly changed.
2. The succeeding maximum deflections decrease logarithmically with the time.

If the deflections be plotted against the time, a curve similar

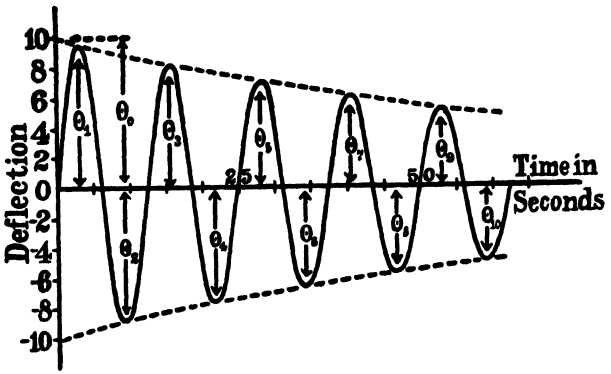


FIG. 37. Motion of a damped ballistic galvanometer.

to that of Fig. 37 is obtained. Let $P'$ be the period of the damped instrument. If the damping constant were zero, the deflections would all equal $\theta_0$, while in practice:

$$\theta_1 = \theta_0 e^{-(D/2M)P'/4},$$

$$\theta_2 = \theta_0 e^{-(D/2M)3P'/4} = \theta_1 e^{-(P'/2)D/2M},$$

and similarly:

$$\theta_n = \theta_{n-1} e^{-(P'/2)D/2M} = \theta_{n-1} e^{-\alpha}. \qquad (1)$$

The magnitude of the deflection as it would be without damping is usually required, and this is:

$$\theta_0 = \theta_1 e^{\alpha/2}.$$

Since $\alpha$ is usually small this may be approximated as:

$$\theta_0 = \theta_1(1 + \alpha/2). \qquad (2)$$

Where $\alpha$ is called the *logarithmic decrement* of the deflections.

When a ballistic galvanometer is used for the same purpose continually, its constant may be determined in place, and then, since the damping constant is dependent only on the instrument itself and the external circuit, the first maximum of the deflection is proportional to the quantity of electricity passed through the coil. But for specifying the characteristics of the instrument, it is necessary to have some standard of reference, and this is usually taken as the undamped condition. Consequently, the $K_b$ of

Eq. (9) of the previous article is obtained for this purpose by using $\theta_0$ in Eq. (6).

**49. Capacitance.**

Whenever a conductor is raised to a potential different from that of surrounding conductors, it acquires a charge. In the case of two conductors, this charge may be shown to be directly proportional to the difference of potential applied, and the constant of proportionality is known as the *capacitance* between the two conductors. In practice, a *condenser* is formed of two sets of conductors, separated by an insulator, known as the *dielectric*. The unit of capacitance is the *farad* which is the capacitance of a condenser which acquires a charge of one coulomb when a potential difference of one volt is maintained across its terminals. As this unit is very large, the *micro-farad*, or one-millionth of a farad, is commonly used.

Due to this simple relation, capacitances are often measured by measuring the charge on a condenser after it has been charged by a source of known electromotive force. Conversely, a condenser of known capacitance may be used to determine the constant of a ballistic galvanometer.

## EXPERIMENT 14

### CALIBRATION OF A BALLISTIC GALVANOMETER WITH A STANDARD CONDENSER

Set up the apparatus as shown in Fig. 38. $G$ is the galvanometer to be calibrated, $K$ is a short circuiting or damping key used to bring the galvanometer to rest after the readings have been taken, $C$ is a condenser of known capacitance, $S$ is a Weston standard cell, and $r$ is a protective resistance of at least 10,000 ohms. First determine the time required to give the condenser its full charge. Throw the key to the charge position and leave it for one second, then discharge and read the first deflection of the

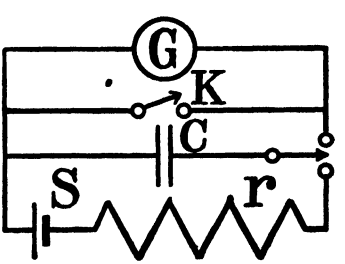


FIG. 38. Calibration of a ballistic galvanometer with a standard condenser.

galvanometer. Bring the galvanometer to rest at the zero point by means of the short circuiting key and repeat for 2, 3, 5, 10 sec. Plot a curve of the deflections against time of charge and in subsequent work use a charging time sufficient to insure that the maximum charge has been obtained.

Now charge the condenser again, and after discharging, allow the galvanometer to swing through four or five complete cycles, reading each maximum deflection in each direction. Repeat this, then reverse the polarity of the standard cell and take the same set of readings with the initial swing in the opposite direction. Calculate the ratios of succeeding deflections (neglecting the signs). Average the set of values thus obtained, and using the average value of $\theta_1$ in Eq. (2) of Art. 48, calculate $\theta_0$, in terms of millimeters deflection. Measure the distance between the mirror and scale and calculate the constant from the equation:

$$K_b = \frac{DQ}{1000\theta_0},$$

where $D$ is the distance from mirror to scale in millimeters, and $Q$ is the charge on the condenser, determined by:

$$Q = CE,$$

in which $C$ is the capacitance in farads and $E$ is the applied electromotive force in volts.

Given the known residual errors in the condenser and standard cell, determine the precision of this constant.

## EXPERIMENT 15

### COMPARISON OF CAPACITANCES WITH BALLISTIC GALVANOMETER

When two condensers are to be compared with each other, it is unnecessary to correct for damping, as the damping correction has the same relative value in the two cases. Connect the apparatus exactly as in Exp. 14, and on discharging, read only the first deflection. The standard cell may be replaced by a dry cell, as the only requirement is that the voltage be constant throughout the experiment. Take at least five determinations of the deflection when the standard condenser is discharged, as the

accuracy of all the results will depend upon this value. Then replace the standard condenser by a laboratory subdivided condenser. Determine the deflection when each section of this is discharged. If this deflection is $\theta_x$ and the average with the standard is $\theta_s$, then:

$$\frac{C_x}{C_s} = \frac{Q_x}{Q_s} = \frac{\theta_x}{\theta_s}.$$

Having determined the capacitances of the individual sections, check by determining the capacitance of all the sections in parallel. This value should check with the sums of the individual capacitances. Also place all the sections in series, and check the result obtained against the reciprocal of the sum of the reciprocals of the individual capacitances.

**50. Inductance.**

Suppose that a wire is moved across a magnetic field, or that the field passing through a coil of wire is changed in value. Then according to the law of induction, an electromotive force is set up in the wire according to the relation:

$$V = N\frac{d\phi}{dt} \tag{1}$$

where $V$ is the induced E.M.F., $N$ is the number of turns in the coil and $d\phi/dt$ is the rate of change of magnetic flux through the coil. Also, whenever a conductor carries a current, there is established around the conductor a magnetic field proportional to the current which changes as the current changes. This change will then produce an electromotive force in the conductor itself and in any surrounding conductors, the direction of the electromotive force being such that it tends to set up a current opposing the change of flux producing it. The phenomenon in the conductor itself is known as *self-inductance*, in surrounding conductors, as *mutual-inductance*. Since this E.M.F. is proportional to the rate of change of current, the unit of both self and mutual inductance is that inductance in which one unit of potential difference is induced when the current is changing at the rate of one unit per second. Thus *the henry* is the inductance in which an E.M.F. of one volt is induced when the current is changing at the rate of one ampere per second.

Now consider the *solenoidal inductor* shown in Fig. 39. The primary circuit consists of a standard solenoid made by winding wire uniformly on a long circular cylinder of non-magnetic material. Let the number of turns per meter length of this primary be $N_1$, its cross-sectional area $A$ and the current it carries $I$ amperes. It can be shown that, if the length is very great compared with the diameter, the field $H$, inside the solenoid and far from the ends, is uniform and the flux (measured in webers) through the coil is *

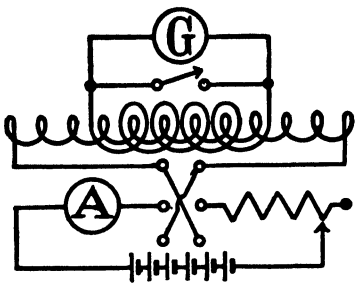


FIG. 39. Calibration of a ballistic galvanometer with a standard inductor.

$$\phi = 4\pi N_1 A I/10^7. \tag{2}$$

Under the same conditions, it can be shown that the return flux outside the solenoid produces a negligible field, so that the amount of this flux passing inside a short secondary coil, of $N_2$ turns, encircling the center of the primary, can be neglected and the flux through the secondary taken to be $\phi$ also. If $\phi$ changes, the voltage induced in the secondary is

$$V = N_2 \frac{d\phi}{dt} \text{ volts.} \tag{3}$$

The mutual inductance of the solenoid, therefore, is

$$M = V \Big/ \frac{di}{dt} = 4\pi N_1 N_2 A(10)^{-7} \text{ henries.} \tag{4}$$

If the total resistance of the secondary circuit is $R$ ohms, the current through the galvanometer will be

$$i = \frac{V}{R} = \frac{N_2}{R}\frac{d\phi}{dt} \text{ amperes.} \tag{5}$$

When the current through the primary is reversed, the flux changes from $-\phi$ to $+\phi$ in a time $T$, so that the total charge

* See Art. 56.

passing through the galvanometer is

$$Q = \int_0^T i dt = \frac{N_2}{R}\int_{-\phi}^{+\phi} d\phi = \frac{2N_2}{R}\phi \text{ coulombs.}$$

Substituting for $\phi$ and using Eq. (4) give

$$Q = \frac{8\pi N_1 N_2 A I}{R}(10)^{-7} = \frac{2MI}{R} \text{ coulombs.} \qquad (6)$$

From the law of the ballistic galvanometer $Q = K_b d$ where $d$ is the deflection. Provided that $Q$ passes before the coil starts to move

$$K_b = \frac{8\pi N_1 N_2 A I}{Rd}(10)^{-7} = \frac{2MI}{Rd}. \qquad (7)$$

The assumptions of perfect flux linkage and instantaneous discharge introduce negligible errors with well made apparatus, so that this is a good method of calibrating a ballistic galvanometer. In fact, in most practical measurements the mutual inductance $M$ is determined experimentally instead of by calculation, by methods such as those to be discussed in Arts. 52, 53 and 55. When this is done the simpler value $2MI/Rd$ of Eq. (7) is used, and errors due to flux leakage are eliminated. Such a method also obviates the necessity of accurately measuring the cross sectional area of the primary winding, the determination of which presents some difficulty.

## EXPERIMENT 16

### CALIBRATION OF A BALLISTIC GALVANOMETER WITH A SOLENOIDAL INDUCTOR OR MUTUAL INDUCTANCE

Set up the apparatus as shown in Fig. 39, using either a solenoidal inductor, the mutual inductance of which may be calculated from Eq. (4), or a mutual inductance which has been calibrated against known standards. In order that the damping factor and the period of the galvanometer may be determined, it is necessary that it be underdamped. To insure this, pass a small current through the primary, so as to start the galvanometer swinging. If it does not swing well past the zero point on the return from the initial kick, it will be necessary to insert a series

resistance between it and the secondary coil. Adjust this resistance until the amplitudes of successive swings of the galvanometer do not decrease by more than twenty per cent. Determine the primary current necessary to give nearly full scale deflection of the galvanometer when the switch is reversed, and use about seven or eight smaller values of the current. Reverse the current in each case, reading the ammeter before and after reversal. Then bring the galvanometer to rest at zero and return the switch to its initial position, so as to obtain a deflection in the opposite direction.

Plot a curve of deflection in millimeters against the current in amperes. Draw an average straight line through the points obtained and determine $I/d$ in Eq. (7) from the slope of this line. Start the galvanometer swinging, measure its attenuation factor as in Exp. 14 and calculate the slope which the $d$ vs $I$ curve would have for zero damping. Inserting this value in Eq. (7) and using for $R$ the combined resistance of the secondary coil, the galvanometer and any added series resistance, calculate $K_b$. Measure the period of the galvanometer by setting it swinging and then *opening the circuit.* Since the beam is moving with a maximum velocity as it crosses the zero point, read the time between alternate transits of this point. From this time in seconds, calculate the figure of merit by Eq. (9) of Art. 47.

**51. The Capacitance Bridge.**

In addition to the methods outlined above, capacitances and inductances may be measured by means of bridge arrangements, similar to the Wheatstone bridge for resistance. Measurements of this type have the decided advantage that they are null methods; that is, they involve the elimination rather than the reading of a deflection, and therefore are not subject to errors of calibration in the galvanometer.

Consider the circuit in Fig. 40, in which $G$ is a ballistic galvanometer. If the key, $K$, is closed on the battery side, an instantaneous flow of current will take place, charging the condensers. After they have been charged, each will have acquired the potential of the battery, no current will flow, and the galvanometer will not deflect, since the points $A$, $B$ and $C$ will all be at the same potential. During the charging, however, the

larger capacitance will draw more current, and in general, the points $B$ and $C$ will be at different potentials. Suppose that conditions in the bridge are such that the galvanometer shows no kick during the charge, then these points remain at the same potential throughout the charging, and we have:

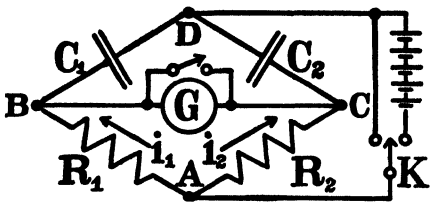


FIG. 40. The capacitance bridge.

$$i_1 R_1 = i_2 R_2 \tag{1}$$

and

$$\frac{q_1}{C_1} = \frac{q_2}{C_2}, \tag{2}$$

where the $i$'s and the $q$'s are *instantaneous* values of the current and charge respectively. But, by definition, the current is the time derivative of the charge, hence:

$$\frac{dq_1}{dt} R_1 = \frac{dq_2}{dt} R_2.$$

Integrating, we find that

$$\frac{q_1}{q_2} = \frac{R_2}{R_1}. \tag{3}$$

Combining (2) and (3):

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}, \tag{4}$$

which is the balance equation of the capacitance bridge. If the key is now thrown to the discharge position, short circuiting the bridge, the energy stored in the condensers will be dissipated in the resistances. The only change in the above derivation is that the current is now the negative derivative of the charge, and the same balance condition applies.

## EXPERIMENT 17

### MEASUREMENT OF CAPACITANCE WITH A BRIDGE

Connect the apparatus as shown, using a battery of about 20–40 volts. Measure the capacitance of the sections of a subdivided

condenser against a standard. For sensitivity of balance, it is well to have the two condensers of about the same capacitance, so, for a one microfarad subdivided condenser, a half microfarad standard will give very good results. With the battery in the circuit, the galvanometer, particularly if it is of high sensitivity, may show a tendency to assume a steady deflection after its first kick. This is due to the fact that the condensers are not perfect insulators, and the bridge may be unbalanced as a Wheatstone bridge. A small amount of practice, however, will enable the experimenter to distinguish between this deflection and the ballistic kick which he wishes to eliminate.

The resistances $R_1$ and $R_2$ should be kept to a value of several hundred ohms as a minimum and will be found to give best results if kept above 1000 ohms. Measure the capacitance of each section, then check against all the sections in series and parallel, as in Exp. 15.

**52. The Inductance Bridge.**

A similar arrangement to that of the last article may be used to measure self and mutual inductance. Such a bridge is shown

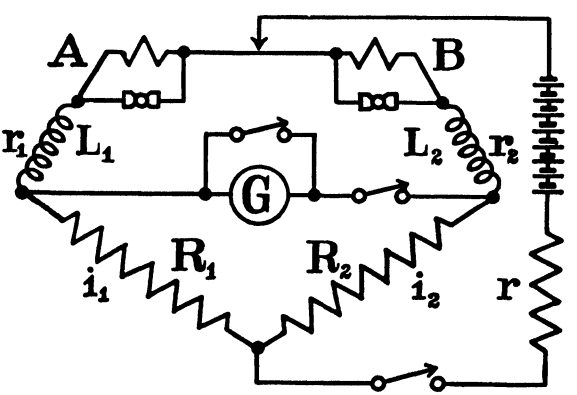


FIG. 41. The inductance bridge.

in Fig. 41. Since the inductances of this bridge have a finite resistance, the bridge must be balanced for steady currents. If $r_1$ and $r_2$ are the resistances of the inductance coils, including that part of the slide wire which is in series with each, the conditions

for a steady current balance are:

$$i_1 R_1 = i_2 R_2, \tag{1}$$

$$i_1 r_1 = i_2 r_2. \tag{2}$$

In addition to these conditions when the current is changing, that is, at the instant of opening or closing the key, the unsteady state condition must be satisfied:

$$i_1 r_1 + L_1 \frac{di_1}{dt} = i_2 r_2 + L_2 \frac{di_2}{dt}.$$

Combining this with Eq. (2):

$$L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt}, \tag{3}$$

and, eliminating the derivatives between (3) and (1):

$$\frac{L_1}{L_2} = \frac{R_1}{R_2}. \tag{4}$$

This is the balance equation of the inductance bridge.

## EXPERIMENT 18

### MEASUREMENT OF SELF AND MUTUAL INDUCTANCE OF TWO COILS

Set up the apparatus as shown in Fig. 41. For $L_2$ the unknown inductance will be used. This will consist of two coils, so arranged that one may slide inside the other. For $L_1$, a standard variable inductance is used. These are made in several forms, all of which have constant resistance and vary the inductance by the relative motion of two coils. Such a variable inductance is shown in Fig. 42. The portion $A$–$B$ of the bridge is a Kohlrausch slide wire, with constant resistances which may be plugged into the circuit at each end. A battery of 20–30 volts should be used.

First set the slide wire somewhere near the middle point and balance the bridge for steady current by varying $R_1$ and $R_2$ until the galvanometer shows no deflection. During this part of the work, keep the variable resistance $r$ at a value of 250 ohms or

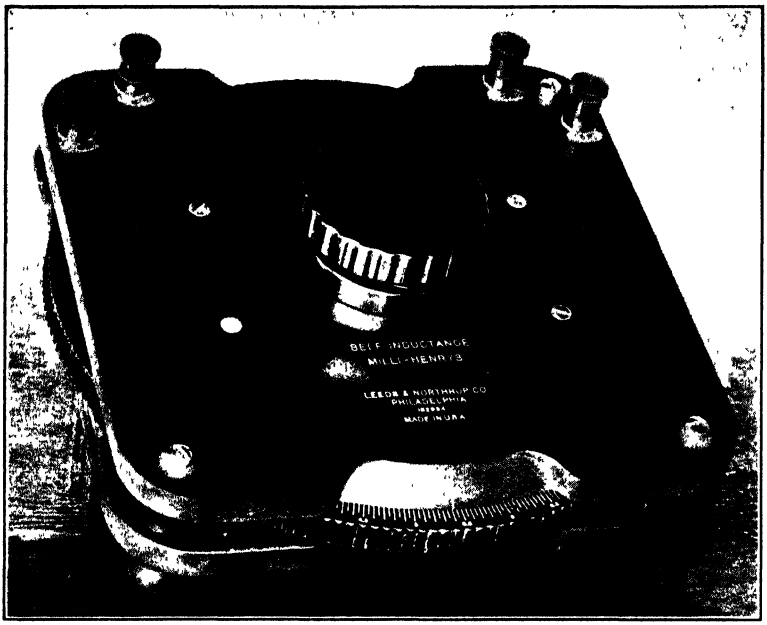


FIG. 42. The Brook's inductometer. (Courtesy of Leeds and Northrup Co.)

more, as the bridge cannot carry steadily the moderately heavy currents which the full voltage across it would supply. As the resistance balance must be made with some nicety, the final balance is obtained by varying the slide wire contact.

When the bridge has been balanced for steady current, decrease the resistance $r$ to zero. On closing or opening the key, the points $C$ and $D$ will not come to the same potential simultaneously, and the galvanometer will kick to the right or left. Eliminate this kick by adjusting $L_1$ until Eq. (4) is satisfied, without disturbing the resistance balance. Do not leave the key closed for more than a few seconds. If the balance point is out of the range of the standard variable inductance, it will be necessary to change the steady state balance. Thus, if $L_2$ is too high, the resistance $r_2$ must be increased. This may be accomplished by inserting the extra resistance available at $B$. Then balance the bridge as before, with steady current, and try again with $L_1$.

Measure the self inductance for the two coils separately, then join them in series and measure the total inductance of the combination. As the two coils act on each other equally, if the mutual inductance is $M$ and the self inductances are $L_a$ and $L_b$, the total inductance $L$ will be:

$$L = L_a + L_b \pm 2M,$$

depending on whether the mutual inductance aids or opposes the self inductance. To check this, reverse one of the coils and make a new determination. From the measured values of $L_a$ and $L_b$ calculate the mutual inductance from each of these values. How closely do the two values check?

**53. Measurements with a Commutator.**

The measurements outlined in the last few articles may be greatly facilitated by the use of a rotating commutator to produce an alternating or an interrupted direct current. For example, in the comparison of capacitances with a galvanometer, a rotating commutator may be used alternately to charge the condenser and discharge it through the galvanometer. If there are $n$ discharges per second, and the condenser of capacitance $C$ has a potential $V$ impressed across it on each charge, then the current through the galvanometer is:

$$CVn = Kd,$$

where $K$ is the sensitivity of the galvanometer and $d$ the deflection.

In a similar manner, a double commutator may be used with either the inductance or capacitance bridge. One commutator is used to connect the bridge to the battery, the other, mounted on the same shaft, connects the galvanometer. This arrangement is known as the secohmmeter. For detailed applications of these methods, the student is referred to the original papers on the subject.

**54. Comparison of Capacitances with an Electrometer.**

One of the most elegant methods of comparing an unknown condenser against a standard is by the use of a quadrant or string electrometer. All condensers used should have a leakage time large compared with the time between readings.

## EXPERIMENT 19

## COMPARISON OF CAPACITANCES WITH AN ELECTROMETER

Obtain a standard condenser of the same order of magnitude as the unknown condensers, whose capacitances are large compared with that of the electrometer and leads. Determine the capacitance by the first method described below. Then take a pair of air condensers, whose capacitances are comparable with that of the electrometer and leads, and compare them by the second method. In either case, set up the apparatus as shown in Fig. 43.

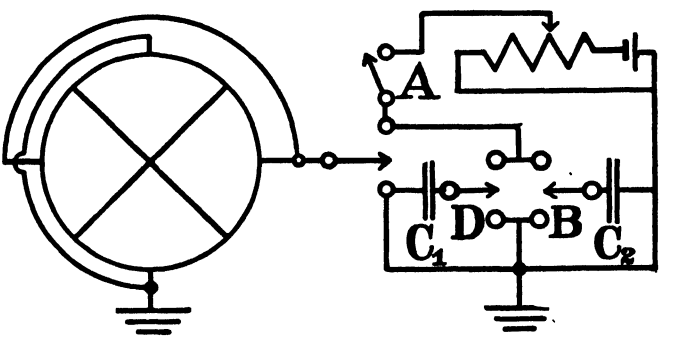


FIG. 43. Comparison of capacitances with an electrometer.

1. With switch $D$ up, earth both sides of $C_2$ using switch $B$. Close switch $A$, thus charging $C_1$ to a voltage $V_0$, measured by the electrometer deflection. Now open $A$ and reverse $B$, so that the original charge on $C_1$ is shared between $C_1$ and $C_2$, and read the new voltage, $V_1$. The charge on the condensers is

$$Q = C_1V_0 = (C_1 + C_2)V_1.$$

Solving for $C_2$, we have

$$C_2 = C_1(V_0 - V_1)/V_1. \qquad (1)$$

2. If it is suspected that the capacitance of the electrometer and leads may affect the result, the following procedure can be used. If $C$ is this additional capacitance, then the above equation

may be written more accurately,

$$C_2 = (C + C_1)(V_0 - V_1)/V_1.$$

After short-circuiting both condensers for a minute to remove residual charges, repeat the above operations, interchanging the manipulations of switches $B$ and $D$ and obtaining the final Voltage $V_2$. This gives:

$$C_1 = (C + C_2)(V_0 - V_2)/V_2.$$

Eliminating $C$ from these equations, gives:

$$C_2 = \frac{V_0 - V_1}{V_0 - V_2} C_1. \qquad (2)$$

**55. The Carey-Foster Method for Mutual Inductance.**

The inductance bridge may be used only for self inductance or for a combination of self and mutual inductance. When mutual inductance is required, the method of Exp. 18 becomes unnecessarily tedious and sometimes impossible. A method which measures mutual inductance directly against a known capacitance has been developed by Carey-Foster and is shown schematically in Fig. 44. The mutual inductance, $M$, of the two coils, $S$ and $P$, is desired. If the key is closed, a current will build up through the primary coil, inducing an E.M.F. in the secondary. A current will also flow through the condenser, charging it. If we let $i_1$ and $i_2$ be the currents in $R_1$ and $R_2$, respectively, and $i$ be that in the primary, we find, by an application of Kirchoff's laws to the circuit that

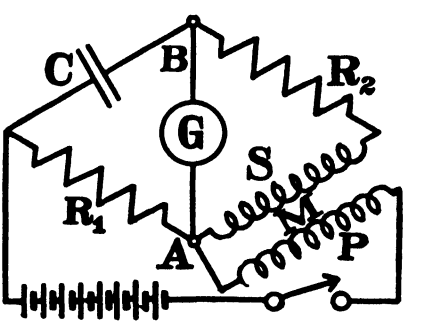


FIG. 44. The Carey-Foster method for mutual inductance.

$$M\frac{di}{dt} + (R_2 + r)i_2 + R_v(i_2 - i_1) + S\frac{di_2}{dt} = 0, \qquad (1)$$

where $S$ and $r$ are the self inductance and resistance of the sec-

ondary. Multiplying by $dt$ and integrating gives

$$Mi + (R_2 + r)\int i_2 dt + R_g\left(\int i_2 dt - \int i_1 dt\right) + Si_2 = 0. \quad (2)$$

The integrals are taken from the time when the switch is closed, that is when all currents are zero, as is the charge on the condenser, until equilibrium is established. At the upper limit of time no current flows in the condenser, hence its charge is equal to $CR_1 i$. At the same time $i_2 = 0$. If the circuit is so balanced that there is no deflection of the ballistic galvanometer, the total charge which has passed through it must be zero, and this is identical with the coefficient of $R_g$. Applying all these boundary conditions we have

$$Mi + CR_1 i(R_2 + r) = 0, \quad (3)$$

$$M = -CR_1(R_2 + r), \quad (4)$$

where $M$ is the mutual inductance in henries and $C$ is the capacitance in farads, while the resistances are in ohms.

## EXPERIMENT 20

## MEASUREMENT OF MUTUAL INDUCTANCE

Set up the apparatus as shown in Fig. 44. The direction of the induced current in $S$ must be that necessary to charge the condenser. To test this, open the circuit at $S$ and close the key, noting the direction of the galvanometer kick. Then open the circuit at $C$ and repeat. The two kicks should be in opposite directions. If they are not, interchange the terminals of either $P$ or $S$.

Use a variable laboratory inductance for $M$ and measure it against a standard condenser. Balance by adjusting $R_1$, $R_2$ and $C$. Since there are any number of combinations of these which will give a balance, get a preliminary balance and then determine the approximate values to give the same degree of accuracy to the determinations of each of the three quantities. Set $C$ and $R_1$ to your calculated values and balance again with $R_2$. Measure $r$ and calculate the mutual inductance. Take a number of readings over a scale of the mutual inductance and plot a calibration curve for it.

**56. The Magnetic Circuit.**

In the *electromagnetic system of units*, on which our definitions of the ampere and coulomb are based, the unit of magnetic field strength is the *oersted*, which is defined as *the field strength existing at a point where a force of one dyne exists on a unit magnetic pole.* The *unit magnetic pole*, in this same system, is defined as *a pole which will repel an exactly similar pole, placed one centimeter from it in vacuo, with a force of one dyne.* The *magnetic potential difference* or *magneto-motive force* around a circuit is obtained by integrating the field strength around the circuit, and its electromagnetic unit is the *gilbert*, equal to an *oersted-centimeter.*

If a magnetic field is due to a current in a closed circuit, the magnetic potential difference for a loop around the circuit is found to be proportional to the current in the circuit times the number of times that the conductor carrying it is surrounded, hence it is more convenient for our purposes to take as a unit of magneto-motive force the *ampere-turn.* The corresponding unit of magnetic field strength in the MKSC system is therefore the *ampere-turn per meter*, which is equal to $0.004\pi$ oersted. This definition results in the field strength near the middle of a long uniform solenoid, placed in a magnetically homogeneous medium, which carries a current of $I$ amperes and has $N_1$ turns per meter, being

$$H = N_1 I. \tag{1}$$

If we consider magnetic and electric potentials to be analogous, then *magnetic flux* corresponds to electric current and *magnetic reluctance* to electric resistance. Reluctance, like resistance, depends upon the material and varies inversely as the cross section $A$ and directly with the length $l$ of the magnetic circuit. If we denote by $\mu$ the constant of proportionality, which is a function of the material, the reluctance of a regularly shaped, homogeneous magnetic circuit is

$$R = l/\mu A,$$

and the magnetic flux through the circuit, if a uniform magnetic field $H$ exists along it, is $\mu HA$, so that the *flux density* is $\mu H$. For reasons extraneous to our present discussion it is convenient to

define $\mu_0$, the *permeability* of a vacuum, as having a numerical value of $4\pi/10^7$. For any other medium the permeability $\mu$ may be expressed as $\mu_r\mu_0$, where $\mu_r$ is the *relative permeability*. Hence the flux density at the center of the solenoid described above is

$$B = 4\pi\mu_r N_1 I/10^7. \qquad (2)$$

The unit of flux density obtained in this way is known as the *weber per square meter*, and the corresponding unit of flux is the *weber*.

**57. Magnetization and Hysteresis.**

The permeability of a ferro-magnetic material, such as iron, is not constant, but depends upon the field strength and the

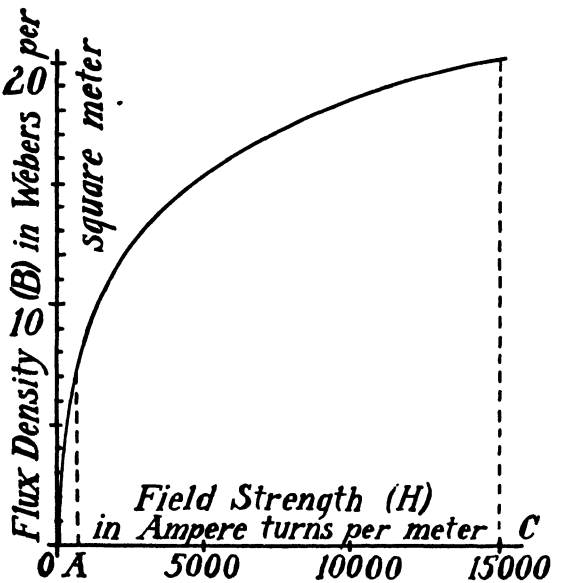


FIG. 45. Magnetization curve.

previous magnetic history of the material. A typical magnetization curve of an iron ring, for example, is shown in Fig. 45. For small values of field strength, from *O* to *A*, the flux density,

$B$, increases nearly linearly with the field, but for larger values, from $A$ to $C$, the slope of the curve approaches a constant value, for which $\Delta B/\Delta H = \mu_0$. The magnetic material is then said to be saturated. "Nonmagnetic" materials, such as air, do not show this effect, or, rather, are saturated at all values of the field strength.

Still another change of permeability is introduced when magnetized material is subjected to a cyclic magnetizing force. The effect of such a force, acting on a previously unmagnetized specimen, is shown in Fig. 46. As the strength increases from zero,

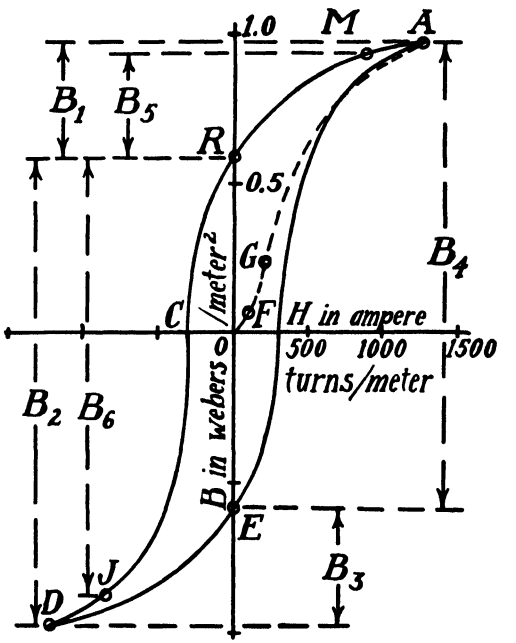


FIG. 46. Hysteresis loop.

the flux density follows the magnetization curve $OA$ but, if the magnetizing force is then decreased from this value, the flux does not drop along this curve, but along a higher curve $AR$. When the field has decreased to zero, the flux density still has the value $OR$, which is known as the *retentivity* of the specimen. If the field is then built up in the opposite direction, the flux follows

the curve *RCD* to a reversed maximum and then, on a reversal, returns through *E* to *A*.

The hysteresis loop of a magnetic substance is particularly interesting because a circuit is constantly put through this cycle when the magnetizing force is supplied by an alternating current, as in a transformer. In such a case, work is done and is converted into heat in the magnetic material. Suppose that a current $I$ is flowing through $N$ magnetizing turns, producing a flux $\phi$ in a closed homogeneous magnetic circuit of length $l$ meters. Now let this current be changed slightly during a time $dt$; then the work done is $dw = VIdt$ where $V$ is the voltage. But the back E.M.F. is that produced by the flux change

$$V = N\frac{d\phi}{dt} = NA\frac{dB}{dt} \text{ volts,} \tag{1}$$

where $B$ is the flux density and $A$ the cross-sectional area of the circuit. Also

$$H = \frac{NI}{l} \quad \text{or} \quad I = \frac{lH}{N} \text{ amperes.} \tag{2}$$

Substituting these values and letting $v$ be the volume, we have

$$dw = AlHdB = vHdB \text{ joules} \tag{3}$$

which is the work done if the circuit is, as we tacitly assumed, regular in shape. If $HdB$ is integrated over the entire cycle, we obtain the area in the loop. Hence the work done per cycle becomes

$$W = v \times \text{Area of hysteresis loop.} \tag{4}$$

Steinmetz carefully investigated the dependence of the area of the loop upon the maximum flux density, $B_m$ and found that the area for any particular specimen is given very closely, under a wide variety of conditions, by

$$W_i = KB_m^{1.6} \tag{5}$$

where $K$ is known as the Steinmetz coefficient of the specimen and $W_i$ is the work *per unit volume per cycle*. This Steinmetz equation is valid for frequencies which are sufficiently low so that

the "skin effect" is negligible and for values of $B_m$ between about 0.1 weber and 1 weber per square meter. Above and below these values of the flux density the exponent becomes greater than 1.6, and may rise as high as 3.0 for the extremely high values of flux density possible with some modern steels. The limitations on the frequency are not serious when ordinary commercial frequencies are used, but they completely invalidate the equation for audio or radio frequency work, in which cases the loop area increases with increasing frequency. *Within the range of its validity*, Eq. (5) allows the computation of the hysteresis loss at any flux density and any frequency, provided that the hysteresis loop has once been determined, since the loss per second is the loss per cycle times the frequency.

A number of methods for the determination of the shape of the magnetization and hysteresis curves are available. The most commonly used methods are those depending upon the change of the mutual inductance of two coils when they are placed upon a magnetic core, and are known as *ballistic measurements*, since they are made with the aid of a ballistic galvanometer.

As an example of ballistic methods, the Rowland ring determination will be treated here. For this measurement, the sample to be tested is made into the form of a ring, on which are wound two coils, a primary for magnetization and a secondary for the induced currents. Such a ring, connected for testing, is illustrated in Fig. 47, where $G$ is a ballistic galvanometer and the coils $N$ and $N_s$ are the primary and secondary, respectively, of a solenoidal inductor or other mutual inductance. The secondary coils of the inductance and the ring are permanently connected in series, so that the resistance of the galvanometer circuit is constant.

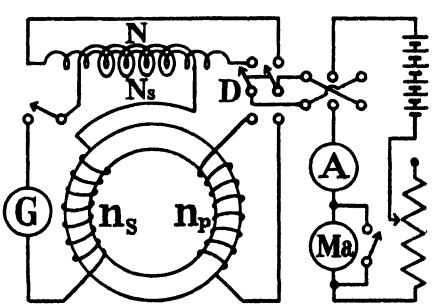

FIG. 47. The Rowland ring circuit.

If the switch $D$ is thrown to connect the primary of the inductance to the source of power, the circuit is identical with

that of Experiment 16, and the galvanometer may be calibrated exactly as in that experiment. Then, by Eq. (7) of Art. 50

$$K_b = \frac{2M}{R} \cdot \frac{I_0}{d_0} \tag{6}$$

where $M$ is the value of the calibrating mutual inductance in henries, $R$ is the combined resistance of the two secondaries and $I_0/d_0$ is the average slope of the current-deflection curve for the standardizing inductance. Now suppose that $D$ has been thrown downward, so that a current is flowing in the primary coil of the ring, producing a flux density $B$. If this current is suddenly changed, the flux is changed from $B$ to $B_1$, the change being $\Delta B$, and it may be shown, exactly as in Art. 50, that the quantity of electricity flowing is

$$Q = \frac{n_s A_r}{R} \Delta B = K_b d \tag{7}$$

where $n_s$ is the number of turns on the secondary coil, $A_r$ is the area of the ring and $d$ is the galvanometer deflection. Hence

$$\Delta B = \frac{2M}{n_s A_r} \cdot \frac{I_0}{d_0} \cdot d. \tag{8}$$

## EXPERIMENT 21

### MAGNETIZATION CURVES WITH THE ROWLAND RING

Set up the apparatus as shown in Fig. 47. *Am* is an ammeter and *MA* is a milliammeter used for measuring low values of current, *always shunted by a knife switch except when these values are actually being read.* Magnetic fields varying from about one hundred to ten thousand ampere turns per meter will be used.

Before attempting to determine the magnetization curve, the experimenter must thoroughly demagnetize the specimen. To do this, start with a current slightly greater than the largest which will be used, and carry the current through a number of reversals, throwing the switch about once each second, and simultaneously reduce the current until a value considerably less than the lowest value which will be used has been obtained. Now connect the mutual inductance and calibrate the galvanom-

eter as in Exp. 16. Do not correct for attenuation, since the damping is the same for the calibration and measurement.

Again connect the primary of the Rowland ring to the circuit and *raise* the current to the value calculated for 100 ampere turns per meter. This will place the specimen at some point such as $F$ on the magnetization curve (Fig. 46). Reverse the current. If the original flux was $B$, the flux will now be $-B$, and from Eq. (8)

$$B = \frac{M}{n_s A_r} \cdot \frac{I_0}{d_0} \cdot d. \tag{9}$$

Again reverse the current, obtaining a second deflection which should be, within the limits of measurement, equal and opposite to the first. Then increase the current to a slightly greater value, and repeat, determining the point $G$ on the curve. *Never decrease the current, even by a small amount,* as a decrease of field strength will cause the specimen to start following a hysteresis loop and will invalidate all the results. Take about fifteen or twenty points in this manner, choosing the points closer together on the lower part of the curve where the curvature is greatest. Plot a curve of flux density in webers per square meter against field strength in ampere turns per meter. Compute $\mu_r$ and plot this against $H$ on the same sheet.

## EXPERIMENT 22

### HYSTERESIS CURVES WITH THE ROWLAND RING

In determining the hysteresis curve, the material is carried through the magnetic cycle a number of times, taking two new points for each half cycle. First set the current at a predetermined value, say at the value necessary to give 1000 to 2000 ampere turns per meter. Then reverse the current a number of times, to make certain that the specimen is following the proper hysteresis loop for this field strength. The last reversal will leave the material in the condition denoted by the point $A$ of Fig. 46. Open the main switch, dropping the field to zero, and carrying the specimen to the point $R$ on the curve. The change in flux determined from Eq. (8) will be that shown as $B_1$. With the main switch open, throw the reversing switch to the other position, then close the main switch, getting $B_2$, open the main

switch for $B_3$, reverse and then close, determining $B_4$ and returning the specimen to the original point $A$.

Now decrease the current to about 0.9 of the original value, placing the material at point $M$ on the curve, then open the circuit, obtaining $B_5$. Reverse and close circuit, getting the flux change $B_6$ and leaving the sample at point $J$. It is now necessary to *increase the current to the original value*, so as to carry the material through point $D$ and keep it on the proper cycle before decreasing the current to about 0.8 of this value and getting at point $K$. Starting from this point, the procedure is repeated until the entire loop is determined. Always be careful to pass through the peaks of the loop by increasing and then decreasing the current.

Plot the hysteresis loop, placing the origin half-way between $R$ and $E$. Measure the area with a planimeter, convert to joules per cubic meter, and compute the Steinmetz coefficient of the sample.

**58. Commercial Instruments for Measurement of Magnetic Properties.**

While the Rowland ring measurements outlined in the last section are identical in principle with most of the methods used for the determination of the magnetic properties of ferromagnetic materials, they are not sufficiently rapid or convenient to be used when a large number of determinations are to be made. Since it is generally too troublesome to form the material to be tested into a ring, these instruments generally make use of a *permeameter* which consists of a yoke of magnetic material against the open ends of which a bar of the material under test can be rigidly clamped. An instrument of this sort is shown in Fig. 48, and the end of the bar, held in place by the clamp, may be seen just to the left of the middle of the picture. The magnetomotive force is generally supplied by two magnetizing windings, a main coil wound over the main bar and an auxiliary coil wound over a portion of the yoke. At first glance this apparatus would seem to be subject to the usual error present in magnetic apparatus not of ring form, namely the fact that poles are set up and that the flux throughout the circuit is not uniform. This difficulty

is avoided in one of two ways, depending on the particular design of the apparatus:

1. A set of distributed compensating coils are mounted to supply additional magnetomotive force to overcome losses of flux at the poles. When this device is used, a number of secondary windings are provided at crucial locations, and the currents in the various compensating coils may be adjusted until the induction in these secondaries indicates a uniform flux throughout the circuit.

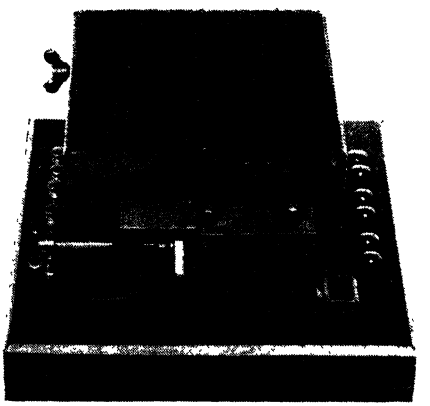

FIG. 48. Permeameter. (Courtesy of General Electric Co.)

2. The magnetic field strength, instead of being calculated from the current and the number of turns on the primary, may be directly measured by means of an auxiliary coil, wound on material of unit relative permeability and mounted inside the main magnetizing coil but not surrounding the test bar. If the effective cross sectional area of this coil is $A_H$ and it contains $N_H$ turns, the charge which will pass through it when the magnetic field strength is changed by $2H$ will be:

$$Q_H = \frac{8\pi N_H A_H H}{R(10)^7}. \tag{1}$$

The constant $N_H A_H/R$ may be obtained either by direct measurement of its factors or by calibrating it by comparison with a mutual inductance, as in Exp. 21. When this arrangement is used, it is necessary that the characteristics of the yoke material be known so that the current in the auxiliary coil can be so adjusted as to just maintain the proper flux in the yoke, without contributing to the magnetization of the test sample.

With the proper adjustments made to either of these types of apparatus, the measurement is essentially the same as that with

a Rowland ring. One complication, however, arises if different cross sectional areas of test samples are to be used, since, if the area of the sample is less than that of the $B$ coil, the total flux measured will represent the sum of the flux through the material and that through the surrounding air. Hence if $A$ is the cross sectional area of the sample and $A'$ is that of the coil, the actual flux density of the metal is:

$$B = \frac{\phi - H(A' - A)\mu_0}{A}, \tag{2}$$

where $\phi$ is the measured flux. For low permeability materials (i.e. $H \approx B/\mu_0$), this correction becomes very serious, and must either be computed or corrected for by means of an auxiliary secondary coil, wound on a non-magnetic core inside the magnetizing coil and connected in opposition to the original secondary.

To avoid the labor of computation, commercial instruments are generally supplied with a *galvanometer control box*, by means of which resistances may be placed either in series or parallel with the galvanometer to lower its sensitivity. The particular combination of series and parallel resistances is determined by the desired damping characteristics of the instrument, and the sensitivity is adjusted until the constant factor $2MI_0/n_sA_rd_0$ in Eq. (8) of Art. 57 has a value which in some multiple of ten, say 0.01 weber per square meter per mm. A simple mental calculation then serves to convert deflections directly into flux densities. For more detailed consideration of the various permeameter arrangements the student is referred to the manufacturers' instructions for particular instruments.

Very frequently in commercial testing one is interested not in the complete magnetic characteristics of the sample, but only in the power consumption when a cyclic magnetizing field is applied. This power is used in overcoming hysteresis, as explained in Art. 57, and in heating the metal by means of *induced eddy currents*. For its direct measurement various modifications of the *Epstein core loss* apparatus are used. In this equipment the simple Rowland ring is replaced by a square frame, along each side of which is placed a bar (usually made of laminated sections) of the test material. Four magnetizing coils and four

secondaries, each set being connected in series, are used, so that complete symmetry is obtained, and the equations deduced for the Rowland ring apply. If a given mass of material is always used in the samples, we find from Eqs. (3) and (4) of Art. 57 that the hysteresis loss per second per unit mass is:

$$P_h = C\int HdB \tag{3}$$

where $C$ is a constant proportional to the mass of the sample and the integral is taken over the number of cycles occurring per second. By means of Eqs. (1) and (2) of the same section, it can be shown that:

$$P_h = C'\int IVdt \tag{4}$$

where $I$ is the instantaneous primary current, $V$ is the instantaneous secondary electromotive force and $C'$ is a constant of the apparatus. But if a wattmeter is connected with its current coils in series with the primary windings and its potential coils across the secondary, it reads the value of the integral directly and therefore allows a simple computation of the loss. Since we have not yet discussed alternating currents, we shall not attempt to prove here that the eddy current losses are likewise measured by this arrangement, but the student familiar with A.C. circuit theory can easily show, by consideration of the phase relationships of the primary current and secondary potential difference, that this will be so. A typical commercial set up for the core loss measurements is shown in Fig. 49.

**59. Measurement of Flux Density in Air.**

Very frequently it is desirable to make a determination of the magnetic flux density in the interior of a solenoid or in an air gap in a magnetic circuit which consists primarily of ferromagnetic materials. A number of methods are available for this, the most common being ballistic measurements. The first of these uses the *flip coil*, which is merely a secondary winding arranged to be rotated rapidly about an axis perpendicular to the magnetic flux. If the current in the primary winding (and therefore the flux) is kept constant, and such a coil is suddenly

turned through 180°, the direction of the flux through the coil is reversed and a change of flux of $2\phi$ is produced exactly as with the Rowland ring. The flip coil may also be applied to a circuit whose magnetic properties are known except in one section, in which the unknown bar sample is fastened. The flip coil encircles this section. If the flux is $\phi$ and this bar is suddenly

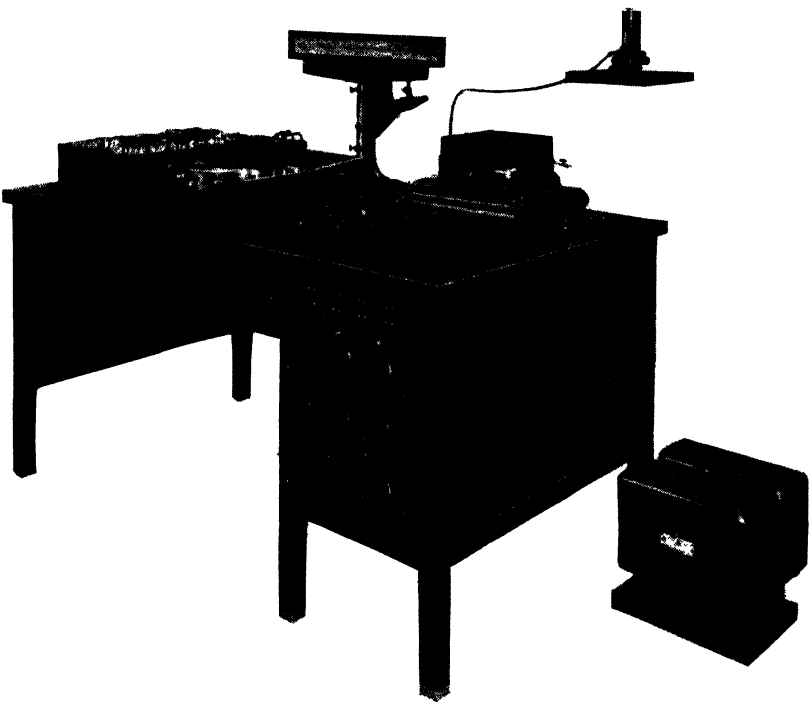

FIG. 49. Core loss apparatus. (Courtesy of General Electric Co.)

withdrawn, allowing the flip coil to drop, a change of flux $\phi$ occurs, which may be measured, as above, with a ballistic galvanometer. From the known constants of the apparatus and this change in flux, the permeability of the specimen may be determined by Rowland's law.

The second apparatus designed primarily for air gap measurements is the *Grassot fluxmeter*, which is a ballistic galvanometer, heavily damped and with no restoring forces. A *search coil*,

which is merely a portable secondary, is connected in series with this galvanometer. This coil is inserted normally in the air gap, then suddenly removed. The deflection of the instrument is proportional to the quantity of electricity generated and, therefore, to the flux cut by the coil.

In addition to ballistic methods, various indirect means may be used to measure magnetization in air. Most metals, and particularly bismuth, experience a change in resistance in the magnetic field, due to the Hall effect, and a bismuth spiral, connected to a Wheatstone bridge, is sometimes used as a secondary instrument to measure flux density, or rather magnetic field strength. The motion of a magnetic liquid has also been used as a secondary method.

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# CHAPTER 6

## THERMIONIC AMPLIFIERS

**60. Thermionic Emission.**

The growth of modern physics dates from the time of the conception of the electrical theory of matter. According to this theory, the chief atomic forces are electrical in nature. There is experimental evidence to show that the normal atom consists of a positively charged nucleus, in which nearly the whole mass of the atom resides, surrounded by a sufficient number of small negative charges, known as *electrons*, to make the whole atom neutral. The smallest known positive charge is carried by the nucleus of the hydrogen atom, which is called a *proton*. The charges on the electron and proton are equal in magnitude to a high order of precision. At present, it seems quite probable that all nuclei are built up of protons and *neutrons*, the latter being elementary particles having essentially the same mass as the proton, but no electrical charge.

For our purpose the electron may be taken as a constant point charge of negative electricity. This charge has been carefully evaluated by direct measurement, by X-ray studies and other means, as $1.600\ (\pm 0.002) \times 10^{-19}$ coulombs. Other measurements have shown the mass of the electron at rest to be $0.910\ (\pm 0.001) \times 10^{-30}$ kilograms. The deviation from this value caused by the motion of the electron is negligible unless its velocity approaches that of light and will be neglected in our future considerations.

An electric current may consist of a movement of electrons, of positively or negatively charged atoms of molecules, or of combinations of these; hence, from the definition of current, the *current density* is

$$i = nev, \tag{1}$$

where $n$ is the number of charged particles per unit volume, $e$ is the charge on each, and $v$ is the average velocity. Suppose that an electron starts from rest at some point in an electric field

where the potential is $V_1$ and falls, under the influence of the field, to some point where the potential is $V_2$. From the definition, Art. 30, of the potential difference between two points as the work required to take a unit charge from one point to the other, it follows that the loss in potential energy of the electron is $e(V_2 - V_1)$. If the electron has not transferred any of its energy to its surroundings, by the process of collision or otherwise, the law of conservation of energy shows that the loss in potential energy is balanced by the gain in kinetic energy so that we have

$$mv^2/2 = e(V_2 - V_1). \tag{2}$$

Since the kinetic energy is proportional to the difference of potential through which the electron has fallen, it is possible, and frequently convenient, to express this energy in "*equivalent volts*" or "*electron volts.*"

In this chapter we shall deal chiefly with electrons liberated by the process known as *thermionic emission.* This term refers to the property possessed by many substances, of which tungsten, thorium, and the alkaline earth oxides (when coated on high melting metals) are important examples, of ejecting electrons from their surfaces when heated to sufficiently high temperatures. This phenomenon can be considered, for convenience, as analogous to the evaporation of molecules from the surface of a liquid. A negative potential gradient away from the metal is then equivalent to a pressure over the evaporating liquid, and tends to decrease the difference between the rate at which the electrons leave and the rate at which they re-enter the surface, which is the net current flowing from the metal. Except at very high fields, the emission itself remains constant. The analogy also holds true to the extent that the electrons come off with a velocity which has a random distribution as to direction and a probability distribution about a certain mean velocity which is dependent upon the temperature. As one would expect from this analogy, the rate of emission of electrons is also a function of the temperature.

Richardson has shown that, if the electrons are removed from the vicinity of the metal (by a positive field) as fast as they are emitted, the *saturation current* per square centimeter of emitting

surface is

$$I = AT^2 e^{-b/T}, \tag{3}$$

where $T$ is the absolute temperature, $A$ is a universal constant, and $b$ is dependent on the nature of the metal and the temperature.

**61. The Two Electrode Thermionic Rectifier.**

The first practical application which was made of the thermal emission of electrons was the two electrode vacuum tube. The electron source, or *cathode*, of such a tube consists of a surface of good emitting material which may be heated electrically. In the older tubes this cathode was generally a tungsten wire (sometimes containing thorium) which could be heated to incandescence by a current passing through the wire itself, but in many modern

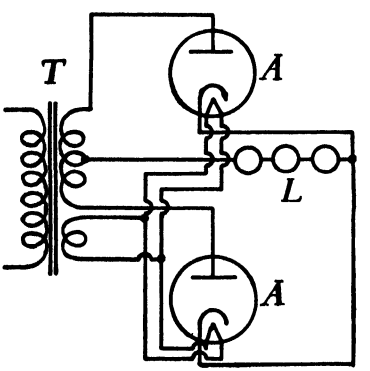


FIG. 50. Full wave rectifier.

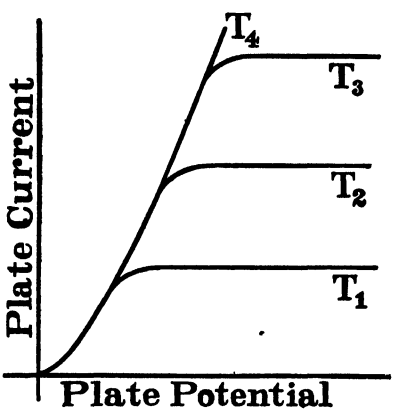


FIG. 51. Characteristics of the two element tube.

tubes the emitting surface is a cylinder of nickel or alloy, coated with alkaline earth or rare earth oxides, which is heated by a tungsten spiral enclosed in and electrically insulated from it. In addition the tube contains an *anode* which is kept at a potential considerably above that of the cathode. Both *electrodes* are enclosed in a metal or glass container in which a high vacuum is maintained. Due to the fact that the cathode is a source of electrons, a positive current may flow from anode to cathode

while, because of the thorough evacuation, no current can flow in the opposite direction. Such a tube, therefore, can be used as a *rectifier* to convert alternating current to direct current. An arrangement for obtaining a pulsating direct current is shown in Fig. 50, where $T$ is a transformer supplying the alternating current, $A$–$A$ are two element tubes, and $L$ is the D.C. load.

The amount of current which such a tube can deliver is limited by two factors. In the first place, it cannot exceed the total emission of the cathode at the running temperature, as given by Eq. (3) of the previous article. Fig. 51 shows a set of curves obtained by varying the potential applied between the cathode and the anode (*or plate*) at various cathode temperatures ($T_4 > T_3 > T_2 > T_1$).

It may be noticed that the plate current does not depend upon the temperature of the cathode at low voltages, but at higher voltages all the electrons which the filament is emitting are removed from its vicinity, and the curve flattens to the saturation value, the magnitude of which is given by the thermal emission equation. If curves of plate current vs. cathode temperature are plotted at various voltages, a similar set of curves will be obtained, and, for each potential, will show a definite maximum value of current. This is due to the second factor, the *space charge*.

Consider a tube which is operating at a high filament temperature and a low plate voltage. The electrons emitted from the cathode will be accelerated toward the anode, but since they have a finite velocity, there will be, at equilibrium, a certain definite number in the space between the two elements. This *electron gas* will be most dense near the filament, where the electrons have not fallen through sufficient potential to acquire a great velocity. This cloud of negative charges will reduce the potential gradient in its vicinity, until, near the cathode, the gradient may actually be slightly negative. In this case, any electrons which reach the plate must have sufficient energy of emission to shoot them through this negative gradient, into the region of positive gradient. It was by consideration of these factors that Child showed that $I \propto V^{3/2}$. As an example of his results, we shall give the formula for the maximum current, per

square centimeter of emitting surface, between a large plane cathode and a similar parallel anode, at a distance $d$ cm., and with a voltage $V$ across the two:

$$I = 2.33(10)^{-3} \frac{V^{3/2}}{d^2} \text{ milliamperes.} \qquad (1)$$

As can be seen from Eq. (1), the current which can be handled by a two element vacuum tube can be made large, assuming that the cathode gives sufficient emission, by increasing the voltage across the tube or by decreasing the distance between the cathode and plate. The latter method is limited by the necessities of mechanical construction and heat dissipation, while the former has the disadvantage of giving a low efficiency tube, as a high voltage drop within the tube means a large power loss. In order to overcome these difficulties, *gas* or *vapor filled rectifiers* are frequently built. If the space between the cathode and plate contains atoms of a gas, some of the electrons will strike these atoms and, if they have sufficient energy at the time, they may *ionize*, or remove one or more electrons from the atom, leaving a positively charged ion behind. Due to the fact that the mass of these ions is much greater than that of an electron, they will acquire less velocity in the field, as indicated by Eq. (2), Art. 60, and so will contribute little to the current. But, also, because they move slowly they will cause a large positive space charge, which can be made, by proper design of the tube, just to balance the negative space charge of the electrons. These gas filled rectifiers, therefore, have only one limitation on their current, that of the emission of the cathode, and are usually more efficient than the vacuum tube types. The gas need be only at a low pressure, and must be of such a nature that it does not attack the cathode surface at the high operating temperature, hence either the noble gases or mercury vapor are generally used.

**62. The Three Element Tube.**

The usefulness of the vacuum tube described in the previous article may be greatly extended by the addition of a third element, a *grid* of fine wires, between the cathode and the plate. We shall consider the simple case with a plane cathode and with the grid

and plate also plane and parallel to it. (We shall make the assumption that the grid, while permeable to electrons, acts electrically like a plane conducting surface. This is not accurately true, but will simplify the considerations, and give qualitatively correct results.) Now suppose that the grid potential is lowered below that of the equipotential plane which normally coincides with it. This will decrease the gradient at the cathode and will consequently decrease the space current flowing from the latter.

The utility of this arrangement is due to two facts:

1. A small change in grid potential will produce the same change in current as a much larger change in plate potential because, due to the smaller distance between cathode and grid, it will produce a proportionately large change of gradient at the former, where the space charge is most appreciable.

2. Since the grid wires occupy only a small part of the area of the plane which we have used to represent them, most of the electrons, even when the grid is positive, will pass through the grid to the plate, and the plate current changes in approximately the same ratio as the cathode current. This is even better in the usual operating case, when the grid is negative with respect to the cathode. In this case only a very few high energy electrons can strike the grid. If the tube is well constructed, so that leakage over the glass surfaces is small and is well evacuated, so that few positive ions will be present to be attracted to the grid, *only a very small current will flow in the grid circuit* and, therefore, practically no power is dissipated in controlling the tube.

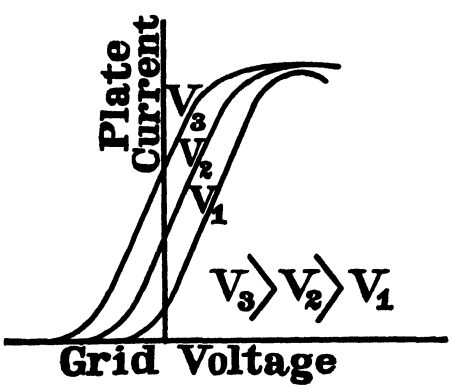


FIG. 52. Characteristics of the three element tube. $V_1$, $V_2$ and $V_3$ refer to plate voltages.

Fig. 52 shows a set of plate current vs. grid potential curves for a three element tube operating at constant cathode tempera-

ture. The decrease in plate current beyond the saturation point is due to the fact that the grid, as it becomes more positive, draws a larger portion of the cathode current, thus allowing less to reach the plate.

## EXPERIMENT 23

### CHARACTERISTIC CURVES OF A THREE ELEMENT VACUUM TUBE

Set up the apparatus shown in Fig. 53 where $B$ is a battery of cells having a voltage equal to or greater than the rated plate voltage on the tube and $C$ is a bank of dry cells. $R_B$ must be a very high resistance, since the battery $B$ is not designed to deliver

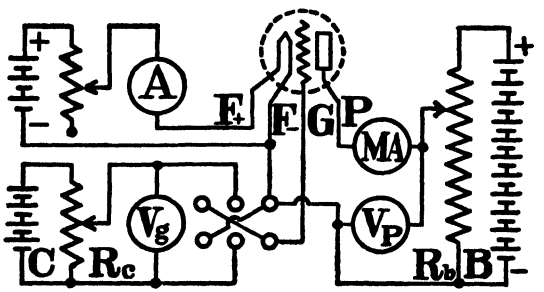


FIG. 53. Characteristic curves of the three element vacuum tube.

an appreciable current. For $MA$ it is advisable to use a milliammeter with several scales, so that the lower parts of the characteristic curves can be determined with some accuracy.

Start with the rated maximum plate voltage across $V_p$ and with the rated current in the filament. Make the grid negative with respect to the filament and increase $V_g$ until the plate current has just dropped to zero, as indicated by the most sensitive scale of the milliammeter. Take this as the first point on your curve and take about five or six points at values of $V_g$ about equally spaced between this value and zero, holding the plate voltage and the filament current constant. Repeat for six plate potentials, about equally spaced between the maximum value and zero. Plot your six curves of plate current against grid potential on a single sheet. On a second sheet, plot six curves of

plate current against plate potential, each at constant grid potential. Read the points for these curves from your first set of curves. These two families of curves are known as the *transfer characteristic curves* and the *plate characteristic curves* of the tube, respectively.

**63. Plate Resistance—Amplification Factor—Transconductance.**

The resistances which were considered in Chapter II were usually measured by an application of Ohm's law. In a vacuum tube, however, the current is not proportional to the voltage, and a determination of the resistance in this way would give a different value for each variation of the operating conditions. Moreover, in dealing with vacuum tubes, we are usually interested in the change of current which results when a small change is made in the plate voltage; hence it becomes convenient to define the *plate resistance* as the derivative of the plate potential with respect to the plate current, i.e.:

$$r_p = \frac{\partial V_p}{\partial I_p}. \tag{1}$$

But this is merely the reciprocal slope of the plate characteristic curves; hence we have, directly, one method of measuring the plate resistance of a tube.

In practice, the plate potential is usually kept nearly constant and the variations are superimposed on the grid potential, so it is desirable that we should be able to give some constant relating the plate current to the variations of the latter quantity. Consequently the *amplification factor* is defined as the negative ratio of a small change in plate voltage to the change in grid voltage which is necessary to compensate exactly for it. Therefore we have:

$$\mu = \left( -\frac{\partial V_p}{\partial V_g} \right)_{I_p = C} \tag{2}$$

and, combining with Eq. (1),

$$g_m = \frac{\mu}{r_p} = \frac{\partial I_p}{\partial V_g}. \tag{3}$$

The ratio $g_m$ is known as the *transconductance* (or *mutual conductance*) of the tube, and is the slope of the transfer characteristic curves. Consequently, if we have previously determined the plate resistance, we can find $\mu$ from these curves. It is of interest that, due to the similarity of the effects of plate and grid voltage on the current, the amplification factor is, for most tubes, essentially a constant (i.e. independent of $I_p$, $V_p$ and $V_g$), depending only on the geometry of the tube. A few specially designed tubes, the so-called super-control amplifiers, are exceptions to this rule.

The characteristics of the three element tube, determined by the methods of the last two paragraphs, are known as the *static characteristics*. Since tubes are generally used with variable potentials imposed on the grid, it is frequently better to determine the characteristics under similar conditions, and a method of doing so has been devised by J. H. Miller. The circuit for this determination is shown in Fig. 54, where $M$ is an audio frequency oscillator * and $H$ is a set of headphones used for detecting the balance condition. $L$ is a low resistance choke, used to by-pass most of the direct current part of the plate current around the headphones, and so to maintain the cathode practically at the potential of the point $O$. The filament current, plate potential and grid potential are first set to the operating values. A fixed current will then flow in the plate circuit, but will not be indicated by the phones, since they respond only to changes in the current through them.

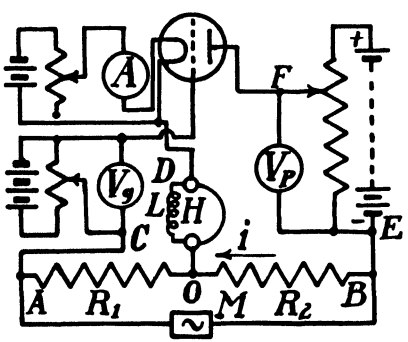


FIG. 54. The Miller bridge for amplification factor.

Now let the tube be operating at plate and grid potentials $V_p$ and $V_g$ respectively, and let the plate current be $I_p$. If the oscillator is started, it will produce a variable current $i$ in the

* See Arts. 87 and 94.

resistances. In general, this will vary the plate current and, consequently, will produce a hum in the phones. There will be, however, one relative value of $R_1$ and $R_2$ for which this hum will disappear. In this case, no current is branched from the circuit at $O$, as it would be indicated by the phones. Therefore, the same current must flow through $R_1$ and $R_2$.

Suppose that this balance has been obtained, and that the current through $R_1$ and $R_2$ is changing at the rate $di/dt$. This will cause the potential on the plate to vary at the rate $R_2(di/dt)$ (since there is no change of current and consequently no voltage drop from $B$ to the plate). The rate of change in plate current due to this is

$$\frac{dI_1}{dt} = \frac{1}{r}\frac{dV_p}{dt} = \frac{R_2}{r}\frac{di}{dt}. \tag{4}$$

At the same time, the grid potential is changing at the rate $R_1(di/dt)$ in the opposite direction, and this produces a plate current change

$$\frac{dI_2}{dt} = \frac{\mu dV_g}{rdt} = -\frac{\mu}{r}R_1\frac{di}{dt}. \tag{5}$$

But the total change of plate current is zero, since any change would be indicated by the phones; hence

$$\frac{dI_1}{dt} + \frac{dI_2}{dt} = \frac{R_2}{r_p}\frac{di}{dt} - \frac{\mu R_1}{r_p}\frac{di}{dt} = 0,$$

$$\mu = \frac{R_2}{R_1}. \tag{6}$$

## EXPERIMENT 24

### STATIC AND DYNAMIC CHARACTERISTICS OF A TRIODE

Set up the apparatus as shown in Fig. 54. Set the plate volts to the rated value and use about six or eight values of grid potential, between the negative cut off value and zero. At each value determine the amplification factor. In order that the voltages at the plate and grid may be essentially those read by the voltmeters, it will be necessary to have $R_2$ negligibly small

compared with the resistance of the plate voltmeter. Therefore the value of this resistance should be just great enough to read the amplification factor with the desired accuracy. If two significant figures are wanted, $R_2$ may be kept between ten and one hundred ohms if it can be read in one ohm steps. For most triodes this would mean a value of five to ten ohms for $R_1$. Using an A.C. voltmeter,* measure the voltage of the oscillator. $R_1/(R_1 + R_2)$ times this voltage will give the *signal voltage* which is applied to the grid.

If a resistance $R$ is inserted between the points $A$ and $C$ in Fig. 54, and $C$ and $D$ are connected by a short wire, the circuit will be that of a Wheatstone bridge, of which three arms are $R_1$, $R_2$ and $R$, while the fourth consists of $r_p$ in series with the low resistance between the points $E$ and $F$ and which responds only to changes of current. Using it in the ordinary way, therefore, the plate resistance may be measured for the same points as were used for the amplification factor. Again keep $R_1$ and $R_2$ small, and adjust $R$ to approximately the plate resistance.

Plot curves of $\mu$ and $r_p$ against grid voltage for the rated plate voltage, specifying on the sheet the value of the signal voltage used. Now determine the slopes of the curves obtained in Exp. 23 for this tube and show, on the same sheet, the values of $\mu$ and $r_p$ as determined from the static characteristics. Note any discrepancies between the two methods and try to explain these by reference to the transfer curves of Exp. 23, taking into account the fact that the maximum variation of the grid voltage is about 1.4 times the recorded signal voltage.†

**64. Triode Direct Current Amplifiers.**

The most widespread use of the three element vacuum tube is as an *amplifier*, in which it is used to control large amounts of energy in the plate circuit with the expenditure of only small amounts of energy in the grid circuit. For example, amplifiers may be used in measuring the exceedingly feeble currents passing through a photo-electric cell by having these currents control

* See Chap. VII.
† See Art. 73.

larger currents which are always proportional to the original. Uses of this sort are found whenever it is desired to measure currents or voltages too small to be read with precision on the instruments available. Amplifiers may be used, for example, when currents of the order of those measured by an electrometer are to be measured by a galvanometer, or when currents small enough to demand ordinarily the latter instrument are to be measured with a portable meter. They therefore enable a given set of instruments to cover a wider range of uses and allow a more rugged and simple instrument to be used for a given measurement than would otherwise be possible.

The principle of all such amplifiers is extremely simple, the unknown potential being impressed on the grid of a three element tube and causing a measurable variation of plate current. It will be seen that the amplifier is essentially a potential measuring device, but it may be used for current measurements in exactly the same way that the electrometer is used:

1. By passing the unknown current through a known resistance and measuring the potential drop produced across it.

2. By allowing the current to build up a charge on the condenser formed by the filament and grid, and by observing the rate of potential difference increase across this condenser.

Just as the choice of the particular galvanometer depends upon whether a high voltage sensitivity or a high current sensitivity is most desired, so the choice of an amplifier tube and its auxiliary circuit depends on the same factors. If a high voltage sensitivity is desired, and if the source being measured can supply the necessary output, a low resistance may be connected between the cathode and the grid, in order to give stability of operation, and a tube of high transconductance must be used. On the other hand, for high current sensitivity as high a value of the cathode-grid resistance as possible becomes necessary. For this purpose it is often possible to use a tube of very low amplification factor (even less than unity in some cases) if its construction is such as to allow a high resistance in this part of the circuit, and yet to obtain very high sensitivity.

A simple set up for the measurement of currents ordinarily in the range of a galvanometer, involving the first method suggested

above, is shown in Fig. 55. The unknown current enters and leaves by the terminals $X$–$X$, and causes a voltage change in the biasing potential on the grid, the main part of which is produced by the battery $C$. The plate current is read by the portable

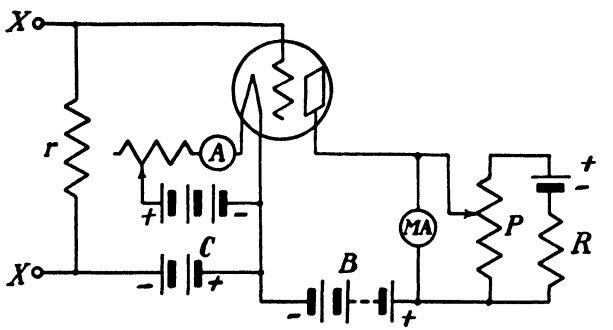


FIG. 55. Three element tube used as a D.C. amplifier — Deflection method.

microammeter $MA$. The potentiometer composed of the resistance $P$ and the high resistance $R$ is used to allow an increase in the sensitivity of the instrument. By use of this potentiometer, when no current is flowing in $r$, the meter may be adjusted to read zero, and only the change in current produced by the unknown is read on the meter. Without this device, the meter would need sufficient range to measure the entire plate current, which may be many times the value of the variations. The meter, of course, should be provided with a series of shunts so that it may be used on a low sensitivity scale during the first adjustments of the potentiometer.

There is a very definite lower value of current which may be measured by this method. The limitation is due to the fact that the grid is never perfectly insulated from the cathode; hence $r$ must always be small compared with this internal resistance, unless it is accurately known and considered as in parallel with $r$. But, even if this is done, it is obvious that the effective value of the resistance can never be made greater than the grid-cathode resistance alone.

While the second method, that of charging, or "rate of drift," may be adapted to three element tubes, it is more often applied

to the special four element amplifying tubes which will be discussed in the next article.

## EXPERIMENT 25

### CALIBRATION OF A D.C. AMPLIFIER

Set up the apparatus as in Fig. 55, using voltages as specified by the instructor for the batteries *B* and *C*. In general these potentials and the cathode heating current will not be the same as those recommended for the operation of the tube used, but will be chosen in such a way (usually low cathode temperature and strongly negative grid) that the cathode-grid resistance is higher than in ordinary operation. Replace *r* by a potentiometer, voltmeter and battery, so that voltages ranging from zero to one volt may be obtained, as in Exp. 9. For *MA* use either a multiscale microammeter or a wall galvanometer with an Ayrton shunt. With this instrument on its least sensitive scale and the tube operating under the specified conditions of the cathode heating current, plate potential and grid potential, set the calibrating voltage to zero and adjust *P* until no deflection is obtained. Go to the next most sensitive scale, adjust for zero again and so on, until the most sensitive scale shows no deflection.

Now apply the calibrating voltages to the grid, exactly as in Exp. 9, returning the voltage to zero and checking the setting of *P* after each reading. Plot a curve of applied voltage against meter reading for the amplifier, specifying the operating conditions on the tube and the scale of the instrument used to measure the plate current. Calculate the microvolt sensitivity and the figure of merit of the amplifier if the resistance *r* is one megohm and compare them with the similar sensitivities of *MA*. (If a portable instrument is used to measure the plate current these sensitivities will be expressed in terms of scale divisions, rather than millimeters deflection at one meter.)

**65. Four Element Electrometer Tubes.**

As was pointed out in Art. 64, the ordinary three electrode tube has a definite lower limit of the currents which it may

amplify, due to the current flow between the filament and the grid. This current may be due to actual ohmic leakage over the insulation, to ionization currents in the gas, or to photo-electric and thermo-electric emission from the grid itself. Tubes (of which the FP-54 of the General Electric Co. and the D-96475 of the Western Electric Co. are examples) have been developed for laboratory amplification of small currents, and have been especially designed to overcome these leakages. The insulation leakage is easily minimized by careful construction, and the photo- and thermo-electric effects are cut down to negligible proportions by the use of thoriated tungsten filaments, running at quite low temperatures. The remaining and major difficulty, that due to positive ions, is removed in two ways:

1. The tube is designed to operate at low voltages (about 4 to 6 volts) so that the electrons never acquire sufficient speed to ionize any residual gas in the tube.*

2. A fourth electrode, *or space-charge grid*, is inserted between the filament and the ordinary, or control, grid, and is operated at a positive potential, so as to repel any positive ions emitted by the filament. This also has another advantage, since it dissipates the space charge in the neighborhood of the filament, and consequently increases the current to the plate at a given voltage, or, in other words, decreases the plate resistance of the tube. Since the changes in the plate current are the quantities measured in most amplifier set ups, any lowering of the plate resistance is effectively equivalent to an inverse change in the amplification factor.

These tubes may be used in exactly the same manner as shown in Fig. 55, the space charge grid being connected to the suitable potential above the negative end of the filament. A more elegant method, however, for either three or four element tubes, is that shown in Fig. 56. This is a null method, and, consequently, its calibration is not dependent on the characteristics of the tube, except for the filament-grid resistance, which, in most work, is large compared with $r$. In using this connection, the galvanometer, $G$, is set to zero under normal operating conditions, as in Art. 64, with the potentiometer composed of $R_1$ and $R_2$ set so

* See Chap. X.

that the voltage measuring device, $V_1$, reads zero. The unknown current is then allowed to flow and the galvanometer returned to zero by adjustment of $R_1$. The voltage drop, $ir$, has then been exactly compensated for, and is read directly on $V_1$. This arrangement requires no calibration other than that of $V_1$. It may be used with currents as small as about $10^{-18}$ ampere.

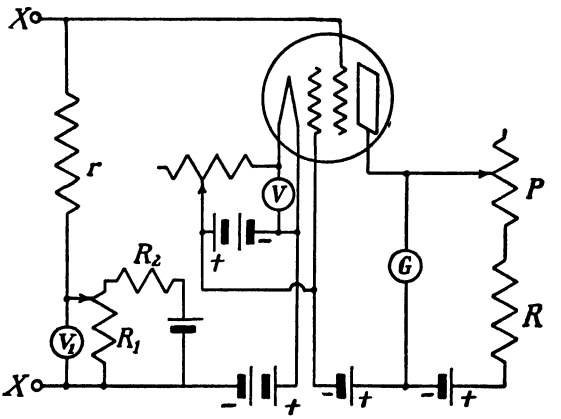


FIG. 56. Four element tube used as a D.C. amplifier — Null method.

For higher sensitivity, the rate of drift method is used. Suppose that, in Fig. 56, $r$ is made infinite. The current will then go to charge the condenser formed by the filament-grid system. If the capacitance of this system is $C$, then the time rate of change of grid voltage will be:

$$\frac{dV_g}{dt} = \frac{dq/dt}{C} = \frac{i}{C}. \tag{1}$$

This change in the grid voltage will produce a change in plate current, and will therefore cause a steady drift of the galvanometer. With no current flowing between the terminals $X$–$X$, only the leakage currents in the tube will contribute to $i$. If the current is then allowed to flow the rate of drift will change. The rates may easily be obtained by measuring, with a stop watch, the times taken for the galvanometer deflection to change by a given amount. Suppose that the times, with and without the current

flowing, are $i_0$ and $i$, respectively; then:

$$\frac{i}{i_0} = \left(\frac{dV_g}{dt}\right)_0 \Big/ \frac{dV_g}{dt} = \frac{i_0}{i_0 + i}, \tag{2}$$

where $i_0$ is the leakage current of the tube. For this method, of course, either $i_0$ or $C$ must be determined in order to calibrate the amplifier. With this arrangement and the General Electric FP-54 pliotron, DuBridge states that currents as small as $10^{-16}$ amperes can be measured, but recommends the "balanced tube" circuit, which will be found in his reference, as more satisfactory for currents of less than $10^{-14}$ amperes. By use of the latter method, he has measured currents as small as $5(10)^{-18}$ amperes.

In using these tubes, great stability may be obtained by using large capacity storage batteries for the control potentials. Due to the low voltages involved, this is entirely practical. It is to be noted especially that sensitive amplifiers are even more sensitive to external fields than are electrometers, and that all shielding precautions outlined in Art. 40 should be followed. It will often be found desirable to inclose the entire amplifying apparatus in a closed metal box.

## EXPERIMENT 26

### MEASUREMENT OF THE GRID RESISTANCE OF AN ELECTROMETER TUBE

The cathode-grid resistance of an electrometer tube may be determined by a variation of the method of Exp. 10, using the tube itself as an electrometer. Set up the apparatus as in Fig. 57. Since the resistance to be measured is extremely high, it will be necessary to insulate and shield very carefully the control grid circuit. Therefore a very well insulated air condenser should be used for $C$, the switch $S$ should be insulated with amber or paraffin and the connecting leads should be carefully shielded by enclosure in a grounded metal case. If the tube used has a guard ring construction, this electrode should be maintained at a negative potential, equal to the rated control grid potential, with respect to the negative end of the filament. For $D$ use a battery which gives a potential somewhat greater than the rated grid bias.

After adjusting the filament heating current to the proper value close $S$ and determine the transfer curve of the tube, exactly as was done in Exp. 23, by adjusting $V_g$. Next, determine the *floating potential* of the grid, that is the potential for which the electron stream reaching it just maintains a steady state, by opening $S$ and waiting until $MA$ comes to a steady value. Record this value and, by reference to the transfer curve, find the potential of the grid when floating. Now adjust $V_g$ in such a way that the rated grid potential is intermediate between it and

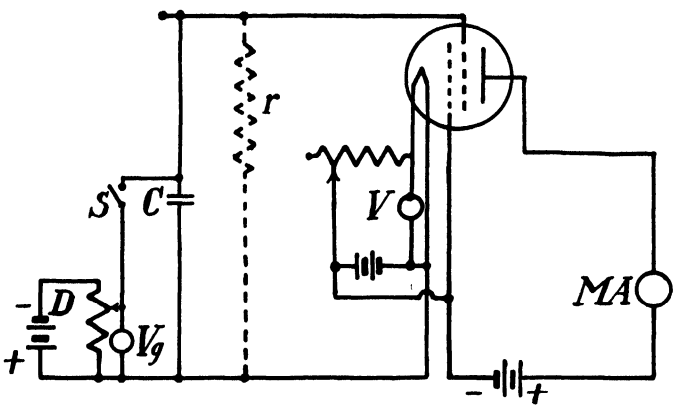


FIG. 57. Measurement of grid resistance.

the floating potential. Close $S$, record the reading of $MA$, then open $S$ and observe the plate current as a function of time after opening. By reference to the transfer curve convert these readings of plate current into values of the grid potential and plot the latter against time.

Since the resistance between the cathode and the grid is dependent upon the grid potential, a straight line logarithmic plot, such as was obtained in Exps. 4 and 10, cannot be expected. As a result, Eq. (3) of Art. 19 must be replaced by the differential Eq. (2). Replacing $Q$ in the latter by $CV$, we have:

$$-\frac{dV_g}{dt} = \frac{V_g}{CR}.$$

Determine graphically the slope of your $V_g$ vs. $t$ curve at the operating value of $V_g$ and calculate $R$.

If it is desired to find the current sensitivity of the tube under operating conditions, using the potential drop method, the resistance $r$, shown dotted in Fig. 57, may be inserted and its resistance, in parallel with the cathode-grid resistance, may be determined by the same method. For stability of operation it is well to keep $r$ to less than ten per cent of the cathode-grid resistance.

**66. Balanced Electrometer Tube Circuits.**

Due to the extremely high sensitivity of electrometer tubes, they are likely to be somewhat unstable in their operation. Even in a very well designed circuit, free from temperature corrections,

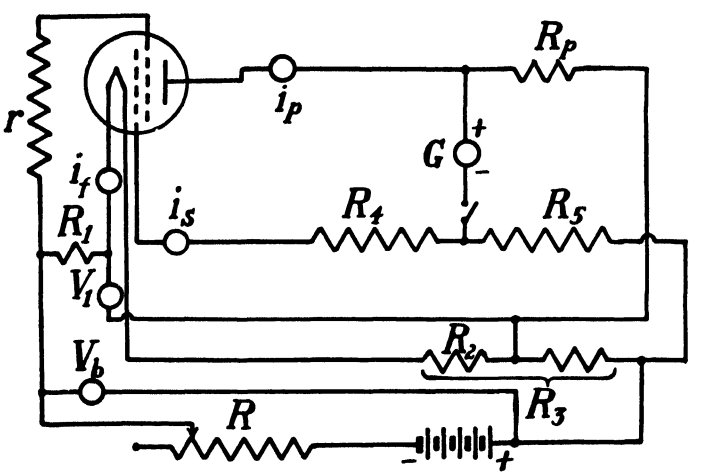


FIG. 58. Balanced tube circuit. (Due to Barth and Penick.)

bad contacts, etc., the zero of the amplifier is likely to shift due to small changes in the battery E.M.F. or in the emission from the cathode. In order to compensate automatically for such disturbances, various circuits have been proposed. All of these take advantage of the fact that all of the voltages which need be supplied to the tube can be obtained from a single low voltage

battery, so that the changes in voltage on different elements can be made dependent on each other. Changes in filament emission are compensated for by recognition of the fact that the current to the space charge grid and that to the plate are practically proportional to each other. Probably the most flexible circuit is one which was developed independently by G. Barth and and D. B. Penick, and which is shown in Fig. 58. If we let $V_f$ be the potential difference across the filament, $V_g$, $V_s$ and $V_p$ be the potentials of the control grid, space charge grid and plate, respectively, with respect to the negative end of the filament and the currents be as marked in the figure, we can write the equations of the circuit. For convenience, we shall write

$$V_s' = V_s + i_s R_4.$$

If the galvanometer is to read zero, for essentially zero current in $r$, we have

$$V_g = -i_f R_1, \tag{1}$$

$$V_f + i_f R_2 = i_p R_p + V, \tag{2}$$

$$V_f + i_f R_3 = i_s R_s + V, \tag{3}$$

where we have written $V$ for the common value of $V_p$ and $V_s'$. In addition to these equations, we require that small changes in the potential of the supply battery or in the filament emission shall not affect the balance as indicated by the galvanometer. To test for this, we shall first expand the $i_p$ and $i_s$ in a Taylor's series, retaining only the first order terms. We have

$$\Delta i_p = \frac{\partial i_p}{\partial i_f}\Delta i_f + \frac{\partial i_p}{\partial V_g}\Delta V_g + \frac{\partial i_p}{\partial V}\Delta V$$

and

$$\Delta i_s = \frac{\partial i_s}{\partial i_f}\Delta i_f + \frac{\partial i_s}{\partial V_g}\Delta V_g + \frac{\partial i_s}{\partial V}\Delta V.$$

Since we have only two sources of variation for which we are compensating, it will be sufficient to use only two independent variables. For convenience we shall choose $i_f$ and $V$. After we

substitute for $\Delta V_g$ from Eq. (1) the above equations become

$$\Delta i_p = \left(\frac{\partial i_p}{\partial i_f} - R_1 \frac{\partial i_p}{\partial V_g}\right) \Delta i_f + \frac{\partial i_p}{\partial V} \Delta V$$

and

$$\Delta i_s = \left(\frac{\partial i_s}{\partial i_f} - R_1 \frac{\partial i_s}{\partial V_g}\right) \Delta i_f + \frac{\partial i_s}{\partial V} \Delta V,$$

and by substituting these values in Eqs. (2) and (3), letting $r_f = \partial V_f / \partial i_f$, we have

$$\left(r_f + R_2 - R_p \frac{\partial i_p}{\partial i_f} + R_p R_1 \frac{\partial i_p}{\partial V_g}\right) \Delta i_f = \left(R_p \frac{\partial i_p}{\partial V} + 1\right) \Delta V$$

and

$$\left(r_f + R_3 - R_s \frac{\partial i_s}{\partial i_f} + R_s R_1 \frac{\partial i_s}{\partial V_f}\right) \Delta i_f = \left(R_s \frac{\partial i_s}{\partial V} + 1\right) \Delta V.$$

Eliminating $\Delta i_f / \Delta V$ from these two equations gives

$$\frac{1 + R_p \dfrac{\partial i_p}{\partial V}}{1 + R_s \dfrac{\partial i_s}{\partial V}} = \frac{r_f + R_2 - R_p \dfrac{\partial i_p}{\partial i_f} + R_p R_1 \dfrac{\partial i_p}{\partial V_g}}{r_f + R_3 - R_s \dfrac{\partial i_s}{\partial i_f} + R_s R_1 \dfrac{\partial i_s}{\partial V_g}}. \tag{4}$$

Eqs. (1)–(4) must be simultaneously satisfied under the operating conditions. But we have five resistances, $R_1$, $R_2$, $R_3$, $R_s$ and $R_p$ at our disposal, so we not only can satisfy the equations but also can choose one resistance arbitrarily. In general it is most convenient to choose $R_3$ in this manner. Since $R_1$ is determined by Eq. (1) and the rated grid bias, and $R_f$, the filament resistance, is a characteristic of the tube, $R_3$ may be so chosen that the product

$$(R_1 + R_f + R_3) i_f$$

gives a convenient battery supply voltage ($i_p$ being small compared with $i_f$), which, for either of the tubes mentioned above, may be anything from twelve volts upward. The control resistance $R$ may then be dispensed with unless large variations

of battery voltage are likely to occur. Solving for the other resistances, we have:

$$R_s = (V_f - V + i_f R_3)/i_s \tag{5}$$

$$R_p = \frac{R_s\left\{\left(\frac{\partial i_s}{\partial V}\right)(V + i_f r_f - V_f) - \left(i_s - i_f\frac{\partial i_s}{\partial i_f} + i_f R_1\frac{\partial i_s}{\partial V_g}\right)\right\}}{\left|\begin{array}{l}\left(\frac{\partial i_p}{\partial V}\right)\left\{(V + i_f r_f - V_f)\right. \\ \qquad \left. + R_s\left(i_s - i_f\frac{\partial i_s}{\partial i_f} + i_f R_1\frac{\partial i_s}{\partial V_g}\right)\right\} \\ \qquad - \left(1 + R_s\frac{\partial i_s}{\partial V}\right)\left(i_p - i_f\frac{\partial i_p}{\partial i_f} + i_f R_1\frac{\partial i_p}{\partial V_g}\right)\end{array}\right|} \tag{6}$$

and

$$R_2 = (V - V_f + i_p R_p)/i_f, \tag{7}$$

as the values of the resistances which will satisfy the balance conditions.

It is possible, by using the rated values of $i_f$, $i_p$, $i_s$, $V_g$ and $V_p$, and by measurement of the partial derivatives involved in Eq. (6), to design with great accuracy a balanced tube circuit for a given electrometer tube. If the rated values of $V_p$ and $V_s$ are different, it will be necessary to choose

$$R_4 = (V_p - V_s)/i_s, \tag{8}$$

but, if they are identical, $R_4$ may be omitted. The measurement of the tube characteristics with sufficient accuracy to enable the derivatives to be known well enough for design purposes requires a great deal of care, however, and will, in general, need to be done by measuring all potentials and currents with a good potentiometer. If the approximate values of the derivatives are known it is possible to avoid this rather tedious set of measurements, as the circuit may be approximately designed and a trial and error method may be used to make the final adjustment. This is possible because with the circuit balanced, the current through the galvanometer from the plate to the space charge grid is a maximum, at the operating value of $i_f$. If $R_2$ is too large this maximum occurs at a lower value of the filament

current, if too small, at a higher value. Hence, as the filament current is increased slightly through the rated value an increase in the galvanometer reading indicates too low a value of $R_2$, and vice versa. The detailed procedure of the balancing, as suggested by Penick, is given in the next experiment.

## EXPERIMENT 27

### BALANCING AND CALIBRATION OF AN ELECTROMETER TUBE CIRCUIT

Using the rated values of currents and potentials on the tube to be used in the circuit, calculate the values of $R_1$, $R_2$, $R_3$, $R_4$, $R_s$ and $R_p$ by use of the above equations. In calculating $R_p$ remember that it follows from the definition of $V_s'$ that

$$\frac{\partial i_s}{\partial V} = \frac{\dfrac{\partial i_s}{\partial V_p} + \dfrac{\partial i_s}{\partial V_s}}{1 + R_4 \dfrac{\partial i_s}{\partial V_s}}$$

and that

$$\frac{\partial i_p}{\partial V} = \frac{\left(\dfrac{\partial i_s}{\partial V_p} + \dfrac{\partial i_p}{\partial V_s}\right)\left(1 + R_4 \dfrac{\partial i_s}{\partial V_s}\right)}{1 + R_4\left(2\dfrac{\partial i_s}{\partial V_s} + \dfrac{\partial i_s}{\partial V_p}\right)}.$$

Set up the circuit as shown in Fig. 58, making $r$ zero to avoid disturbances during adjustment, and leaving the galvanometer switch open. Fixed resistors of the proper value may be used immediately for $R_1$ and $R_4$, so these may be adjusted at once with the help of a Wheatstone bridge and placed permanently in the circuit. For $R_p$ and $R_s$ variable resistors, such as dial boxes, should be used for the first adjustment. The resistances chosen should allow an adjustment of at least fifty per cent on each side of the calculated value. Since $R_3$ is a low resistance, a slide wire rheostat is probably best suited for it. Choose one which has a resistance slightly higher than that calculated and connect a high resistance in parallel with its entire length. By adjustment of this high resistance the value of $R_3$ may be brought

to the calculated one and the slide will serve to separate $R_2$. Having set all resistors and having adjusted $V_p$ to the calculated values, observe $i_s$ and $i_p$, and the potential drop, $V_2$, then calculate the drop across $R_s$ and $R_p$. By successive trials (since the current will change with the resistances) adjust these two resistances until the calculated potentials on the two sides of the galvanometer are the same and are equal to the rated plate potential, with respect to the negative end of the filament. It is now safe to close the galvanometer circuit. This instrument will probably not read zero. Decrease $R_p$ slightly and notice the direction in which the galvanometer is deflected; this direction corresponds to a positive current from the plate toward the space charge grid and we shall call it positive. Now adjust $R_p$ until the galvanometer deflection is as close to zero as possible. By varying $R$, change the filament current slightly and, by noticing the change of the galvanometer deflection, determine whether $R_2$ is too high or too low. Return the filament current to the rated value, change this resistance by a small amount in the proper direction, readjust $R_p$ until the galvanometer again reads zero, and once more vary $R$ to test for the proper value of $R_2$. Repeat this procedure as many times as necessary until a change of $R$ in either direction causes a small galvanometer deflection in a negative direction. When the current is nearly balanced, it will be necessary to be sure that sufficient time is allowed, after each change of $R$, for the filament to come to equilibrium.

If the tube is to be used with a high value of $r$ it must be shielded very carefully. This generally necessitates the enclosure of the tube and all the auxiliary resistances in a closed metal container, with leads coming through for the battery supply, galvanometer and from the ends of $r$, the latter being extremely well insulated. When extreme nicety of measurement is required, the container is often made tight and evacuated, to cut down leakage over surfaces, or it is made reasonably tight and a drying agent is enclosed. Since the dial boxes used for $R_p$ and $R_s$ are not well suited to this construction, and since the slide wire rheostat used for $R_3$ is likely to develop extra resistance at the sliding contact, these resistances will need to be replaced

by small resistances which can be fixed at the proper values and be soldered into the circuit. Therefore, if the tube is to be used under these conditions, measure the final values of these resistances and $R_2$ with a Wheatstone bridge and adjust new resistors of the required type until they are identical with those used during the test. The meters may, of course, be omitted in the final arrangement. On the other hand, if a small value of $r$ is to be used in the final circuit, the tube will not need shielding and no changes need be made.

With the circuit in final form arrange to apply a variable small potential across $r$ by means of a potentiometer and a dry cell, as was done in Exps. 9 and 25. For each voltage applied, read the galvanometer deflection. Plot a calibration curve. Calculate the microvolt sensitivity and the figure of merit of the circuit. The instrument is now ready for use, and may be used in the same method as an electrometer for any of the measurements for which that instrument has been suggested.

**67. A.C. Amplifiers.**

While the D.C. amplifiers discussed in the last few articles have come to have a rapidly increasing use in the laboratory, the older and wider use of amplifying tubes is in connection with alternating currents. The most common examples of this type of amplification are the audio frequency and radio frequency amplifiers used in radio reception, and the laboratory instruments are in no essential way different from these.

The simplest circuit for alternating current amplification is quite similar to that shown in Fig. 55. The microammeter is merely replaced by an alternating current measuring device and the balancing potentiometer may be dispensed with, since the meter used will not respond, ordinarily, to the direct current in the plate circuit, but only to the amplified alternating current superimposed upon it. For these purposes, the amplification factor of a single tube is often not large enough to give the desired amplification. In such cases, two or more *stages* of amplification are used, the output from the plate of one tube being fed into the grid circuit of the following tube. There is not sufficient space available here to enter into the details of design of such

amplifiers, but two simple types of coupling between tubes are shown in Fig. 59. As stated in Art. 62, it is necessary that the grid be made negative with respect to the filament. This is ordinarily done in one of three ways:

1. By placing a *biasing battery* ("C battery") between them, as shown by the solid lines in Fig. 59.

2. By the use of a *grid leak resistor*, as shown by the dotted lines of the figure. When such a resistance is inserted and the

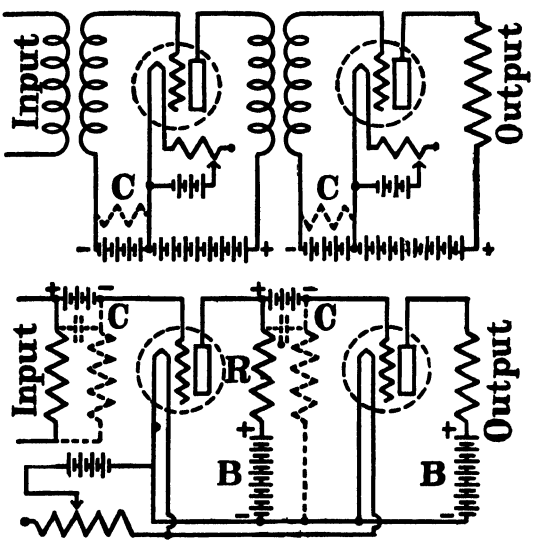


FIG. 59. Coupling of amplifiers.

C battery omitted, electrons striking the filament charge it to a negative potential such that the leakage through the resistance just balances this electron current.

3. By the use of a *self-biasing resistor*. Such a resistor, of considerably smaller magnitude than the one necessary for a grid leak, can be placed in series with the cathode. The cathode current, flowing in the resistor, makes the remote end negative and the grid is connected to this point.

In the first diagram of Fig. 59, the tubes are coupled by means of transformers, so that variations in plate current of the first tube superimpose variations in voltage on the constant grid

potential of the second tube. The second diagram shows *resistance coupling* in which variations in the plate current of the first tube, passing through $R$, produce variations in the grid potential of the second. Note that in this case the $C$ battery must have a larger voltage than the battery $B$ to make the grid negative, and that if a grid leak is substituted, a condenser is necessary.

An almost unlimited number of variations on the simple circuits of Fig. 59 are possible and find use in practice. The choice of the particular circuit depends on the use to which it is to be put. For example, if it is desired to amplify low frequency alternating currents for the operation of an audible recording device, it may be essential to use a circuit which passes the desired frequencies and cuts out their harmonics, by a *filter* composed of inductances and capacitances. In some cases, a *regenerative circuit* in which part of the output of one tube is fed back to the grid circuit of one of the earlier tubes is desirable; in other cases such "feed back" must be eliminated by special "neutralizing" devices. Such cases will be found treated in the standard works on the subject, some of which are found in the references at the end of the chapter.

**68. Voltage Amplifiers—Voltage Gain.**

Alternating current amplifiers are used either to amplify voltage or to amplify power, and the design requirements of the two types differ somewhat. When it is desired that a large voltage variation in the plate circuit shall be obtained from a small voltage variation in the grid circuit, the tubes of the amplifier are generally operated at such a grid bias as to use the linear portion of their transfer curves. Operation of this type is technically known as *Class A amplification*, and gives a voltage in the plate circuit which is proportional to the signal voltage. If a variable voltage, of amplitude $\Delta V_g$, is impressed on the grid of a tube, a change of the plate current of amplitude $g_m \Delta V_g$ will be produced in the plate circuit, assuming, as a first approximation, that the plate voltage remains constant. In order to make a voltage available in this circuit, a high impedance is connected in series with the plate and the variable part of the voltage drop

across this impedance is taken as the output of the tube and is supplied either to the measuring device or to the grid of a second stage. The amplitude of the output voltage will be to this approximation:

$$Z\Delta i_p = Zg_m\Delta V_g,$$

Where $Z$ is the impedance in the plate circuit. Actually, however, the plate voltage is decreased by the increased current in the impedance, and to compute the change in the plate current we need to make use of the relations:

$$\Delta i_p = g_m\Delta V_g + \Delta V_p/r_p, \tag{1}$$

$$\Delta V_p = -Z\Delta i_p. \tag{2}$$

Combining these two equations and substituting for $g_m$ the value $\mu/r_p$ we have:

$$Z\Delta i_p = Z\mu\Delta V_g/(Z + r_p), \tag{3}$$

so that the voltage amplification, or *voltage gain*, is:

$$k_V = Z\Delta i_p/\Delta V_g = Z\mu/(Z + r_p). \tag{4}$$

As Eq. (4) shows, the voltage gain of a single stage amplifier is always less than the amplification factor of the tube used, but approaches this value as the plate current impedance becomes very large. However $Z$ is limited in value by the fact that it must be small enough so that, at the extremes of the swing, $r_p$ remains essentially constant, i.e. that the tube continues to operate on the linear portion of the characteristic curves. In using Eq. (4) it must be remembered that $Z$ is the effective impedance of the plate circuit and not just the impedance in series with the plate and B battery. For example, in the first stage of the resistance coupled amplifier of Fig. 59, $Z$ is very nearly the impedance of $R$ in parallel with the resistance of the grid resistor of the second stage.

**69. Tetrodes and Pentodes.**

As was pointed out in the last article, the voltage gain which can be obtained from a triode is limited by the plate resistance. Another limitation is imposed by the capacitance between the

control grid and the plate. As the signal impressed on the grid swings it positive, the electrostatic field between it and the plate causes the latter to acquire a negative charge and thus decreases the plate voltage. Not only does this decrease the voltage gain in the same way as the voltage drop in the plate impedance, but it decreases it by an amount which depends on the frequency of the impressed signal, since the displacement current between the control grid and the plate depends upon the frequency. Both of these difficulties can be removed by the ingenious device of inserting a second grid, the *screen grid*, between the control grid and the plate. If this grid is maintained at a positive potential with respect to the cathode it will decrease the space charge near the latter in exactly the same way as does the positive plate in a triode. Since it is this space charge which principally affects the plate current, this current will be almost independent of the plate voltage, and $r_p$ will be very large, enabling a corresponding gain in amplification. In addition, since the screen grid maintains an approximate equipotential plane between the control grid and the plate, the electrostatic capacitance between these two electrodes is greatly reduced, giving a further gain. The screen grid, like the control grid, is an open structure but, since it is maintained at a positive potential it draws some current. To prevent this current from causing variations of the screen grid potential, a by-pass condenser is usually placed between the screen grid and the cathode to offer a very low impedance path to the alternating screen grid currents. Tubes containing a screen grid are known as *tetrodes*.

In all tetrodes part of the gain introduced with the screen grid may be compensated by *secondary emission* from the plate. Just as the electrons within a metal may be given sufficient energy to escape by heating, so they may acquire this energy from an impinging electron which strikes at high velocity. It is clear that these secondary electrons, when liberated from the plate, will be a negative current from it, and as such will decrease the plate current. This is particularly true in a tetrode if an attempt is made to operate it for high voltage gain. In that case, at the maximum value of the plate current, the potential of the plate may drop to below that of the screen, causing the latter to serve

as an effective collector of the secondary electrons. As a result the plate impedance of a tetrode must always be kept small enough so that the plate remains positive with respect to the screen at all times. To overcome this limitation a third grid, known as a *suppressor*, is often added to make the tube a *pentode*. The suppressor is mounted between the screen grid and the plate and is generally maintained at the potential of the cathode. It therefore serves to repel the secondary electrons from the plate and to return them to their source, and so allows a high output impedance (sufficient to cause the plate to swing negative with respect to the screen) to be used without harmful effects. A secondary advantage of the suppressor is that it improves further the electrostatic shielding of the control grid and the plate.

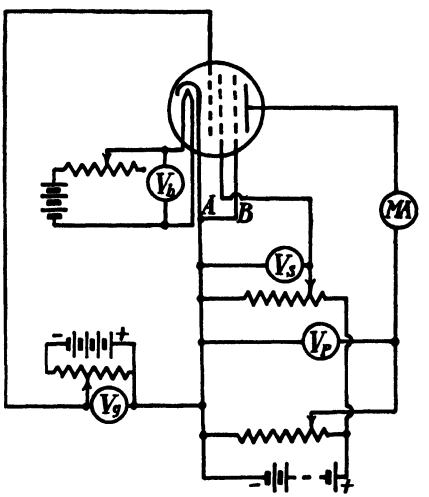


FIG. 60. Characteristic curves of a pentode.

A second type of pentode, the *beam power tube*, replaces the suppressor grid by two electrodes so placed as to produce a space charge near the screen, and uses this space charge to return the secondary electrons to the plate. This type of tube also uses a careful geometrical design to keep the screen current very small.

## EXPERIMENT 28

### CHARACTERISTIC CURVES OF A PENTODE

The circuit for the tests of the characteristic curves of a pentode is shown in Fig. 60. Since most pentodes are designed for operation with alternating current for the cathode heater, they are of the separate heater type as shown. If a directly heated cathode is used, all connections shown to the cathode in the

diagram will be made to the negative side of the filament. In many tubes the connection *A–B* between the cathode and the suppressor is made in the base of the tube at the time of manufacture.

Set and maintain the voltage across the heater and that between the cathode and the screen at the rated values, since these do not vary under operating conditions. Then proceed to vary the plate and control grid voltages exactly as in Exp. 23, to determine the characteristic curves of the tube. Calculate the amplification factor, the transconductance and the plate resistance of the tube for the linear portions of the characteristic curves, and calculate the maximum voltage gain possible using the tube as a Class A voltage amplifier.

**70. The RCA Ultra Sensitive D.C. Meter.**

An interesting application of pentode tubes to laboratory measurements is furnished by the ultra sensitive portable micro-

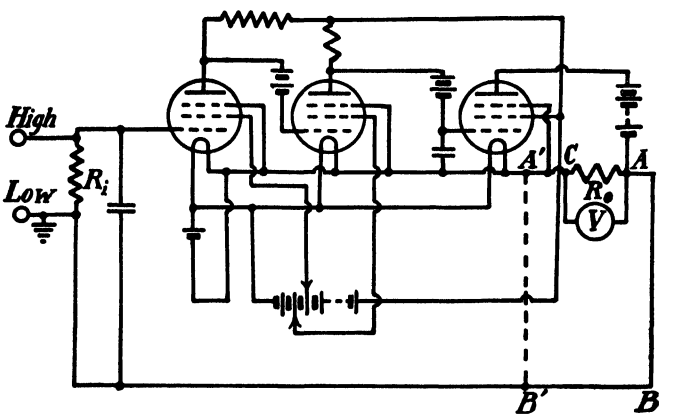


FIG. 61. Simplified circuit diagram of the RCA ultra-sensitive meter.

ammeter manufactured by the RCA Manufacturing Company. This instrument is essentially a voltmeter of high and adjustable input resistance and in it the voltage gain of an amplifier is sacrificed in order to give extreme stability. A simplified circuit diagram is shown in Fig. 61. In this diagram the three tubes

used are Type IB4 pentodes. It will be noticed that, if a current is passed through the input resistance $R_i$ in such a direction as to make the terminal marked *High* positive with respect to that marked *Low*, the plate of the first tube swings negative, that of the second positive, that of the third negative, and so on. Hence if any odd number of tubes are used, a positive current will flow from $C$ to $A$ when a current in this direction flows through $R_i$. Now suppose that a voltage difference $V_i$ is impressed across the input resistor and that the connection $A$–$B$ is replaced by the wire $A'$–$B'$, shown dotted in the figure. Then a voltage difference:

$$V_0' = k_V V_i$$

will be read by the meter $V$, provided that it was set to read zero with *High* and *Low* at the same potential and that $k_V$ is the overall voltage gain of the amplifier. If, however, the connections are as shown by solid lines in the figure, the output voltage will be subtracted from the input voltage. If the former in this case is $V_0$, the new potential of the control grid of the first tube with respect to the negative filament will be:

$$V_i' = V_i - V_0, \tag{1}$$

and, since:

$$V_0 = k_V V_i', \tag{2}$$

we have:

$$V_0 = \frac{k_V}{1 + k_V} V_i. \tag{3}$$

If $k_V$ is large (in the actual instrument under discussion, which uses three pentodes, it is about 1000), the effective voltage gain, $V_0/V_i$, is practically unity and retains its value within very close limits even for changes of fifty per cent or more in the value of $k_V$. The instrument, therefore, is practically independent of the tube characteristics, and will retain its calibration for long periods. While its voltage sensitivity is the same as that of the voltmeter used in the plate circuit of the last tube, its current sensitivity is extremely high, as the value of $R_i$ is limited only by the grid-filament resistance of the first tube.

In the commercial instrument the diagram of Fig. 61 has added to it a set of switches enabling a wide range of full scale

values to be used, as well as switches for checking the filament supply potential and for reversing the deflection of the meter. In addition, an arrangement for supplying a known voltage across an unknown resistance and thus measuring this resistance

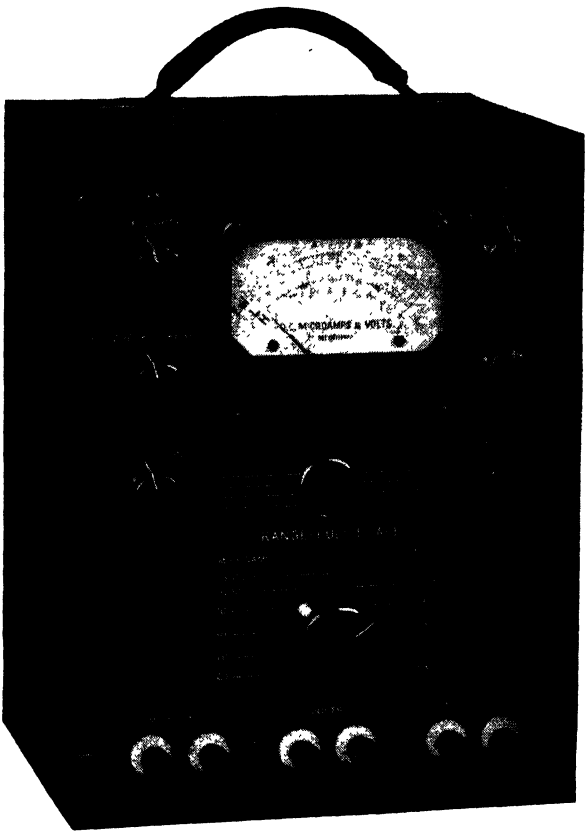

FIG. 62. The RCA ultra-sensitive meter. (Courtesy of RCA Manufacturing Co.)

by Ohm's law, is included. On the most sensitive scale an input resistance of $5 \times 10^6$ ohms is used, with a resulting sensitivity of $4 \times 10^{-10}$ amperes per division. The instrument is shown in Fig. 62, where the series of ranges obtainable may be seen marked around the multiple point selector switch.

**71. Power Amplifiers—Impedance Matching.**

In the amplifiers discussed in Art. 67 the object of design was to obtain as large a voltage gain as possible. Very often we are more interested in the alternating current component of the power supplied to the output than we are in the voltage gain. For example, in radio receiving sets we want as much power as possible to be supplied to the loud speaker after the last stage of audio frequency amplification. *Power amplifiers* may be used on the linear portion of the characteristic curves, that is as Class A amplifiers, but, since a tube operated in this way carries a large direct current component of plate current, with subsequent heating of the tube, they are often operated with sufficient negative grid bias to decrease this part of the current to a very small amount or zero. Amplifiers operated in this way are classed as:

*Class B amplifiers*, in which the grid is biased approximately to cut off so that current flows in the plate circuit only during one half of the cycle, and

*Class C amplifiers*, in which sufficient negative grid bias is applied so that current flows in the plate circuit only during a small portion of the cycle. In both Class B and Class C amplifiers linearity of amplification is sacrificed to large power output, and they therefore do not find as wide application in measuring circuits as do those of Class A.

In designing and testing power amplifiers it is necessary to remember that the conditions to be imposed on the plate circuit are different from those required by voltage amplifiers. Since power amplifiers are generally used to supply a device which is essentially a resistance load, we shall consider only pure resistance in the plate circuit in order to investigate the optimum conditions. Suppose that a tube of plate resistance $r_p$ and amplification factor $\mu$ is used as an A.C. amplifier and is connected with a load resistance $R$ in series with the plate. We wish to know the value of $R$ which will result in a maximum power transfer to the load. Let $i_p$, $v_g$ and $v_p$ be the alternating components of the plate current, control grid voltage and plate voltage, respectively, then by proceeding as in Art. 68, we find that

$$i_p = \mu v_g/(R + r_p). \qquad (1)$$

The useful power supplied to the load is $i_p^2R$, and if we test this for a maximum by differentiating with respect to $R$ and equating to zero we find that

$$R = r_p \tag{2}$$

represents the optimum condition. It will be noticed that in making this choice of the load resistance, voltage gain is sacrificed.

As a general rule, since the load resistance and the plate resistance are likely to be determined by other conditions, Eq. (2) cannot be satisfied directly. As a result, *impedance matching transformers* are frequently used in the plate circuit of a power amplifier. Thus, even if $R$ and $r_p$ have radically different values, it is in general possible to design a transformer of which the primary impedance is approximately equal to the latter, while its secondary impedance is approximately equal to the former. If such a transformer is inserted between the plate of the last tube and the load, it will lead to a maximum power transfer. Such a transformer has the further advantage of cutting out of the load circuit the direct current component of the plate current, but, if, as is frequently the case, the load resistance is less than that of the plate, voltage gain is further sacrificed. Because of these various limitations on the voltage gain of power amplifiers, a power stage of amplification is generally preceded by one or more stages of voltage amplification.

Because of the particular applications of power amplifiers, the amplification is generally measured in terms of a power ratio rather than a voltage ratio. The usual definition of the *power gain* is the ratio of the power delivered by the amplifier to the power supplied to it. Because of the wide use of power amplifiers in acoustic reproduction, an acoustic unit, the *decibel* is used to express power gain. The *power amplification* in decibels is defined as ten times the logarithm, to the base ten, of the ratio of the output to the input power. Thus an amplification of one decibel means a power ratio of 1.25; of two decibels, a ratio of 1.59; of ten decibels, 10.00, etc. Because the power input to the amplifier frequently is negligibly small and is not equivalent to the power which could be delivered from the source to the output instrument if they were directly connected, the *voltage amplification* of a power amplifier is often a better index of its performance

than the power amplification. In order that both may be measured in the same units, the *voltage amplification* is commonly taken as identical with the power amplification which would be obtained if the input and output impedances of the amplifier were identical. Since under these circumstances the power ratio would be the square of the voltage ratio, the voltage amplification in decibels is

$$\alpha = 20 \log_{10} k_v,$$

where $k_v$ is the voltage gain.

**72. Special Purpose and Multi-Electrode Tubes.**

With the rapid development of the radio and acoustical fields, the triodes, tetrodes and pentodes discussed above are being supplemented rapidly by new, special purpose tubes which contain more electrodes. Most of these tubes do not differ in principle from the simpler tubes which we have considered, but are so designed as to combine the functions of two or more tubes in particular circuits. While there is not sufficient space here to discuss these tubes in detail, particularly in view of the fact that new tubes are being brought out very frequently, they find a wide application in the laboratory as well as in commercial applications. The student who wishes to make efficient use of amplifying tubes is referred to the standard texts on the subject, to the periodical literature and to the manuals issued periodically by the manufacturers.

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# CHAPTER 7

## ALTERNATING CURRENT INSTRUMENTS

**73. Conventional Values of Alternating Quantities.**

If we apply to a D'Arsonval meter a varying potential, of low frequency, the pointer will follow the variations of the supply. If, however, the frequency is increased, the fluctuations occur more rapidly than the mechanical system of the meter can follow, and the pointer will assume a steady position. Now suppose that the imposed potential is of the form

$$e = E_0 \sin 2\pi ft,$$

then, since the deflecting torque on the meter is proportional to the voltage across its terminals, the average torque will be proportional to the average of this voltage, or proportional to

$$\frac{\int_0^{1/f} E_0 \sin 2\pi ft dt}{\frac{1}{f}},$$

where the integral, it is to be noticed, is taken over one complete cycle. But this integral, as direct integration or a glance at Fig. 63 will show, has the value of zero. Hence, on a sinusoidal

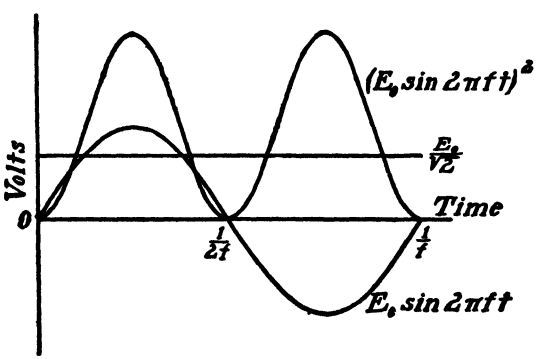



FIG. 63. Conventional values of alternating potentials.

alternating potential, the average value is always zero, and has absolutely no significance.

In choosing a conventional definition of the magnitude of an alternating current or potential, it is necessary that two requirements be met:

1. We must choose some function of the magnitude which is always positive, so that its average value will not vanish for a symmetrical wave.
2. The alternating current or potential unit should be equivalent in as many ways as possible to the corresponding direct current unit.

Both of these requirements are met by using the *root-mean-square* (r.m.s.) of an alternating magnitude, since

1. The square of a real number is always positive.
2. The energy dissipated in an electrical circuit containing pure resistance is

$$\int I^2 R\,dt \qquad \text{or} \qquad \int E^2/R\,dt.$$

The value of an alternating potential, then, will be defined as

$$\left\{\frac{\int_0^{1/f} E^2 dt}{\frac{1}{f}}\right\}^{1/2} = \left\{\frac{\int_0^{1/f} (E_0 \sin 2\pi f t)^2 dt}{\frac{1}{f}}\right\}^{1/2} \tag{1}$$

for a pure sinusoidal wave.

Now, letting $2\pi f t = \theta$, we have

$$E_{rms}^2 = \frac{\int_0^{2\pi} E_0^2 \sin^2\theta\, d\theta}{2\pi} = \frac{E_0^2}{2\pi}\left(\frac{\cos\theta \sin\theta}{2} + \frac{\theta}{2}\right)_0^{2\pi} = \frac{E_0^2}{2},$$

$$E_{rms} = E_0/\sqrt{2}. \tag{2}$$

In Fig. 63, the instantaneous values of the squared voltage are represented. The horizontal line represents the root mean square value.

**74. The Electrodynamometer.**

It is very desirable that an instrument used on alternating currents should measure directly this root-mean-square value, since its determination involves considerable difficulty, and depends upon the wave form of the current. One instrument frequently used for measuring this quantity directly is the electrodynamometer. In this instrument, as in the D'Arsonval type, the current is measured by the amount a coil is rotated against a restoring torque by electromagnetic forces. Here, however, the moving coil carries an alternating current and the magnetic field is produced by an alternating current of the same frequency passing through a fixed coil. If the two fields are in series the magnetic field is proportional to the current, and from Art. 23 the instantaneous value of the deflecting torque is proportional to the product of the current by the field at that instant, and hence to the square of the current. If the inertia of the system is great enough, a steady deflection will be reached which is proportional to the square of the current. If the scale be marked off in intervals corresponding to equal increments of current, so that it reads root mean square values, it will be found that the divisions are much smaller near zero than at the other end of the scale.

This is not true in electrodynamometer wattmeters. In these the current through the device consuming power traverses the fixed coil and the potential across it is applied across the movable coil and a series resistance. The torque is

$$T = kiB = k\frac{E_0 \sin (2\pi ft + b)}{Z} I_0 \sin (2\pi ft + a) \qquad (1)$$

where $E_0$ and $I_0$ are the maximum values of the voltage and current respectively, $Z$ is the impedance of the moving coil circuit, $b$ is the phase difference between the current in this coil and the applied voltage, and $a$ is the phase difference between the current and voltage in the device in which the power consumption is being measured. Integrating this over a cycle, we find the deflection, which is proportional to the average torque, to be

$$\theta = K\frac{E_0 I_0}{2}\cos (a - b). \qquad (2)$$

If $b$ is small this deflection is therefore proportional to the power consumed, giving a linear scale. Since the constant involves the impedance of the moving coil circuit, and since this depends on the frequency, the calibration for such an instrument only holds over a limited frequency range. Owing to the presence of $b$ the readings are only strictly accurate at one power factor. However, if the inductance is small compared with the resistance, both errors are negligible over a considerable range.

In the more accurate electrodynamometer type of instrument no iron cores are used and an appreciable current is required to produce the forces necessary to operate the indicator. For this reason, the voltmeter of this type usually draws much more current than a D.C. instrument of the same quality. For some work this makes it necessary to keep it permanently in the circuit as removing it changes conditions enough to invalidate the readings. In order to attain great current sensitivity, it is necessary to strengthen the alternating magnetic field by introducing a laminated soft iron core in the fixed coil. If sufficient power, of

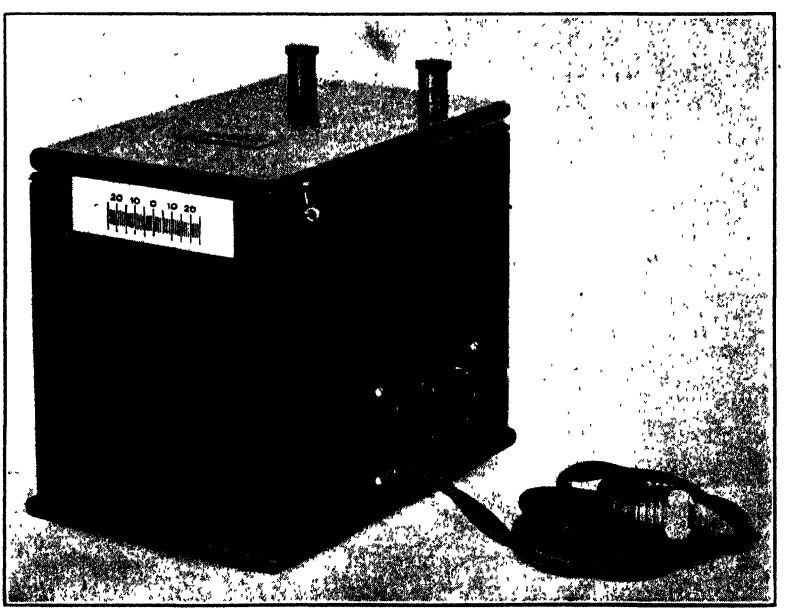



FIG. 64. Portable A.C. galvanometer. (Courtesy of Leeds and Northrup Co.)

the same frequency and phase as that to be measured, is available to operate this field a sensitivity of the same order as that attained in D.C. galvanometers can be obtained. At the same time, however, so many uncertainties are introduced with the iron core that such A.C. galvanometers are rarely calibrated, being used almost exclusively as null instruments.

When an electrodynamometer galvanometer is used in the manner just described it has one serious disadvantage. The magnetic coupling between the current which excites the field coils and the moving coil results in an induced E.M.F. in the latter, and this may be fed back into the circuit on which measurements are being made, causing disturbances. This trouble can be eliminated if the moving coil is placed in the output circuit of a vacuum tube amplifier whose input is supplied by the potential difference which is to be measured. Since the moving coil is necessarily of moderately low impedance this arrangement, if a single stage is used, has low voltage gain. With several stages, high voltage gain may be obtained if extreme constancy of calibration is not required, or gain may be sacrificed to stability by the use of an inverse feed-back arrangement similar to that described in Art. 70.

A portable instrument of the electrodynamometer type, manufactured by the Leeds and Northrup Company, is shown in Fig. 64. This instrument, which is intended for commercial frequencies, operates directly from a 110 volt line, as it contains a small transformer which supplies the current for the measurement as well as for the galvanometer lamp. The scale, which is mounted on the front of the box, is of ground glass, and the beam may be observed distinctly in a fairly well lighted room.

The great majority of portable precision meters for A.C. are built on the electrodynamometer principle. Here again, the mechanical construction is nearly identical with that of the direct current meters described in Chapter III.

### 75. Soft Iron Instruments.

If a piece of soft iron is placed in an alternating magnetic field alternating magnetic poles are induced in it whose strength is nearly proportional to the field strength over a wide range. It is

possible to arrange these pieces of soft iron so that the forces between them due to these induced poles operate an indicating device. Since the direction of the force between magnetic poles is unchanged when the sign of both poles is changed, these forces are unidirectional, and may be balanced against a restoring torque. This principle is frequently used in the construction of switchboard meters. They do not have linear scales and possess considerable reactance, but are simple and robust in construction and rarely require repairing. They are not suitable for precision work.

**76. The Electrostatic Voltmeter.**

The instruments discussed in the previous article were electromagnetic in nature. A second type of instrument may be constructed, utilizing the forces which exist between charged conductors. While any type of instrument which balances the force of attraction or repulsion against a restoring force may be used in this way, the usual arrangement is similar to that of the quadrant electrometer, discussed in Art. 42. Suppose that one pair of quadrants is connected to ground while the other pair is connected to the needle and to the potential to be measured. Then, since the potential of the quadrants and the needle reverse simultaneously, the torque will be rectified, and substituting the unknown potential $v$ in Eq. (5) of Art. 42

$$T \propto v^2. \tag{1}$$

When this deflecting torque is averaged over the entire cycle, we have

$$D = K \frac{\int_0^t v^2 dt}{\int_0^t dt}, \tag{2}$$

so that the electrostatic voltmeter gives a deflection proportional to the mean square voltage, in the same way as an electrodynamometer. The constant is, moreover, independent of frequency, and the instrument may be calibrated on direct current.

For low voltages, these meters have an arrangement very

similar to that of the quadrant electrometer, although, in some cases, two of the quadrants are omitted. For higher voltages (up to thousands of volts) the axis of the needle is usually mounted horizontally and the restoring torque is furnished by a weight at the end of an adjustable lever arm. As with electrometers, care

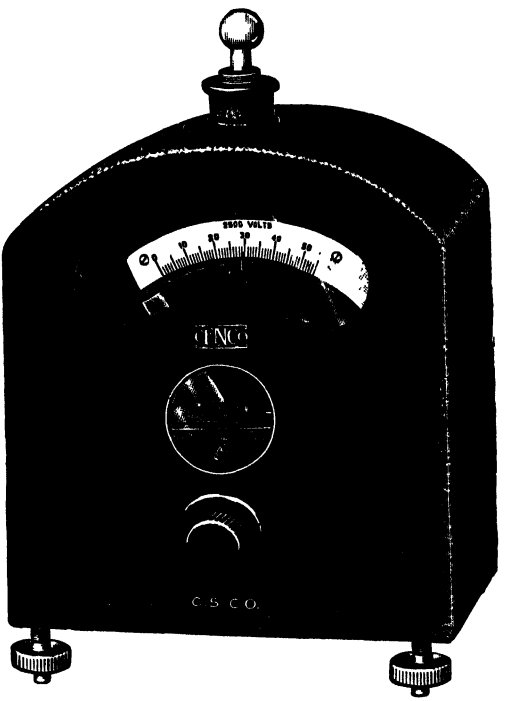



FIG. 65. Electrostatic voltmeter. (Courtesy of Central Scientific Co.)

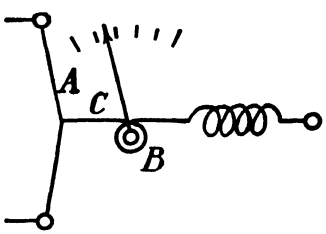
must be taken in the insulation of electrostatic voltmeters, and, as they must be kept free from moisture, drying materials of some sort are usually kept inside the case for this purpose. A typical design of instrument for high voltages is shown in Fig. 65.

**77. Thermal Instruments.**

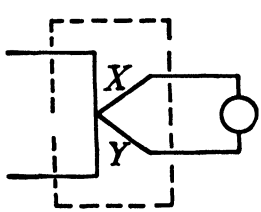
Most of the instruments described thus far have been characterized by fairly high values of inductance or capacitance and

their calibrations, therefore, are distinctly dependent upon the frequency. They are thus limited in application to the range of commercial frequencies, that is to frequencies roughly below one hundred cycles per second. For the measurement of *audio frequencies* (approximately 100–20,000 per second) and *radio frequencies* (approximately 100,000–1,000,000,000 per second) it is necessary to use other types of instruments. One such type is the class of thermal instruments, which has two principal subdivisions, the *hot-wire meter* and the *vacuum thermocouple*, both of which depend on the heating effects of a current.

The arrangement of the hot-wire ammeter is shown in Fig. 66*A*. *A* is a wire through which the current to be measured passes. It is usually of some fairly high resistance, high melting point metal, such as a platinum or a platinum alloy. Connected to the center of this wire is the wire *C*, which winds around the pivoted bobbin *B* to which the needle is attached. The current through the wire causes it to heat and expand, allowing the spring to move the needle across the scale. This arrangement represents about the simplest type of mechanical coupling of the wire and needle. In practice, various modifications of this system are used. Since the temperature rise of the wire with a given current passing through it is a quantity which it is difficult to determine theoretically, and since the scale depends on the type of coupling, no general derivation of the law of the instrument can be given. For any particular instrument, the scale is empirically determined.




*A - Hot Wire Ammeter*




*B - Vacuum Thermocouple*

FIG. 66. Thermal meters.

The effect measured in this case is proportional to $i^2R$, so it is evident that the instrument will read root-mean-square values of current, and that its calibration will be independent of frequency.

The essentials of the vacuum thermocouple instrument are shown in Fig. 66 *B*. The part of the apparatus enclosed by the dotted line is sealed into a glass bulb and thoroughly evacuated. This reduces the convection losses from the wire to a negligible amount, and thus increases both the sensitivity and the reliability of the instrument. The temperature rise of the wire is measured by the thermoelement of two metals, *X*–*Y*, joined to each other and the wire. Since the heat losses from the wire are very nearly proportional to the difference of temperature, $\Delta T$, between the wire and its surroundings, we have

$$i^2R = k\Delta T$$

or

$$\Delta T = \frac{i^2R}{k}. \tag{1}$$

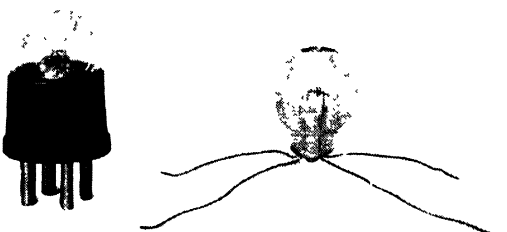
The thermoelement, moreover, produces a voltage across the galvanometer which is proportional to the temperature rise, so the deflection of this instrument

$$D = k_1\Delta T = Ki^2.$$

The vacuum thermocouple, consequently, reads root-mean-square current, and its calibration is nearly independent of frequency.

Due to the intrinsically simple construction of the vacuum thermocouple and to its small physical dimensions, the capacitance and inductance of the instrument may be kept extremely low. For audio frequency measurements no special precautions are necessary, and the tube may be mounted rigidly by means of a standard vacuum tube socket or other mount. Such a couple is shown in Fig. 67*A*. In order to keep the capacitance between leads small when measurements are to be made at radio frequencies, the solidity of mounting is sacrificed and unmounted couples, as shown in Fig. 67*B*, are substituted. It may be advantageous to insert the couple in a case in order to shield it from light and from temperature disturbances. By proper mounting

the capacitance and inductance can be kept so low that the limiting factor in going to high frequencies is the *skin effect* in the heater element. Since this effect becomes small as the diameter of a wire decreases, the heaters are generally made of short lengths of extremely fine wire. The General Electric Company



(*A*) (*B*)

FIG. 67. Vacuum thermocouples with mounts. (Courtesy of General Electric Co.)

makes couples of the types shown in Fig. 67 which give a five millivolt output to the direct current instrument for currents varying between 1.3 ma. and 125 ma., the heater resistances extending from 800 ohms for the low current instrument to one ohm for the high range. Another advantage of the vacuum thermocouple instrument is indicated by these values, since it can be used as a fairly high resistance voltmeter, the maximum resistance value of 1000 ohms per volt being comparable with good direct current instruments and much higher than is obtainable with electrodynamometer or soft iron instruments.

## EXPERIMENT 29

### CALIBRATION OF A VACUUM THERMOCOUPLE

Since the wave form, at moderate frequencies, has no effect on the calibration, the thermocouple will be calibrated on direct current against a standard milliammeter. Connect a dry cell in series with the milliammeter and a resistance sufficiently high to limit the current to well below the rating of the couple. The galvanometer will be of a sensitivity and type suited to the couple, and will usually be specified by the manufacturer. Pass currents of 1/5, 2/5, 3/5, 4/5, and 1 times the rating through the

couple and note the galvanometer deflections. Reverse the terminals and repeat. Average the values thus obtained. From the known voltage sensitivity of the galvanometer, calculate the voltage across its terminals at each current, and plot a curve of potential across the couple against current. On this curve, also specify the *resistance of the galvanometer and leads*, as this will affect the sensitivity.

**78. Rectifying Instruments.**

One method for the measurement of alternating currents and potential differences which immediately suggests itself involves the rectification of the alternating current and its subsequent measurement with a D'Arsonval type meter. Any rectifier would be suited to this purpose, but the principal form used commercially is the copper oxide rectifier. If a contact is made between copper and copper oxide, it is found that the resistance offered to a flow of current from the pure copper to the oxide is reasonably low, while the resistance in the opposite direction is so large as to be essentially infinite by comparison. If four such rectifying elements are connected as shown in Fig. 68, where *A*, *B*, *C* and *D* are the elements, *M* is a D'Arsonval meter and the arrows indicate the direction of allowed current flow, an alternating potential difference impressed at *O*–*P* will cause current to flow through *A*–*M*–*C* during one half of the cycle and through *B*–*M*–*D* during the other half, giving full wave rectification. If the effective resistance through two rectifier elements and the meter in the allowed direction is $r$ and a sinusoidal E.M.F., $E_0 \sin 2\pi ft$, is impressed at *O*–*P*, the average current through the

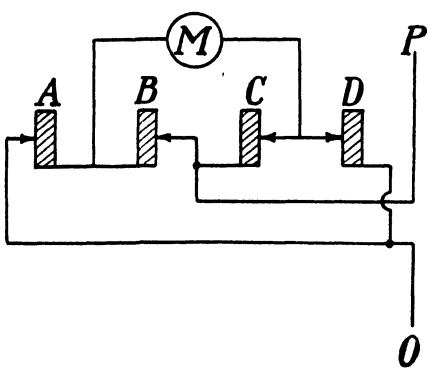



FIG. 68. Arrangement of a rectifying meter.

meter during each half cycle will be

$$\frac{\int_0^{1/2f} (E_0 \sin 2\pi ftdt/r)}{1/2f}$$

and, since the meter reads this average value of the current, its readings will be

$$i_{av} = -(E_0/\pi r)[\cos 2\pi ft]_0^{1/2f} = 2E_0/\pi r = (2\sqrt{2}/\pi)(E_{rms}/r), \quad (1)$$

a value which is independent of the frequency. If, however, a direct current potential difference is applied, the current read by the meter will be

$$i_{dc} = V/r. \quad (2)$$

Equations (1) and (2) show that the meter will not retain its direct current calibration for alternating currents, since it reads *average* value in each case. If calibrated on direct current and used to measure a pure sinusoidal voltage it will read $\pi\sqrt{2}/4$ times too high, unless an additional resistance

$$R = \left(\frac{\pi\sqrt{2}}{4} - 1\right) r \quad (3)$$

is inserted in series with it. When a rectifying instrument is to be used for both alternating and direct current such a resistance is usually arranged to be cut into the circuit. Even then the calibration will be valid only for E.M.F.'s which approximate very closely a sine curve, since the derivation of the factor connecting the two resistances in Eq. (3) depends upon the wave form assumed.

For a given wave form the calibration is independent of frequency up to values at which the capacitive current flow through the rectifying elements becomes appreciable compared with the conductive flow considered above. The instrument can therefore be used only when

$$2\pi fCr \ll 1,$$

$C$ being the capacitance of the series-parallel combination of rectifying elements. For instruments of low sensitivity (small values

of $r$), therefore, the frequency range is great and the rectifying meter may be used in the audio-frequency range, while for high sensitivity microammeters a large frequency error occurs at frequencies appreciably above those for which they have been calibrated.

A full wave copper oxide rectifier, arranged for simple connection to a portable D.C. meter, as manufactured by the Westinghouse Electric and Manufacturing Co., is shown in Fig. 69.

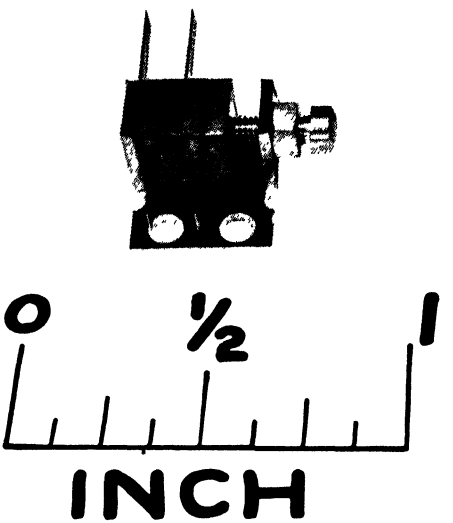



FIG. 69. The cuprox full wave rectifier. (Courtesy of Westinghouse Electric and Mfg. Co.)

The complete meters of this type are furnished with sensitivities of from 200 to 5000 ohms per volt.

## EXPERIMENT 30

### DETERMINATION OF THE FREQUENCY RANGE OF A RECTIFYING VOLTMETER

If the meter has not already been calibrated on alternating current, it will be necessary to calibrate it by comparison with a precision electrodynamometer at 50–60 cycles, arranging for a

variable voltage across the two meters as in Exp. 8. After calibration place the meter in parallel with a good vacuum thermocouple with sufficient *non-inductive* resistance in series with the latter so that the heater element carries rated current when the rated voltage of the rectifying meter is applied. Supply the voltage to these instruments from a high resistance potentiometer rheostat as in Exp. 8, except that the source of potential difference will be an audio-frequency oscillator * instead of a direct current battery. This oscillator should be one which gives a wave form † which is independent of the frequency setting, in order that the errors due to this factor shall not appear as frequency errors. Setting the oscillator for a frequency of 60 cycles per second, adjust the potentiometer rheostat until the rectifying meter reads at about the middle of its scale and then read the voltage from each instrument. Take the ratio of the two readings as a standard and repeat this measurement at about 120 cycles, 240 cycles, 480 cycles, etc., until a frequency is reached at which the ratio has changed by more than a fixed amount, which has been previously set as the limit of the allowable error of the instrument. (This limit will ordinarily be taken somewhere between one and five per cent, depending on the use to which the meter is to be put.) Record the highest frequency at which you have found the error to be less than this limit as the upper limit of frequency within which the meter is to be used to give the specified accuracy.

**79. The Vacuum Tube Voltmeter.**

The fact that a three element vacuum tube, when the grid is made sufficiently negative, will not pass a current to the plate enables this instrument to be adapted to measurements of voltage. In fact, the vacuum tube voltmeter is one of the most elegant of voltage measuring devices, for all frequencies.

The instrument ordinarily takes one of two forms, the *deflection type* or the null *peak-value type*. The wiring diagram of both of these is shown in Fig. 70 *A*. The latter type may easily be understood by reference to Fig. 70 *B*. Suppose that the

* See Art. 94.

† See Art. 96.

voltage to be measured has a maximum value of $V_x$, and that the plate current, under the particular operating conditions used, becomes zero when the grid has a negative potential of $V_0$ volts.

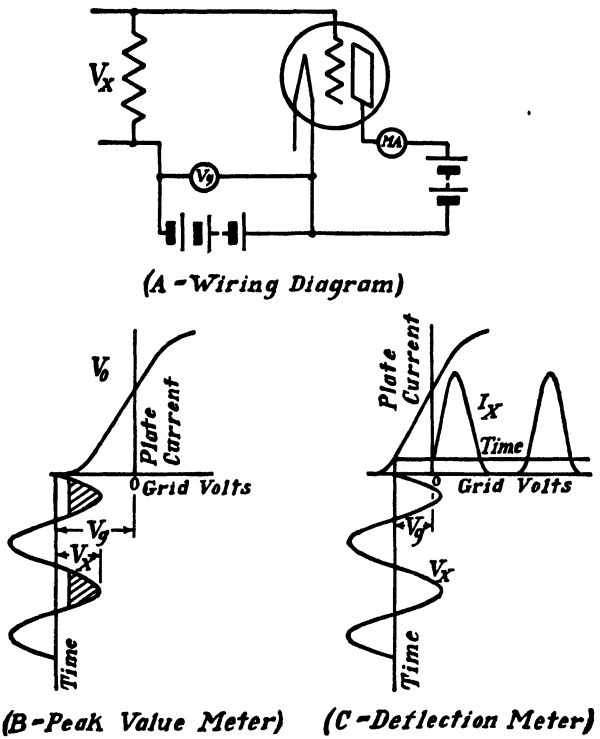



FIG. 70. The vacuum tube voltmeter.

Then, if $V_g - V_0$ is less than $V_x$, the conditions are as shown, and during the shaded part of the cycle a current will flow to the plate, and be recorded by the milliammeter. If $V_g$ is increased until this current just reaches zero, then

$$V_x = V_g - V_0,$$

where $V_g$ is read by the direct current voltmeter and $V_0$ is a constant determined in the tube calibration.

The principle of the deflection instrument is shown in Fig. 70 *C*. The steady potential $V_g$ in this instrument is not sufficient to stop the plate current entirely, but does move the operating range of the tube below the linear part of the characteristic curve. Now suppose that the unknown voltage follows some wave form such as $V_x$. The variations in grid potential caused by this will cause the plate current to vary along some curve $I_x$, and since we are operating below the linear portion of the curve, this wave will not be symmetrical, the positive half of the voltage wave being more effective than the negative half in producing plate current. The difference between the positive and negative half cycles will be recorded by the milliammeter. Any increase in the amplitude of the unknown voltage will cause very little change in the negative half of the current cycle, while it will cause a decided increase in the amplitude of the positive half. Hence the current is a function of the voltage. The exact nature of this function depends upon the tube characteristics, and must be determined by calibration.

The reading of either of these types of vacuum tube voltmeter depends upon the wave form. In the peak value type the true amplitude of the voltage is read, but the ratio of this value to the root-mean-square value depends upon the wave form. In the deflection type, since the reading is proportional to an integral of the current over a cycle, the wave form also enters quite decidedly. Consequently, these voltmeters should be calibrated on a wave of essentially the same form as that to be measured. They possess the advantage, however, of being adaptable as high resistance voltmeters, since the resistance across which $V_x$ is applied is limited only by the grid-cathode resistance of the tube used. When very high values of this resistance are used, it will be necessary to shield the circuit very carefully, as is done when using an electrometer or an electrometer tube. The frequency limitations are imposed by the distributed inductance and capacitance of the various resistances in the circuit and by the interelectrode capacitances in the tube itself. By proper choice of resistances and tube the meter may be used into the radio frequency range.

**80. The Double Tube Vacuum Tube Voltmeter.**

The advantages of the vacuum tube meter discussed in the last article are its comparative independence of frequency and its characteristically high impedance, while its most serious disadvantage is the serious dependence of the readings on the wave form. By sacrificing slightly the maximum obtainable accuracy it is possible to build a vacuum tube meter which gives readings proportional to the square of the voltage impressed across the terminals, and is therefore independent of wave form. This meter depends for its operation on the fact that for some commercial tubes there is a portion of the transfer curve, near the cut-off value of the grid voltage, over which the plate current, $I_p$, and the grid voltage, $V_g$, other factors being constant, are related by the equation

$$I_p - I_0 = k_2(V_g - V_0)^2 + k_3(V_g - V_0)^3, \tag{1}$$

$V_0$ being a fixed value of the grid bias slightly above cut off and $I_0$ being the plate current which flows when this bias is applied. For suitable tubes $k_3 \ll k_2$.

The circuit of the double tube meter is shown in Fig. 71. If $k_{21}$ and $k_{31}$ are the values of the two coefficients in Eq. (1) for the upper tube, $k_{22}$ and $k_{32}$ are the coefficients for the lower tube, and $\mu$ is the common amplification factor, the resistances are so chosen that

$$R_{p1}/R_{g1} = R_{p2}/R_{g2} = \mu, \tag{2}$$

$$R_{g1} = k_{31}/2k_{21}^2 \quad \text{and} \quad R_{g2} = k_{32}/2k_{22}^2. \tag{3}$$

The potential difference across $V_A$ is so chosen that

$$V_A = -V_0 - V_p/\mu, \tag{4}$$

where $V_p$ is the plate potential. Now suppose that a voltage $V$ is applied across the points $B$–$C$, $B$ being positive. The lower tube, in general, will be biased to below cut off, so it will make no contribution to the plate circuit current. The new grid potential on the upper tube (Tube 1) will be

$$V' = VR_1/(R_1 + R_2) + V_0 - I_pR_g = V_1 + V_0 - I_pR_g,$$

if we assume that the change in the plate potential, $(I_p - I_0)R_g$, is negligible. Substituting this expression into Eq. (1) and neglecting squares and higher powers of $k_3V_1/k_2$ and of $(I_p - I_0)R_g/V_1$ and all terms in $I_0R_g/V_1$ (which will be justified except for very small values of $V$ which have already been excluded by assuming cut off on Tube 2) we find

$$I_p - I_0 = k_{21}V_1^2 - 2k_{21}V_1(I_p - I_0)R_g + k_{31}V_1^3$$
$$= k_{21}V^2R_1^2/(R_1 + R_2)^2 = KV^2, \quad (5)$$

by Eq. (3), so that the galvanometer readings are proportional to the square of the voltage and this instrument may be calibrated

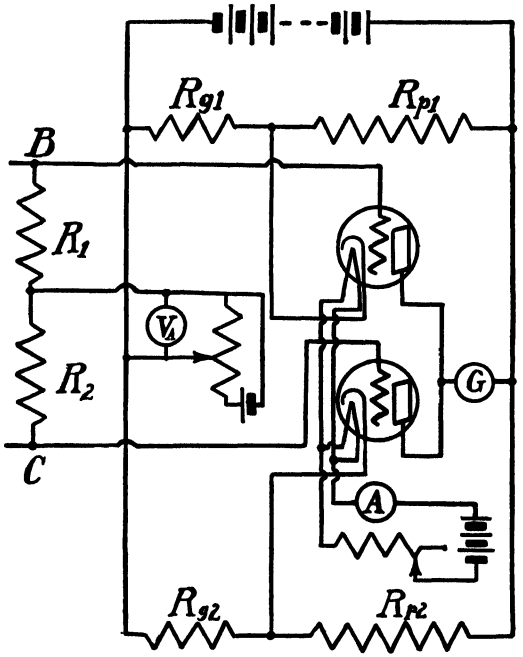



FIG. 71. The double vacuum tube voltmeter. (By permission of Review of Scientific Instruments)

to read root mean square values. By considering the polarity of $B$ and $C$ to be reversed, we derive an exactly similar expression for Tube 2, and to insure that the meter shall read the same

direct and reversed we must choose

$$R_1^2/R_2^2 = k_{22}/k_{21}. \tag{6}$$

Equations (2), (3) and (6) determine all the resistances in the circuit except the input resistance $R_1 + R_2$ which may be chosen at will, subject to the limitation of the grid-cathode resistances of the tubes and to shielding difficulties. Since the zero input grid voltage $V_0$ to give a constant $I_0$ varies with the plate voltage according to the relation

$$V_0 = -V_p/\mu - V'',$$

where $V''$ is a constant, it will be seen that Eqs. (2) and (4) adjust the resistance in such a way that variations of the supply voltage for the plate circuit are automatically compensated.

Due to the approximations made in the theory and the inexactitude of Eq. (1) for available tubes, the double tube voltmeter is apt to be in error by about five per cent. This precision has been obtained with the circuit of Fig. 71 using two Type 37 tubes and choosing the values $V_p \approx 75$ volts and $V_A = -1.66$ volts, to allow operation from a 90 volt "B" battery. If the circuit is shielded and non-inductive resistors are used, the instrument is good into the radio-frequency range.

## EXPERIMENT 31

### CALIBRATION OF A VACUUM TUBE VOLTMETER

The apparatus will be set up as in Fig. 72. Check the filament current and plate potential frequently, and hold them constant throughout the experiment. The meter will be calibrated on the commercial supply, which should have a good sinusoidal form, and the standard will be a good electrodynamometer voltmeter calibrated at the frequency of the supply. By means of the potentiometer, vary the voltage from zero to three volts by ½ volt steps. At each voltage start with a value of $V_g$ which allows some plate current to flow, and increase this potential until the milliammeter reading just drops to zero. From the reading of the electrodynamometer and Eq. (2) of Art. 73, calculate the maximum value of the voltage. Plot

two curves, one of grid voltage necessary to reduce the current to zero against maximum voltage, the other against root-mean-square voltage for a sinusoidal supply. The intercept of the former will give $V_0$.

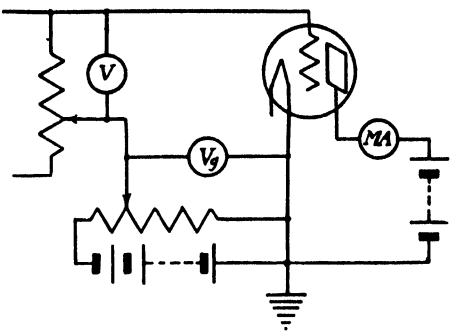



FIG. 72. Calibration of a vacuum tube voltmeter.

Next, calibrate the meter for a deflection instrument on a sine wave. Hold the grid potential constant at the value given by the instructor, and vary the unknown voltage as before. Plot a curve of milliamperes to the plate against unknown root-mean-square voltage, specifying, of course, the values of the quantities which were held constant.

**81. Attenuators.**

When the alternating potential of a low resistance source is to be measured, and only meters of lower range than that necessary for the measurement are available, it is possible to extend the range of these instruments by devices similar to the volt box of Art. 39 or by series resistors, as used with direct current meters. In many alternating current measurements, however, the source is a vacuum tube or other high impedance device and, as has been shown in Arts. 68 and 71, the potential difference in the output depends very appreciably on the impedance of the voltmeter used. When it is desired to measure the voltage output at a critical load impedance, it is therefore necessary to use some sort of impedance matching device. While transformers may be used for this purpose, they seldom possess characteristics independent of frequency in the audio frequency range, and as a result most precise measurements at these frequencies are made with the help of an *attenuation network* or *attenuator* composed of pure resistances. While many combinations of resistors may be used for this purpose, they are all reducible to the relatively

simple "T" network shown in Fig. 73, and only that form will be treated here. Suppose that we are designing such a network so as to satisfy the conditions for maximum power transfer for a source of internal resistance $r_i$, as derived in Art. 71. It is then

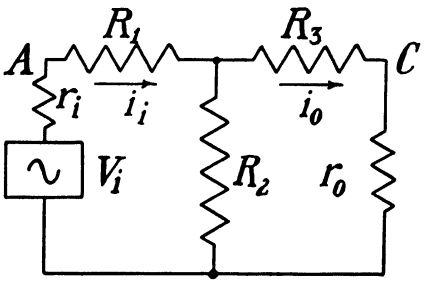



FIG. 73. The T-pad attenuator.

desirable that its resistance as measured between the terminals $A$–$B$ shall be equal to $r_i$, and as measured between $B$–$C$ shall be equal to $r_0$, the impedance of the voltmeter. Moreover, let us suppose that we desire that the *attenuation* shall be

$$\alpha = -10 \log_{10} \frac{i_0^2 r_0}{i_i^2 r_i}. \tag{1}$$

Applying Kirchoff's laws to the circuit as shown, we obtain

$$\begin{aligned} i_i(r_i + R_1 + R_2) - i_0 R_2 &= V_i, \\ -i_i R_2 + i_0(R_2 + R_3 + r_0) &= 0. \end{aligned} \tag{2}$$

which on solution for the input resistance yields

$$R_{AB} = \frac{V_i}{2i_i} = \frac{Z^2}{2(R_2 + R_3 + r_0)} = r_i \tag{3}$$

where

$$Z^2 = (r_i + R_1 + R_2)(R_2 + R_3 + r_0) - R_2^2$$

and where the final equality is given by the condition for power matching. Since interchanging $R_1$ with $R_3$ and $r_i$ with $r_0$ leads to the interchange of $A$–$B$ with $C$–$B$, as far as the T-pad itself

is concerned, the condition for output matching is

$$R_{BC} = \frac{Z^2}{2(r_i + R_1 + R_2)} = r_0. \tag{4}$$

If we now solve Eqs. (2) for $i_i$ and $i_0$ and substitute into Eq. (1), we obtain a third relation

$$\alpha = -10 \log_{10} \left[ \frac{R_2^2 r_0}{(R_2 + R_3 + r_0)^2 r_i} \right] = -L \log_e \left[ \frac{R_2^2 r_0}{(R_2 + R_3 + r_0)^2 r_i} \right]$$

where $L = -10 \log_{10} e = 4.3429$. Hence

$$\frac{R_2^2 r_0}{(R_2 + R_3 + r_0)^2 r_i} = e^{-\alpha/L}. \tag{5}$$

Solving this equation for $(R_2 + R_3 + r_0)^2$ and substituting into Eq. (3) after squaring both sides, we find

$$r_i = \frac{Z^4 e^{-\alpha/L}}{4R_2^2 r_0}. \tag{6}$$

But Eqs. (3) and (4) give

$$(r_i + R_1 + R_2)(R_2 + R_3 + r_0) = (r_i/r_0)(R_2 + R_3 + r_0)^2 = Z^2 + R_2^2.$$

Hence from Eq. (5)

$$Z^2 = -R_2^2(1 - e^{\alpha/L}). \tag{7}$$

Substituting this value into Eq. (6) yields

$$R_2 = \frac{2\sqrt{r_i r_0}}{e^{\alpha/2L} - e^{-\alpha/2L}} = \frac{\sqrt{r_i r_0}}{\sinh(\alpha/2L)}. \tag{8}$$

If this value of $R_2$ is now inserted into Eq. (7) and the resulting value of $Z$ is substituted into Eq. (4), we find that

$$R_1 = r_i \coth(\alpha/2L) - R_2. \tag{9}$$

Similarly, substituting in Eq. (3), we are led to

$$R_3 = r_0 \coth(\alpha/2L) - R_2. \tag{10}$$

Eqs. (8)–(10), with the help of a table of hyperbolic functions, determine the resistances for a T-pad attenuator (and consequently for any other form) which gives the desired attenuation for any combination of $r_i$ and $r_0$. The only limitation on the design is that $\alpha$ must be chosen of sufficient magnitude so that $R_1$ and $R_3$ do not assume negative values, and this means that there is a minimum value of the attenuation for a given pair of matching impedances.

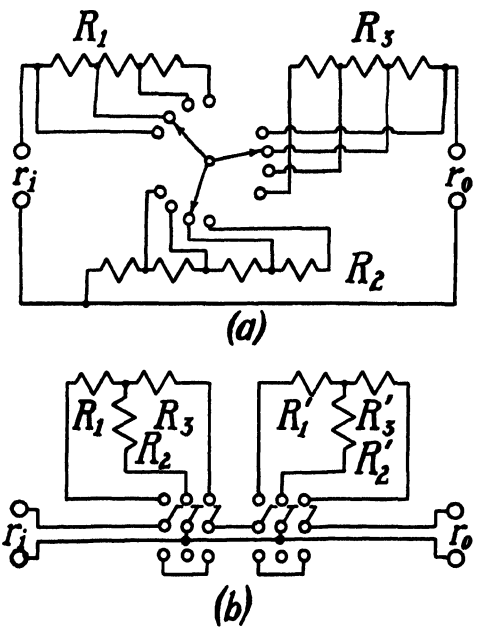



FIG. 74. Attenuation networks.

For variable attenuation, networks are built either with adjustable resistances, as shown in Fig. 74*a*, or as a series of simple pads which may be connected together, as in Fig. 74*b*. When the latter arrangement is used it is necessary, of course, that the input and output impedances be identical for all stages. Fig. 75 shows several mounted attenuators, of the types described. Such instruments have constants independent of frequency for values

of the latter below those for which the effect of the inductance and distributed capacitance of the windings becomes appreciable. Commercial instruments of the type shown are carefully wound to minimize these effects. If it becomes necessary to build an

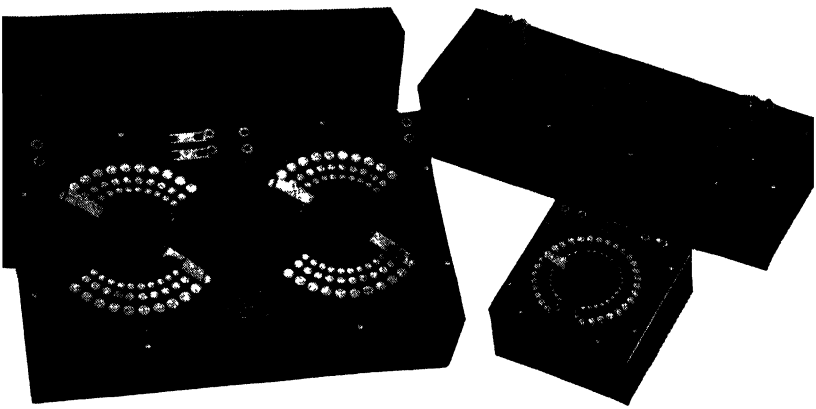


FIG. 75. Commercial attenuation boxes. (Courtesy of General Radio Co.)

attenuator from laboratory resistances for alternating current work, only shielded, non-inductively wound resistance boxes should be used.

## EXPERIMENT 32

### MEASUREMENT OF THE VOLTAGE AMPLIFICATION OF AN AMPLIFIER WITH AN ATTENUATOR

Set up the circuit of Fig. 76, using an audio frequency oscillator * as a source of supply. The output of this oscillator is fed to an attenuator which may be either of a commercial type or may be set up using three adjustable resistance boxes of the type described in Art. 101. Since the input impedance of the amplifier will probably be much higher than the output impedance of the attenuator, the resistance $R_1$ must be inserted to make the effective input resistance correct. The output impedance of the amplifier must also be matched (cf. Art. 71). If the instrument

* See Art. 94.

under test is a power amplifier the resistance of the meter circuit, i.e. between the points $P$ and $Q$, should be equal to the output impedance; if it is a voltage amplifier this resistance should be the same as the input impedance of the instrument with which it is intended to operate. As the indicating meter, $V$, any sensitive A.C. instrument, such as a vacuum tube voltmeter, may be used. With most amplifiers the circuit can be set up as shown,

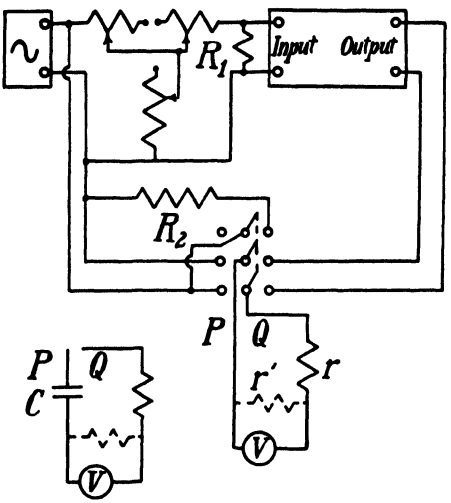



FIG. 76. Measurement of amplification with an attenuator.

with either $r$ or $r'$ chosen in such a way that the resistance between the points $P$ and $Q$ is of the correct magnitude. The parallel resistance $R_2$ is equal to the impedance of the meter circuit and is inserted so that the load on the oscillator is independent of the position of the double throw switch. If, however, the amplifier is so designed that any direct current may be supplied from its output terminals, it will be necessary to connect the meter as shown in the inset, where $C \gg 1/2\pi f_m r_0$, $f_m$ being the maximum frequency at which the amplifier is to be tested.

With the oscillator adjusted to a frequency $f_1$, the lowest for which the amplifier is to be used, and with the switch thrown to the left, adjust the input voltage to the attenuator to a fixed value, in the range of those that the amplifier will be expected to

supply from its output. Throw the switch to the right and adjust the attenuator until the meter, as nearly as possible, reproduces its former reading, then check the input voltage. Repeat at frequencies $2f_1$, $3f_1$, etc., until $f_m$ is reached, keeping the same reading on $V$ for each frequency.

Now suppose that the input voltage is $V_i$ and that the corresponding output voltage is $V_0$. If $k_v$ is the voltage gain of the amplifier, $V_0 = k_v V_i$, and the power supplied to the attenuator is $k_v^2 V_i^2/r_i$, where $r_i$ is its input impedance. If the attenuation factor, in decibels, is $\alpha$, the power supplied to the amplifier is

$$\frac{k_v^2 V_i^2}{r_i} 10^{-\alpha/10} = \frac{V_i^2}{r_0},$$

the equality being insured by the equal readings of the meter. Hence

$$k_v = \sqrt{\frac{r_i}{r_0}} 10^{\alpha/20},$$

and the voltage amplification

$$\alpha_g = 20 \log_{10} k_v = 10 \log_{10} (r_i/r_0) + \alpha.$$

Plot the amplification against the frequency, using a logarithmic scale for the latter. On the curve specify the equivalent output impedance used with the amplifier, the uncertainty in $\alpha_g$ and the input level in volts.

**82. Null Method Indicators.**

As was seen in the earlier chapters of this book, a great many direct current measurements are made most easily and with the greatest accuracy by balancing two quantities against each other, and thus reducing the quantity measured by the deflection instrument to zero. The same is true of alternating current measurements; hence it is desirable to study instruments particularly adapted to this type of work.

Any of the instruments discussed above may, of course, be used as null indicators. However, several instruments which are not commonly used for measuring currents are quite widely used for bridge measurements.

The first of these, used throughout the audio frequency range, is the *telephone receiver*. This is particularly well adapted for circuits in which there is direct current flowing in addition to the alternating current to be balanced, as it records only variations in current, and hence only the A.C. component will be detected. When a source which is reasonably free from harmonics is used, this simple indicator is probably as good as any available for most work. Trouble arises, however, when harmonics are present, as, in general, the harmonic frequencies will not be balanced at the same time as the fundamental, and the minimum of sound will be quite difficult to judge. Even in this case, however, the difficulty may be partly overcome by the use of a tuned telephone receiver; that is, one which may be adjusted to respond to the desired frequency with a much greater sensitivity than it has for harmonic frequencies. Another method sometimes used is to have a fundamental of about 5000 cycles, so that the response of the ear to the second harmonic is low.

When greater sensitivity than is obtainable with the ordinary head piece is required, one or more stages of amplification may be inserted between the bridge and the receiver. If a single stage is used it should be a power amplifier matched to the receiver impedance, if two or more stages are used all except the last should be voltage amplifiers. Transformer coupling is in general satisfactory. Under some conditions a loud speaker may be preferable to a receiver, and in this case the permanent magnet types are the better unless a D.C. source without appreciable fluctuations is available for field coil excitation. Difficulties with harmonic frequencies and with the general "noise" background due to small induced voltages, Brownian motion of electrons in the resistors of the amplifier, etc., may be minimized if the amplifier is tuned to a narrow band of frequencies * around that being supplied to the bridge.

Other instruments, such as the vibration galvanometer or vacuum tube devices, are also good null indicators and will be discussed in the next few pages.

* For the principles of this tuning see Art. 88.

**83. The Vibration Galvanometer.**

A nearly ideal instrument for null indication at low frequencies is the *vibration galvanometer*. The arrangement of this instrument, as manufactured by the Leeds and Northrup Co., is shown in Fig. 77. The arrangement is essentially that of a D'Arsonval galvanometer, with a very short period. This period may be adjusted in two ways,—first, by adjusting the effective length of the suspension by moving the screw at the top of the diagram; second, by adjusting the tension of the suspension by the screw at the bottom. The natural period may thus be adjusted to coincide with the period of the current. Now, combining Eq. (2) of Art. 23 with Eq. (1) of Art. 25,

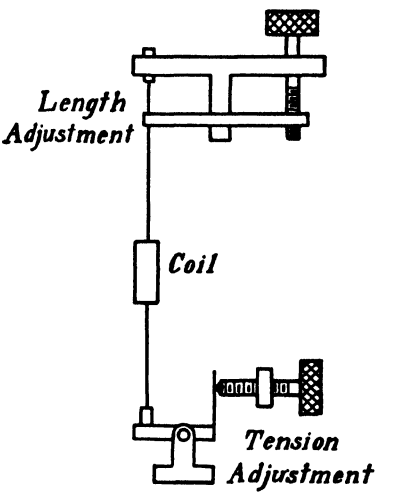


FIG. 77. The vibration galvanometer.

$$M\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + k_1\theta = TI_0 \sin 2\pi ft \qquad (1)$$

where $T$ is the constant of proportionality between the current and the torque, while $I_0$ is the maximum value of the imposed sine wave. In the vibration galvanometer the damping is always made very small. Under these circumstances the vibrating system will have a natural period, as shown by Eq. (4) of Art. 25, which is

$$f_0 = \frac{1}{2\pi}\left(\frac{k_1}{M} - \frac{D^2}{4M^2}\right) \approx \frac{1}{2\pi\sqrt{M/k_1}}. \qquad (2)$$

If we try, as a particular solution of Eq. (1)

$$\theta = \frac{TI_0}{k} \sin (2\pi ft - \alpha), \qquad (3)$$

we find, upon substituting and separating terms in sin $2\pi ft$ and cos $2\pi ft$, that the differential equation is satisfied if

$$\cot \alpha = \frac{2\pi fD}{4\pi^2 f^2 M - k_1}$$

and

$$k = \{(4\pi^2 f^2 M - k_1)^2 + 4\pi^2 f^2 D^2\}^{1/2}. \tag{4}$$

Consequently the deflection is a periodic variable with a frequency equal to that of the impressed current. If the inverse of this frequency is small compared with the time of persistence of vision, the effect of an alternating current in the coil will be to stretch out the spot of light reflected from the galvanometer mirror onto the scale into a band of width proportional to the amplitude of the current. This band will have a decided maximum when the impressed frequency equals the natural frequency as given by Eq. (2), since under these circumstances the first term on the right of Eq. (4) vanishes. Strictly speaking, we should add to the expression for $\theta$ in Eq. (3) the solution of the homogeneous equation from Eq. (4) of Art. 25 but in practice this is not important because we deal with the steady state which is established after the negative exponential term has reduced this part of the solution to a negligible magnitude.

To investigate the response at harmonic frequencies we may take

$$f = nf_0 \approx \frac{n}{2\pi\sqrt{M/k_1}}$$

which, on substitution into Eq. (4) gives

$$k^2 \approx k_1^2(n^2 - 1)^2 + 4\pi^2 f^2 D^2.$$

In practice $D$ is always so small compared with $k_1/f$ that the second term is completely negligible with respect to the first unless $n \approx 1$. Consequently the ratio of the amplitude $A_n$ of the response to the $n$th harmonic to that for the fundamental, the currents being equal in the two cases, is

$$\frac{A_n}{A_1} \approx \frac{2\pi fD}{(n^2 - 1)k_1}.$$

Wenner, for one galvanometer, gives the values

$$k_1 = 5.7 \times 10^{-4} \text{ newton meters/radian, and}$$
$$D = 1.8 \times 10^{-9} \text{ newton meter seconds/radian,}$$

so that for this instrument the sensitivity for the second harmonic is about 1/1500 of that for the fundamental (100 cycles).

**84. Visual Electronic Indicators.**

In all of the instruments thus far discussed the final indication has been given by some sort of a pointer which is moved mechanically by the magnetic or thermal effects of an electric current.

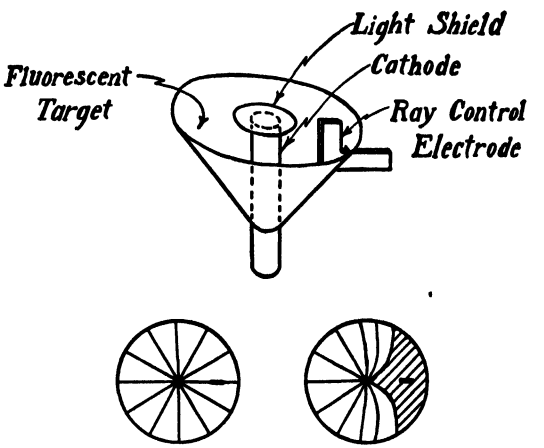



FIG. 78. The electron ray tube.

In a distinctly different type of indicating device a beam of electrons is controlled by electrostatic or electromagnetic fields and the intersection of this beam with a fixed surface is made visible by the fluorescence excited by the impact of the electron beam on zinc sulphide or a similar material. Chief among these instruments is the *cathode ray oscillograph*, which will be discussed in the next chapter. Another, in which we are more immediately interested, is the *electron ray tube*.

The essential elements of an electron ray tube are shown schematically in Fig. 78. A cylindrical cathode emits electrons which are collected by a conical target acting as a cathode. The

inside of this target is coated with fluorescent material and glows with a green light when struck by electrons. On one side of the cathode is a control electrode which is operated at a potential between that of the cathode and that of the target. When this electrode is approximately at the potential of the target the field is nearly symmetrical and the electrons will flow as shown in the sketch on the left, causing the whole target to glow. If, on the other hand, the control electrode is appreciably negative with respect to the target the electrons will be repelled from it, the lines of flow will be similar to those shown on the right, and the area shown cross hatched will be shadowed. The angle of the shadow varies from 0° to about 100°, depending on the potential of the control electrode.

In practice the electron ray tube is enclosed in the same envelope as a triode and the usual connection is indicated in Fig. 79. The control voltage is applied to the grid of the triode element through the terminals *A–B* and the triode plate current flows through a resistance *R*, the drop across which determines the difference of potential between the control electrode *C* and the target *T*. These two electrodes will have potentials sufficiently close to give zero shadow angle only if the grid potential is nearly at cut-off or is more negative than this. Any change of the potential in a positive direction will then cause the control electrode to assume a negative value with respect to the target and will produce a shadow of angular size depending on the difference between the grid voltage and the cut-off potential. Consequently the tube may be used as an approximate voltmeter, of limited range and variable sensitivity.

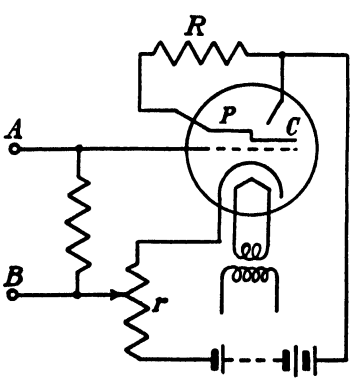



FIG. 79. Electron ray tube circuit.

As an alternating current null indicator the tube operates in a manner very similar to that of the peak value vacuum tube voltmeter of Art. 79. By adjustment of the self biasing resistor *r*

the grid may be biased just to the closing potential. Any alternating potential applied to $A$–$B$ will then be effective only during the positive half cycle, during which time it will cause a shadow. If the inverse of the frequency is small compared with either the time of persistence of vision or the decay period of the fluorescence this shadow will have sharp edges. It will reach a minimum width when the amplitude of the signal is zero.

Koehler has measured the characteristics of a 6E5 electron ray tube used with one pentode stage of amplification between it and the bridge and has obtained sensitivities of $2 \times 10^{-7}$ volt per degree of shadow. Such an arrangement may be operated from a rectified supply and has the advantage over auditory methods that alternating current components in this source, resulting from insufficient filtering, cause very little difficulty.

**85. The Lock-in Amplifier.**

One instrument which possesses distinct advantages as a bridge detector is the lock-in amplifier proposed by Cosens, which is shown in Fig. 80. The difference in the plate currents of two nearly identical vacuum tubes is measured by means of the galvanometer $G$. The voltage to be observed is supplied to the grids of these tubes, in parallel, through the transformer $T_1$ and the phase shifter $S$. Their plate potentials are supplied from two secondaries of the transformer $T_2$ so connected that they are 180° out of phase. The primary of $T_2$ is fed from a source of the same frequency as the voltage being measured, the usual arrangement, when this is the potential difference across the galvanometer terminals of a bridge, being to use the same source that supplies the bridge. If the grids of the two tubes were maintained at a constant potential $V_c$, the plate current flowing through tube $I$ during that half of the cycle when its plate is positive would be identical to that flowing through tube $II$ during the other half, and if the period of the galvanometer is long compared with the inverse frequency no deflection would result. Now suppose that the alternating voltage applied to the grids is positive during that half cycle during which tube $I$ is delivering current, then its plate current will be increased. But during the next half cycle the grid voltage will be negative and the current of tube $II$ will be decreased. Hence an average difference of

potential will build up across the galvanometer and a reading will be obtained. The condenser $C_0$ increases the effective period of the galvanometer since it averages out any fluctuations which take place in times appreciably less than the characteristic time $\tau = C_0(R_1 + R_2)$ of the output circuit. Adjustment of the resistance $R_2$ compensates for small differences in the character-

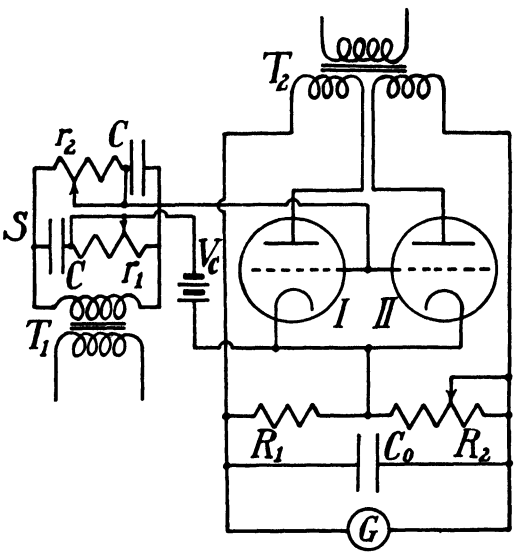



FIG. 80. The Cosens lock-in amplifier.

istics of the two tubes. With no signal applied to the grids, this resistor may be adjusted until a zero reading of $G$ is obtained. The grid biasing potential $V_c$ prevents the grids from swinging positive and drawing current, which might be fed back into the bridge network.

The above discussion tacitly assumes that the grid and plate potentials are in phase,* and the maximum sensitivity is obtained only if this condition is satisfied. Since the ordinary A.C. bridge in general has an output voltage which is not in phase with the input, the phase shifter $S$ is provided. In this part of the circuit the contacts are mechanically connected so that $r_1 = r_2$ at all times, and with the circuit as shown phase shifts of about 120°

* For amplification of this paragraph see Chap. IX.

can be brought about by changing the resistance. This is particularly convenient for bridge work, where it is often desirable to balance separately the in-phase component and that which is 90° separated from it.

One of the distinct advantages of the lock-in amplifier is that it eliminates "noise," due to causes given in Art. 82. The tuned amplifiers suggested in that article have one serious disadvantage. They are maximally effective only if the band of frequencies to which they respond is narrow, yet under these conditions they require a very constant frequency source if they are to maintain constant sensitivity. The Cosens amplifier, however, is always locked to the supply frequency of the bridge. If this frequency is $f_1$ and some other frequency $f_2$ is applied to the grids, the phase relationship between the two will be continuously changing and a beat note of frequency $f_1 - f_2$ will be set up. If $\tau = C_0(R_1 + R_2) \gg 1/|f_1 - f_2|$, the meter will be unable to follow this beat frequency and will read zero. Hence for frequencies far from that applied to the plate there is no sensitivity. On the other hand, if the grid and plate frequencies are nearly equal so that $\tau < 1/|f_1 - f_2|$, the reading of $G$ will fluctuate between positive and negative values with the beat frequency. For a random distribution of noise frequencies, however, the average effect will still be zero.

The instrument as described is manufactured by the Cambridge Instrument Company.

**86. The A.C. Potentiometer.**

The potentiometer, which found such wide applications in direct current measurements, has also been applied to alternating current standardization work. Since two alternating potentials, in order to balance, must be alike in both amplitude and phase angle, it is necessary to use some sort of *phase shifter* with these instruments. Since their application is somewhat limited, and the operation varies considerably with the different instruments, no detailed discussion will be given here, but the experimenter is referred to the manufacturers' bulletins.

The principal application of A.C. potentiometers is in the standardization of commercial frequency electrodynamometer laboratory standards.

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# CHAPTER 8

## OSCILLATORS AND OSCILLOGRAPHS

**87. Laboratory Sources of Alternating Current.**

A great many measurements, some of which have been discussed in the preceding chapter, demand a power supply of alternating current at a fixed or adjustable frequency. Since the type of apparatus used for this supply varies with the desired frequency, it may be worth while to summarize the range of frequencies most commonly used, together with the wavelengths of the associated electromagnetic radiations:

| | Frequencies (cycles/sec.) | Wave Lengths (meters) |
|---|---|---|
| Commercial | 15–100 | $2 \times 10^{7}$–$3 \times 10^{6}$ |
| Audio | 30–15000 | $1 \times 10^{7}$–$2 \times 10^{4}$ |
| Supersonic | 15000–300000 | $2 \times 10^{4}$–$1 \times 10^{3}$ |
| Radio | $3 \times 10^{5}$–$3 \times 10^{10}$ | $1 \times 10^{3}$–$1 \times 10^{-2}$ |

In the range of commercial frequencies no essential problem is presented, as the various types of rotating machinery supply power in large amounts, with very constant frequency and very little harmonic content.* When large amounts of power are required, alternators may be used throughout the audio frequency range, but the mechanical requirements become very difficult to meet at frequencies above these. Fortunately for most laboratory measurements only relatively small amounts of power are needed, but it is desirable that the frequency be held quite constant. These requirements are generally met by various types of *oscillators*. The simplest of these, perhaps, is the *microphone hummer*. This instrument, as manufactured by the General Radio Company, is shown schematically in Fig. 81. The frequency control is obtained by means of a tuning fork, which is driven at its natural period. This is accomplished by a field coil, permanently connected across the D.C. supply, and an armature coil supplied by the secondary of the input transformer. The

* See Art. 96.

primary of this transformer is connected across the battery in series with the "button," which consists of a movable diaphragm, driven by the fork and pressing on carbon granules, similar to a telephone transmitter. When the fork is vibrating, the pressure on these granules varies, and since the resistance of the button varies nearly linearly with the pressure, a sinusoidal wave is

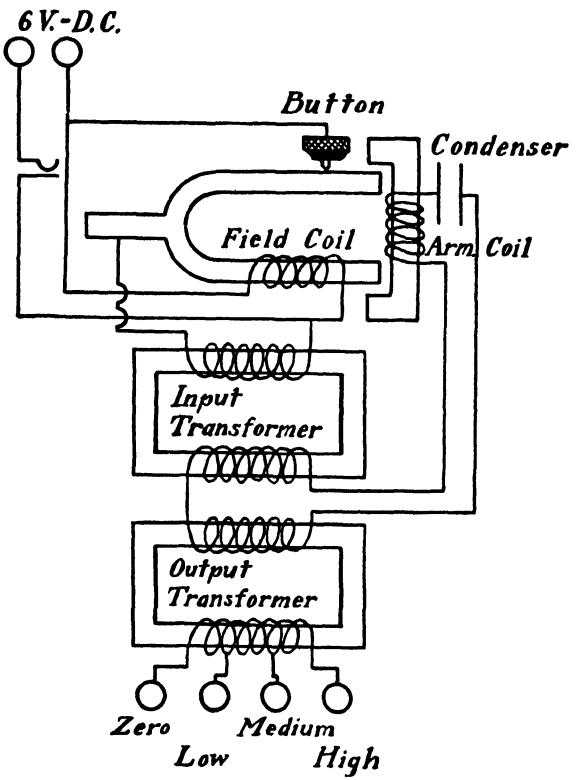


FIG. 81. The microphone hummer.

superimposed on the D.C. supply. This, in turn, induces an alternating current in the secondary circuit, and thus supplies energy for continuing the oscillations of the fork, as well as supplying the output transformer. In this particular instrument, three taps are supplied, for output voltages of 0.5, 1.5, or 5.0 volts. The capacity is 0.06 watt, at 1000 cycles.

The microphone hummer is in general characterized by rather bad wave form. When a purer wave or larger amounts of power than it will supply are required, or when the desired frequencies are above those at which a mechanical system can easily be designed to operate, various types of *electron tube oscillators* are used, and these instruments will be discussed in the following pages.

**88. The Oscillating Circuit.**

Consider the circuit of Fig. 82, in which $L$ is a pure inductance, $R$ is a pure resistance and $C$ is a pure capacitance. If the condenser is charged initially, we may set up a current by closing the switch. Taking the direction of the current, $i$, as positive when the condenser charge, $q$, is increasing, we have $i = dq/dt$. By definition the voltages across the condenser, resistance and inductance are respectively $q/C$, $iR$, and $Ldi/dt$ so that, applying Kirchoff's second law that the voltage drop around such a circuit is zero, we have:

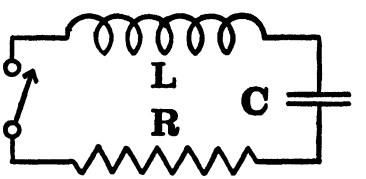


FIG. 82. The oscillating electrical circuit.

$$\frac{q}{C} + L\frac{di}{dt} + Ri = 0, \tag{1}$$

or, taking the derivative with respect to time,

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0. \tag{2}$$

But, with proper substitutions for coefficients, and with a change of variable from $i$ to $\theta$, this equation is identical with Eq. (1) of Art. 25. Hence, in the case where

$$\frac{R^2}{4L^2} - \frac{1}{CL}$$

is negative, the solution is of the form

$$i = ae^{-(R/2L)t}\cos(\mu t + b), \tag{3}$$

where

$$\mu^2 = \frac{1}{CL} - \frac{R^2}{4L^2}.$$

If we compare this equation with that of a sinusoidal wave

$$i = a \cos (2\pi ft + b)$$

where $f$ is the frequency, we see that the current follows a sine wave with logarithmically decreasing amplitude, and has the frequency defined by

$$(2\pi f)^2 = \frac{1}{CL} - \frac{R^2}{4L^2}.$$

The only case which is of practical interest is that in which the damping constant, $R^2/4L^2$, is small compared with $1/CL$. In this case

$$f \approx \frac{1}{2\pi\sqrt{LC}}. \tag{4}$$

If we now open the circuit of Fig. 82 at any point and insert a source of voltage of amplitude $V$ which varies sinusoidally with time, Eq. (1) is replaced by

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = \frac{dv}{dt} = 2\pi fV \cos 2\pi ft. \tag{5}$$

Since this equation, with appropriate change of variable, is identical with Eq. (1) of Art. 83, its solution is

$$i = \frac{V}{k} \cos (2\pi ft - \alpha), \tag{6}$$

where

$$k^2 = R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2 \tag{7}$$

and

$$\cot \alpha = 2\pi fRC(4\pi^2 f^2 LC - 1)^{-1}. \tag{8}$$

In obtaining these expressions we have, as in Art. 83, neglected transient terms, so our solution is valid only in the steady state. From Eq. (6) we see that a sinusoidally varying E.M.F. will

produce a sinusoidal current in the circuit, but that this current will be out of phase with the voltage as shown by Eq. (8).

The dependence of the amplitude of the current, $V/k$, on the impressed frequency, $f$, is of some interest. Obviously $k$ has a minimum value, and $V/k$ a maximum, when the term in parenthesis in Eq. (7) is zero, or when $f = f_0$, hence the maximum amplitude will occur when the frequency impressed on the circuit is the natural resonant frequency as given by Eq. (4) and for this frequency $k = R$. If the impressed frequency is not that of resonance, but differs from it by a small amount so that

$$f = f_0 + \Delta f,$$

the magnitude of $k$ will be greater. To investigate this case we shall introduce a new constant of the circuit

$$Q = \frac{2\pi f_0 L}{R}. \tag{9}$$

If we eliminate $L$ and $C$ from Eq. (7) by the use of Eqs. (4) and (9) we find, neglecting the cube and higher powers of $(\Delta f/f_0)$, that

$$k = R\left[1 + 4Q^2\left(\frac{\Delta f}{f_0}\right)^2\right]. \tag{10}$$

This equation shows that the sharpness of resonance is determined by the $Q$ of the circuit. For example, when $\Delta f/f_0 = 1/2Q$, the magnitude of $k$ will be twice that which it has at resonance. Consequently, if the resistance of the circuit is low compared with the inductive reactance, $2\pi f_0 L$, the current through the circuit for a given impressed voltage will fall off very rapidly as the frequency departs from that of resonance.

**89. The Radio Frequency Oscillator.**

The results obtained in the last article point the way to the construction of the vacuum tube oscillator for supplying continuously an alternating current of high frequency. The simple circuit of such an oscillator is shown in Fig. 83. If $R_0$ is the plate resistance of the tube, $\mu$ its amplification factor, $e_g$ the variable component of the grid potential and $M$ the mutual inductance between $L$ and $L'$, then we have the relations,

taking $E_c$ and $E_B$ as the grid and plate potentials:

$$\frac{q}{C} + Ri_L + L\frac{di_L}{dt} = 0, \tag{1}$$

$$\mu e_g = R_0 i_P + \frac{q}{C} + E_B, \tag{2}$$

$$e_g = -M\frac{di_L}{dt} - E_c, \tag{3}$$

$$i_C = i_L + i_P. \tag{4}$$

If the value of $q/C$ from Eq. (2) is substituted in Eq. (1), and $e_g$ and $i_P$ are then eliminated by Eqs. (3) and (4), the resulting

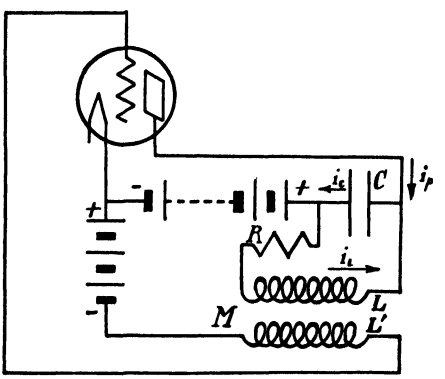


FIG. 83. The vacuum tube oscillator.

equation may be solved for $i_c$. If Eq. (1) is then differentiated and this value of $i_c$ used in place of $dq/dt$, it will be found that

$$L\frac{d^2i}{dt^2} + \left(\frac{L - \mu M + CRR_0}{CR_0}\right)\frac{di}{dt} + \frac{R + R_0}{CR_0}i = \frac{\mu E_c + E_B}{CR_0}, \tag{5}$$

where $i$ is the $i_L$ used above.

Now, in order to produce continuous oscillations, the damping constant must be equal to or less than zero. Equating it to zero, to get the conditions for which oscillation can just take place, we have

$$\frac{L - \mu M + CRR_0}{CR_0} = 0,$$

so that the condition for oscillation is

$$L - \mu M + CRR_0 \leqq 0. \tag{6}$$

This quantity may be negative either with $L$, $C$, $R$ and $R_0$ all positive and $\mu M$ large, or with $\mu M$ small and $R_0$ negative. The first case applies to a *regenerative circuit* in which sufficient energy is fed back to the oscillating or tank circuit from the B-battery and through the tube to supply the losses, while the second applies to negative resistance oscillators in which the slope of the plate current-grid voltage curve, as obtained in the complete circuit, is negative. If the damping constant is less than zero, oscillations will start in the circuit and the amplitude of $i$ will increase with time. This, obviously, cannot continue indefinitely and in practice the time rate of increase of the amplitude rapidly decreases to zero due to a change of the tube characteristics $\mu$ and $R_0$ with increasing amplitude. In the steady state of oscillation which then prevails, the damping constant is maintained at its upper limit of zero.

Examining the oscillator in a more qualitative fashion, we see that the oscillating current flows in the tank circuit *L-C-R* and has its frequency determined by that circuit, which has a very low impedance for $f = f_0$ and for frequencies near this value. In order to maintain the amplitude of oscillation it is necessary that a relatively small alternating potential, of the correct frequency and phase, be impressed on the circuit, and the fact that these conditions may be satisfied is shown by Eqs. (5) and (6). However, since the impedance of the tank circuit is low not only for the exact resonant frequency but also for frequencies near it, oscillations may be set up for such frequencies and an oscillator of the type discussed may vary its frequency over a finite range. Eq. (10) of the previous article shows that the extent of the possible variation is determined largely by the $Q$ of the tank circuit.

Many variants of the circuit of Fig. 83 may be used for radio frequency oscillators. For example the tank circuit may be connected to the grid instead of the plate, or may interconnect these two elements. Multigrid tubes offer special advantages. For details of most of these circuits the reader is referred to texts dealing with communication applications of the vacuum tube.

One type, however, is sufficiently different from the simple circuit to deserve mention here. It is the *push-pull oscillator*, used principally for the generation of high frequencies. The circuit of one such oscillator is shown in Fig. 84. In this arrangement the grids of the two tubes are coupled to the plates through the condensers *C–C* and the resistor *R* in such a way that a positive change in the plate potential of one tube produces a negative change on the grid of the other. The combined effect, provided that the amplification factor of the tubes is sufficiently high, is

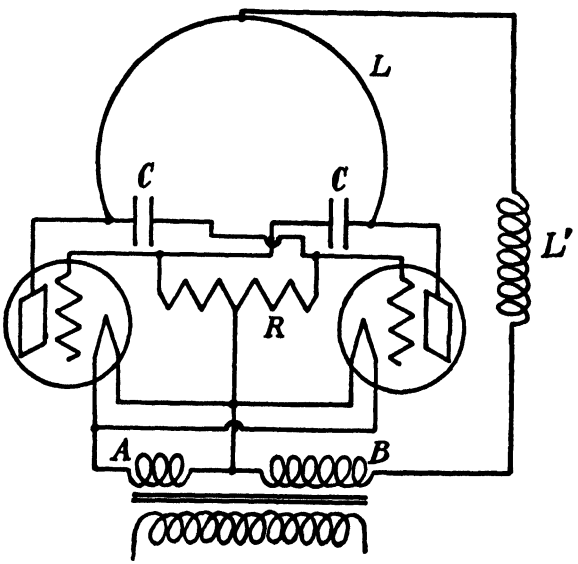


FIG. 84. Short wave push-pull oscillator circuit.

to provide a negative resistance circuit. *L'* is a radio frequency choke of such inductance that *L* is connected to the supply transformer by a low impedance path for commercial frequencies but is practically isolated from it at radio frequencies. One short wave oscillator using this circuit, with standard tubes and giving reasonable power output is manufactured by the Central Scientific Co. The assembled oscillator, with an adjacent wave meter (see Art. 90) is shown in Fig. 85. Here *L* is a single loop of copper. The supply is taken from the commercial 110 volt A.C. line through a transformer, *A* being a low voltage secondary for

the cathode heating current and $B$ a high potential winding for the plate supply. This apparatus oscillates at a wave length of 3.4 meters.

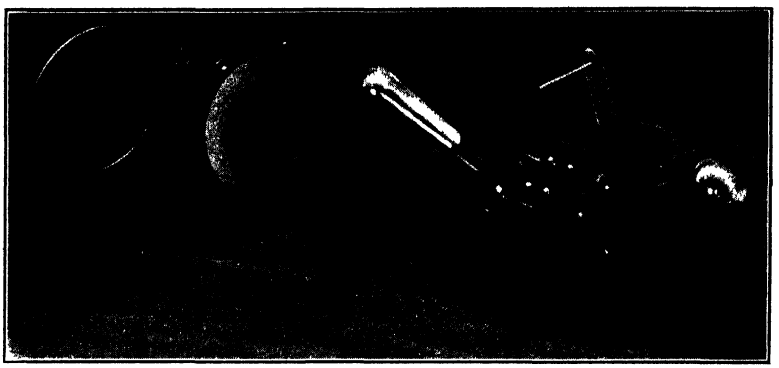

FIG. 85. Short wave oscillator. (Courtesy of Central Scientific Co.)

**90. Radio Frequency Measurement.**

Another application of the principles of resonant circuits is the wave meter, used for measuring the wave lengths (or frequencies) of high frequency oscillations. It consists of an inductance and an accurately calibrated variable condenser, together with a thermo-galvanometer or other means of detecting alternating currents of small magnitude. If the inductance is coupled with an oscillating circuit, the E.M.F. induced across it will act as the supply voltage and the current will depend on the relation between the impressed and resonant frequencies, reaching a sharp maximum at the resonance condition, as shown in Art. 88.

The circuit of the Type 724-A precision wave meter of the General Radio Co. is shown in Fig. 86. The resonant circuit of this instrument consists of the variable precision condenser $C$ and the inductance $L$, which is a plug-in coil. Actually seven such coils are supplied, for various parts of the range. The voltage across the condenser is applied to a vacuum tube voltmeter through the voltage divider consisting of the condensers $C_1$ and $C_2$. The vacuum tube (a triode used as a diode) serves the double purpose of rectifying the radio frequency signal so that it can be indicated by a D.C. meter and of acting as a current

limiting device to protect this meter in case too close coupling to the oscillator is obtained. The complete instrument, with a set of inductances, is shown in Fig. 87. One convenient mechanical feature of the 724-A instrument is the provision for rotating the inductance so as to bring it into the best position for coupling while the wave meter as a whole is stationary. The seven inductances allow frequencies from $16 \times 10^3$ to $5 \times 10^7$ cycles per second to be measured.

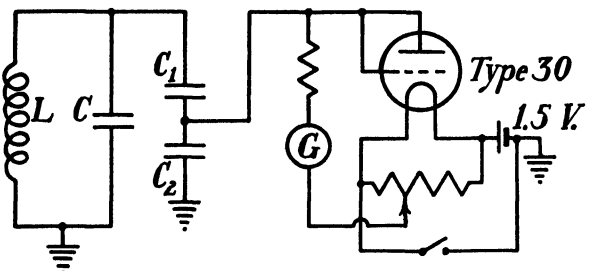


FIG. 86. Circuit of the Type 724-A wavemeter.

When using a wave meter care must be taken that the mutual inductance between the resonant circuit and the oscillator is not too great, as in such circumstances a double peak may be observed, with the true resonance lying between the two maxima. If this is avoided a single peak will be found, and the frequency of the radiation is given by Eq. (4) of Art. 88 as

$$f = \frac{1}{2\pi\sqrt{LC}}.$$

If the wave length is desired it may be obtained from the general equation of electromagnetic waves as

$$\lambda f = c,$$

where $\lambda$ is the wave length and $c$ is the velocity, equal to $2.998 \times 10^8$ meters per sec. In practice, of course, the frequency or wave length is read from the calibration curve of the instrument.

A more direct method of determining wave lengths, particularly in the short wave region (up to about ten meters) is furnished by *Lecher wires*. Suppose that a wave is started travelling along a wire. When it reaches the end of the wire, it encounters a

sudden change in conductivity, and, consequently, is reflected back along the wire. This is exactly analogous to the reflection of sound waves in an organ pipe, and all the phenomena of standing waves occurring in the acoustic case are duplicated here.

FIG. 87. Type 724-A precision wavemeter. (Courtesy of General Radio Co.)

Since the current and voltage are 90° out of phase if the circuit is resonant, as may be seen from Eq. (8) of Art. 88, the voltage loops will occur at current minima and vice-versa. The exact phenomena are, perhaps, best understood by reference to an experiment.

## EXPERIMENT 33

### CALIBRATION OF A WAVE METER WITH LECHER WIRES

Connect the apparatus as shown in Fig. 88. The coupling inductance is a single turn which couples with the inductance loop

on the short wave oscillator. The two Lecher wires are mounted taut and parallel to each other for a distance of at least twice the wave length. A flashlight bulb is mounted in series with the wires. Care should be taken that this bulb is so mounted that the lengths of the wires on each side of it are equal. The length of the lower wire includes, of course, the length of the inductance loop. Adjustment of this may be made by changing the lengths of the sections $M$ and $N$.

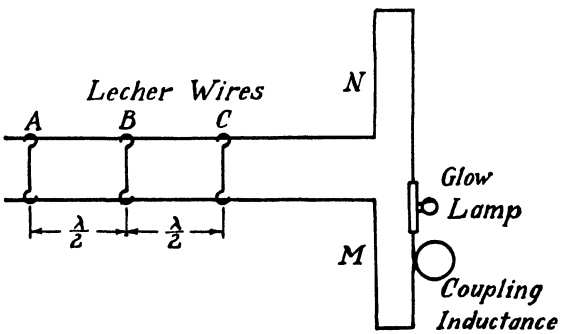


FIG. 88. Measurement of wave length with Lecher wires.

Now start the oscillator. If oscillations are occurring, a small neon tube, brought near to the loop, will glow brightly.

Near the end of the wires connect the short circuiting wire $A$. As this is moved along the wires, the current flowing to the wires will vary, until it reaches a decided maximum for one point, at which the length of the wires is an exact multiple of the wave length, and $A$ is at a current loop of the standing wave. This will be indicated by the increased brightness of the flashlight lamp, which is also at a current loop. Bring the wave meter near the oscillator and tune it to resonance. The wave length should be checked frequently throughout the rest of the experiment, and *should not change.*

Locate the next current loop with a second short circuiting wire, $B$, etc., until all the loops along the wire have been found. The wires should then be at intervals of one-half the wave length. Measure the intervals and, taking an average, calculate the correction to be applied to the meter at this wave length.

**91. Distributed Capacitance.**

In the discussions above we have considered throughout that the inductances, capacitances and resistances are entirely separated from each other. Such ideal conditions are never obtainable. The most serious deviation from the assumed condition

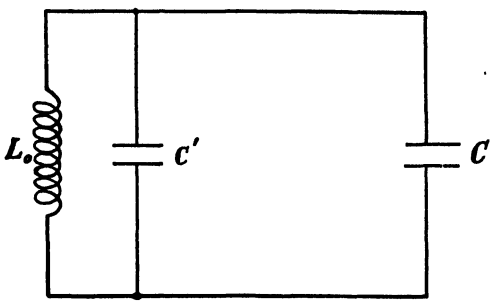


FIG. 89. Effect of distributed capacitance.

is in the inductance coils, which always possess resistance and capacitance. This capacitance is due to the electrostatic effects between turns and between the coil as a whole and the rest of the circuit. As a very simple first approximation, we shall consider that it acts like a condenser in parallel with the inductance. The resonant circuit will then be as shown in Fig. 89.

If the circuit is brought into resonance with a known frequency, $f$, by varying the capacitance, $C$, the existing relation is:

$$f^2 = \frac{1}{4\pi^2 L_0(C + C')}.$$

If $L$ is taken as the apparent inductance of the coil, then

$$f^2 = \frac{1}{4\pi^2 LC}.$$

Combining these equations,

$$LC = L_0(C + C'),$$

$$L = L_0 + L_0 C' \frac{1}{C}. \qquad (1)$$

Hence, if the apparent inductance is plotted against the inverse of the capacitance which gives resonance, an approximately straight line should result. The intercept at $1/C = 0$ is the true inductance of the coil, and the slope of the line is this inductance times the distributed capacitance. For a rigorous determination of these quantities the theory must be elaborated to take into account the separate effects of capacitance between turns, capacitance to ground, etc.

## EXPERIMENT 34

### MEASUREMENT OF INDUCTANCE AND DISTRIBUTED CAPACITANCE OF RADIO FREQUENCY INDUCTANCES

A vacuum tube oscillator, such as that shown in Fig. 83, will be used as a source of radio frequency supply, and its frequency will be measured with a wave meter. $C$ should be a variable condenser and there should be arrangement for varying the mutual inductance between $L$ and $L'$. A number of different coils should be used for $L$, so as to give a great frequency range. Adjust $C$ and the mutual inductance until the instrument starts oscillating. This condition will usually be denoted by a sudden change in the reading of the milliammeter in the plate circuit.

Now bring the inductance coil of the wave meter near to $L$ and measure the frequency as explained in Art. 90. As soon as an approximate adjustment is obtained, move the wave meter to a greater distance from $L$ and obtain the final balance. This insures sufficiently little coupling between the two circuits.

Next, tune the resonant circuit, including the inductance under test, with the oscillator, by varying the condenser, which must be a high grade variable standard, similar to that used in the wave meter. Read the capacitance which gives resonance and calculate the inductance from Eq. (4) of Art. 88. Repeat this for four or five different wave lengths, and, by the method of the preceding article, determine the true inductance and the distributed capacitance of the coil.

**92. Frequency Stabilization and Control.**

For many laboratory or commercial applications it is essential that the frequency of a radio frequency oscillator be held within

a very narrow range. The frequency of oscillation of any vacuum tube oscillator may vary due either to changes in the tube characteristics or to changes in such external factors as temperature, supply voltage, etc. As explained in Art. 89, variations due to such causes may be partially avoided if the tank circuit is one of high $Q$. Since it is very difficult to build electrical resonating circuits with this constant sufficiently high for extremely nice frequency control, recourse is frequently had to a mechanical resonance device, the *piezo-electric crystal.*

Certain crystals, of which quartz and Rochelle salt are the most notable examples, have the property of developing charges on their surfaces when they are subjected to mechanical strain. Conversely, when these crystals are subjected to an electric field mechanical stresses proportional to the field are produced. In a *crystal controlled oscillator* the tank circuit is replaced or supplemented by a piezo-electric crystal, usually of quartz. This crystal is used as the dielectric of a condenser which is so connected that changes in the plate current of the vacuum tube produce changes in its charge which, in turn, produce mechanical stresses. Since the equation of motion of an elastic solid is essentially identical with Eq. (1) of Art. 88, provided that the effects of restoring forces, mass and internal friction replace those of capacitance, inductance and resistance, respectively, such a solid may be made to oscillate and will have a characteristic frequency. The internal friction in a natural, nearly perfect crystal is very small, so the equivalent $Q$ of the system may be made very high, with a consequent gain in frequency stability. For a detailed quantitative treatment of the methods of cutting, mounting and connecting the crystals, the reader is referred to the standard texts on radio communication, piezo-electricity and elasticity.

Another device which is commonly used to assist frequency stability is a *buffer amplifier* connected between the oscillator and the load. Since the effective parameters of the oscillatory circuit vary with changing load, the frequency of a simple oscillator may change with changing load conditions. If, on the other hand, the oscillator supplies the grid of a vacuum tube amplifier which, in turn, supplies the load, the load impedance of the former remains very large and practically constant.

While the crystal controlled oscillator has the advantage of

extremely constant frequency, it has the marked disadvantage that the period of oscillation is not readily variable, so that it cannot be used for applications which demand adjustment of this quantity. When not too great a range of frequencies is desired, the simple circuit of Fig. 83 or one of its variants, with either $L$ or $C$ (or both) adjustable, serves very well, the only condition on their variation being that the inequality of Eq. (6) of Art. 89 shall not be upset. If, on the other hand, a wide range of frequencies is needed, the necessary range of $L$ and $C$ becomes too great, and a *beat frequency oscillator* is generally used. In this instrument there are two oscillators, one of which usually operates at a fixed frequency considerably above the highest

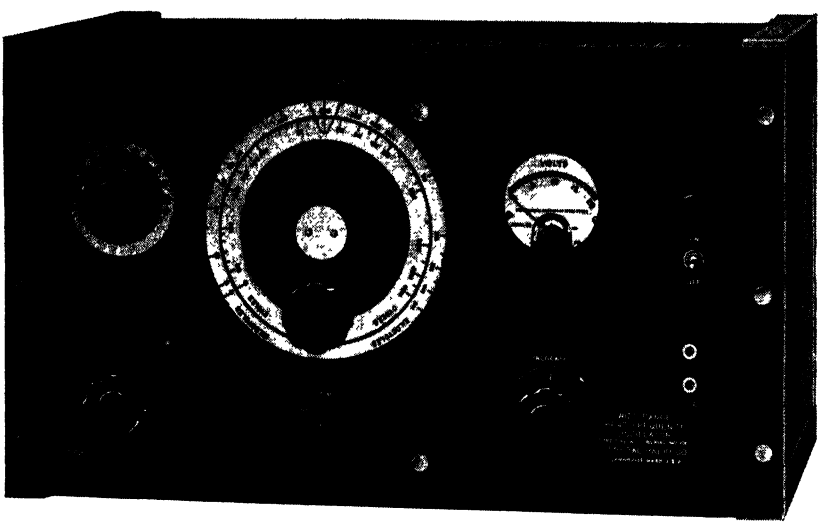

FIG. 90. Beat frequency oscillator. (Courtesy of General Radio Co.)

which will be needed, while the second is adjustable. The outputs are combined by a detector tube, either directly or through buffer amplifiers which prevent interaction of the two oscillators, and this tube has an output consisting of the sum and difference frequencies. The former of these may be removed by appropriate filters and the resulting beat frequency amplified for supply to the load. In order that the calibration of such a device may remain fixed it is necessary that the two oscillators be of similar design

and be mounted in such a way that temperature changes and other disturbing effects will make nearly identical changes in their frequencies. A typical beat frequency oscillator which covers a range from 50 cycles to five megacycles is shown in Fig. 90. For details regarding the characteristics and operation of this and similar instruments the manufacturers' bulletins are the best sources.

**93. Ultra Frequency Oscillators.**

The radio frequency oscillator with a conventional tank circuit is distinctly limited as to its maximum frequency for two reasons: first, because the capacitance and inductance of the connections become appreciable and, second, because the time of transit of the electrons through the vacuum tube itself must be taken into account. When oscillators are being designed for frequencies appreciably above $3 \times 10^8$ sec.$^{-1}$ ($\lambda < 1$ meter), these considerations cannot be neglected and it becomes necessary to include all elements of the oscillating circuit within the tube. In such oscillators the capacitance between the tube electrodes becomes important and the mass of the electron takes the place of inductance, at least in part. While a detailed analysis of the action of transit time oscillators is beyond the scope of this text, a qualitative discussion of the more successful types may be worth while.

In the *Barkhausen-Kurz oscillator* a triode is operated with a positive grid and a negative plate. Electrons emitted from the cathode pass through the grid, are retarded and finally reversed between it and the plate. A fraction of these again pass through the grid and are once more reversed between the grid and the cathode. In this way a cloud of electrons is set into oscillation about the grid, with subsequent periodic variation in the electrode potentials. Losses to the grid are made up by emission from the cathode. A second, and distinctly different, type of oscillator is the *magnetron*, which consists of a tube with concentric cylindrical electrodes. The anode is maintained at positive potential but electrons are prevented from reaching it by the action of a magnetic field parallel to the axis. This field has a value sufficiently great so that emitted electrons follow a curved path not quite touching the anode. Here, again, the effect is of an electron cloud oscillating between the cathode and anode. A third type

has been developed by Llewellyn and Bowen, who have constructed a diode with parallel anode and cathode and with an inductance composed of two concentric cylinders built into the vacuum tube. In this case the tank circuit is a parallel resonant circuit similar to that discussed in the previous articles, but the analysis differs in that transit times of the electrons become important. In the analysis of any of these oscillators the electron space charge is naturally important.

With the ultra frequency oscillators it is generally impossible to make conventional connection to external circuits by means of leads, since the capacitance and inductance of these leads would upset the circuit. The coupling between the oscillator and any circuit under test is made either through the radiation field or through carefully tuned connections in which standing waves are set up in resonance with the oscillator. (Cf. Lecher wires, Art. 90.)

**94. Audio Frequency Oscillators.**

While the simple vacuum tube oscillator as used at radio frequencies may be used to produce oscillations in the audio range, two objections to it arise in practice. The first is that the large inductances and condensers required make the cost high and the instrument cumbersome; the second, that the adjustment for various frequencies requires considerable manipulation and is quite slow. These objections are met by the use of a beat frequency oscillator of essentially the same type as that described in Art. 92. The same difficulties occur here as in the radio frequency range, in that the harmonic content of the output wave and its frequency stability are likely to be worse than with a single oscillator, but these difficulties can be partially overcome by careful design. A portable oscillator which will deliver about 50 milliwatts at frequencies between 30 and 15000 cycles is shown in Fig. 91. This instrument is arranged so that the frequency calibration may be checked by beating the oscillator frequency against the commercial supply frequency, the beat note being indicated by a small neon lamp. An auxiliary variable condenser in the tank circuit of the fixed frequency oscillator allows the calibration to be corrected.

When extreme frequency control is desired the beat frequency

oscillator becomes troublesome. If only a single frequency is needed, the two radio frequency oscillators may be separately controlled by crystals. Crystal control of a single oscillating element in this range is not practicable because the resonant frequency of a crystal varies inversely with its dimensions and the size and cost of a crystal vibrating in the audio frequency range becomes prohibitive. Consequently at these frequencies the

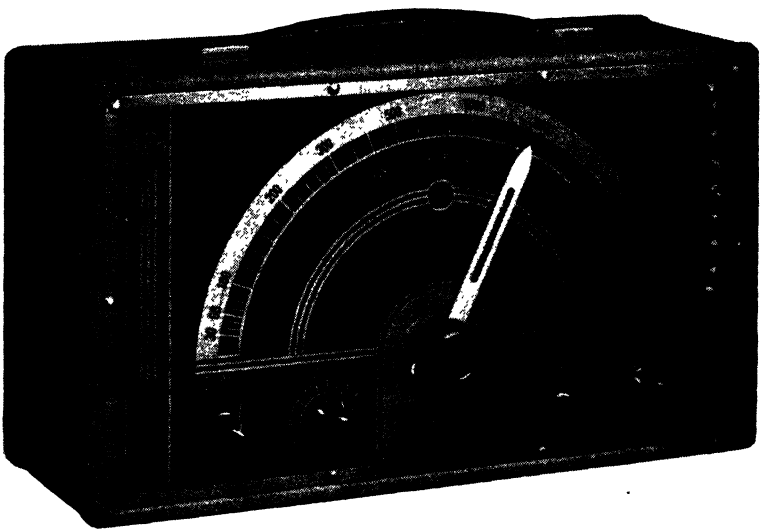

FIG. 91. Audio frequency oscillator. (Courtesy of RCA Manufacturing Co.)

*magnetostriction oscillator* is commonly used. It follows from elementary magnetostatics that a ferromagnetic material, when subjected to a magnetic field, has forces acting at the induced poles. If a bar of such material is in a periodic field along its axis, these stresses will cause a periodic change in length and the amplitude of this vibration will be a maximum for a given field if the frequency of the forced oscillation coincides with a resonant frequency of longitudinal vibration of the bar. These oscillations may be used to stabilize the frequency of an audio frequency oscillator in much the same way that those of a piezo-electric crystal are used in the radio range. The advantage of the magnetostriction oscillator is that the velocity of sound in metals is

of the order of $3 \times 10^4$ meters per second so that bars of reasonable length have resonant frequencies in the audio and supersonic regions of the spectrum. A typical circuit for a magnetostriction oscillator is shown in Fig. 92. For very close control, thermostating of the oscillating element is necessary.

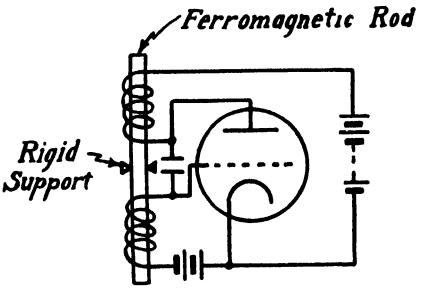


FIG. 92. Magnetostriction oscillator.

Any of the oscillators described above may be used in both the audio and supersonic ranges, as may the relaxation oscillators to be described in the next article.

**95. The Relaxation Oscillator.**

All of the vacuum tube oscillators which we have discussed thus far are based on resonant systems, either electrical or mechanical, in which there is a restoring force proportional to the displacement and they consequently give a wave which is, to a first approximation, simple harmonic. There are certain inherent advantages possessed by the *relaxation oscillator*. The action of this instrument is based on the idea that in some circuits it is possible to have two equilibrium sets of operating conditions, such that a certain critical change in circuit parameters will cause a change from one to the other. As an illustration of a very simple electrical relaxation oscillator we may take the circuit of Fig. 93. Here the condenser $C$ is charged through a resistance $R$, the charge on the condenser varying logarithmically with time. Across the condenser there is connected a gas filled *glow tube*, $G$. Such tubes have the characteristic that with increasing voltage the current remains very small until a critical potential difference is reached. At this *breakdown voltage* the current increases very

suddenly. If the voltage is decreased after breakdown the current does not drop along the curve on which it increased but remains appreciable until the *extinction potential*, somewhat below the breakdown voltage, is reached. As a result of this characteristic the voltage across $C$ will increase at a rate determined by $C$ and $R$ until the breakdown voltage is reached, then it will suddenly decrease to the extinction potential, after which it will start to increase again, so that the voltage-time curve will have the form shown in the lower diagram.

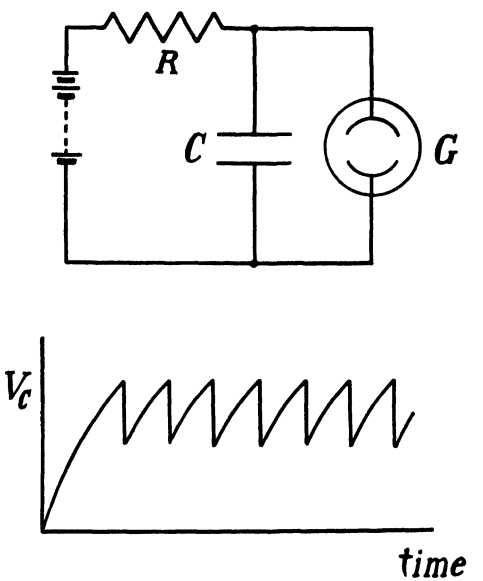


FIG. 93. Glow tube relaxation oscillator.

The glow tube relaxation oscillator is satisfactory only for low or moderate frequencies, as the extinction of the discharge takes an appreciable time. More satisfactory oscillators can be constructed using vacuum tubes, and one of the best of these is the *multivibrator* whose circuit is shown in Fig. 94. The action of this circuit is due to the fact that the current transfers from one tube to the other. To see that this is true, consider that at some instant the two tubes are carrying equal currents. Then the circuit is completely symmetrical, and the plate voltage of the

tubes is across each condenser. Now imagine that a small increase occurs in the plate current of tube *A*. This will cause an additional potential drop across $R_1$ and a consequent decrease in the potential across $C_2$. As a result, this condenser will start to discharge, producing a current in $R_4$ in such a direction as to swing the grid of tube *B* negative. This, in turn, decreases the plate current of tube *B*, with a resultant decrease in the drop across $R_2$, and increase in the charge on $C_1$ and a positive swing of the grid of tube *A*, leading to a further increase in the plate current of this tube. This discussion would seem to indicate that the process would go on cumulatively until tube *B* reached cut-off and only tube *A* carried current. If the two condensers were

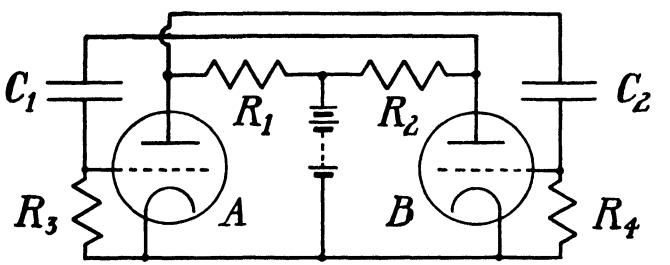


FIG. 94. The multivibrator.

replaced by resistors this would indeed be the case, but with the circuit as shown another factor enters. It may be noticed that the elements $C_1$–$R_2$–$R_1$–$C_2$–$R_4$–$R_3$ form a series circuit in which, at equilibrium, the two condensers would have equal charges. During the process just described, however, the charge on $C_1$ has been continually increasing, while that on $C_2$ has been decreasing. Consequently there is a reverse process going on continually in which charge flows from $C_1$ to $C_2$, causing a current in $R_3$ and $R_4$ in the opposite direction from those mentioned above. As tube *B* approaches cut-off the magnitude of this effect will become larger than that of the current transfer between the tubes and $C_2$ will begin to increase its charge, so that the action reverses and the plate current of tube *B* begins to build up. The time taken for the process depends, of course, on the values of the capacitances and resistances, decreasing with decrease of *C* or *R*. The output, which may be taken across either pair of resistors or the

condensors, consists of a series of pulses, of shape and duration which depend upon the parameters of the circuit.

Either of these relaxation oscillators may be forced to oscillate with a fundamental frequency controlled by another oscillator, by the application of a locking-in voltage. If a potential difference varying periodically with time is applied in series with the D.C. supply of Fig. 93 or to the grid of either tube of Fig. 94, say by means of a transformer, and the frequency of the relaxation oscillator is slightly greater than that of the control voltage, the additional positive impulse coming just before the circuit would be ready to "trip" will act as a trigger and cause the oscillator to operate. In this case the fundamental frequency of the relaxation oscillator will be the same as that of the control. The scheme works equally well, moreover, if the natural frequency of the relaxation circuit is adjusted to just over any integral divisor of the control frequency. Suppose that the approximate ratio is $n$, then, just after the oscillator has tripped, the first $n - 1$ pulses will reach it at a time when they are insufficient to give a trigger action, but the $n^{\text{th}}$ will cause it to operate. Thus the relaxation oscillator may be used as a *frequency divider*. It is by means of such frequency division that the frequencies of standard piezo-electric crystals are determined. By the use of successive dividers they may be made to control an audio frequency. This may be used to operate a clock which can be directly compared with the period of rotation of the earth.

One great advantage of the relaxation oscillator as a frequency changer is that its output is far from sinusoidal and is very rich in harmonics having frequencies which are integral multiples of the fundamental. By the use of resonant circuits any one of these may be selected, so that, if $f_c$ is the frequency of the control oscillator the relaxation oscillator may be made to deliver a frequency

$$f = (m/n)f_c$$

where $m$ and $n$ are any reasonably small integers.

**96. Wave Form.**

In a large number of alternating current measurements, the frequency of the source is of great importance. It is well known,

however, that in the electrical case, as in the acoustic, the frequency is seldom an exactly determined quantity, certain *harmonic* or multiple frequencies being present. For example, when we speak of a fifty cycle alternating current, we mean that the *fundamental* period for one complete cycle is 1/50 second, but there may be, superimposed on this, a variation with a 1/100 second period, there almost certainly is one with a 1/150 second period and so on. If the amplitude of these harmonics is at all appreciable compared with that of the fundamental wave, they may become quite troublesome, and consequently it is desirable to have some means of determining this quantity.

The way to do this was pointed out by Fourier, who in 1812 showed that any finite, single-valued function of a single variable, which has only a finite number of finite discontinuities and a finite number of maxima and minima, can be represented in a finite interval by a series of sine and cosine terms. That is, in the electrical case, if $v$ is the instantaneous value of voltage and $x$ is defined as $2\pi ft$ where $f$ is the fundamental frequency of the oscillation, then

$$v = A_0 + A_1 \sin x + A_2 \sin 2x + \cdots + A_n \sin (nx) + \cdots + B_1 \cos x + \cdots + B_n \cos (nx) + \cdots. \quad (1)$$

The values of the coefficients are quite easily found, for, if the series is multiplied by $dx$ and integrated, term by term, from 0 to $2\pi$, all the terms except the first disappear, and we have

$$\int_0^{2\pi} v\,dx = 2\pi A_0 \qquad \text{or} \qquad A_0 = \frac{\int_0^{2\pi} v\,dx}{2\pi}.$$

Again, multiplying each term by $\sin (mx)dx$ and integrating, we get the series

$$\int_0^{2\pi} v \sin (mx)dx = A_0\int_0^{2\pi} \sin (mx)dx + A_1\int_0^{2\pi} \sin x \sin (mx)dx + \cdots + A_n\int_0^{2\pi} \sin (nx) \sin (mx)dx + \cdots + B_n\int_0^{2\pi} \sin (mx) \cos (nx)dx \cdots.$$

In this expression all the terms of the form sin $(mx)$ cos $(nx)$ will be found, on integration between the specified limits, to give zero. Also, all of the form sin $(nx)$ sin $(mx)$, except one, will give zero. The exception is the one for which $m = n$, for which the integral is

$$\int_0^{2\pi} \sin^2 (nx)dx = \int_0^{2\pi} (\tfrac{1}{2} - \tfrac{1}{2} \cos 2nx)dx = \pi \qquad (2)$$

and, therefore,

$$A_n = \frac{\int_0^{2\pi} v \sin (nx)dx}{\pi} . \qquad (3)$$

Similarly, multiplying by cos $(mx)dx$ and integrating:

$$B_n = \frac{\int_0^{2\pi} v \cos (nx)dx}{\pi} . \qquad (4)$$

If we know $A_n$ and $B_n$, then we have as the equation of the $n$th harmonic

$$v_n = A_n \sin (nx) + B_n \cos (nx) = C_n \sin (nx + c_n), \qquad (5)$$

where $c_n$ gives the phase difference between the harmonic and the fundamental waves, if we choose $B_1 = 0$, and has the value $\tan^{-1} B_n/A_n$, while the amplitude, $C_n$, has the value $(B_n^2+A_n^2)^{1/2}$.

In general, the problem is to find $C_n$ (and sometimes $c_n$), knowing $v$ as a function of $x$. If the functions $v$ cos $(nx)$ and $v$ sin $(nx)$ are plotted and integrated with a planimeter, the values of the coefficients can be obtained. Such a method, however, would be unduly laborious, and a simpler method is commonly used in practice. If the value of $v$ is taken at frequent intervals, say every fifteen degrees (or twenty-four values for the entire cycle), then the sum of these values, multiplied by sin $(nx)$ and divided by the number of intervals, gives, to a good approximation, the average height of the $v$ sin $(nx)$ curve. This average height, however, is by Eq. (3) half the value of $A_n$ (except for $A_0$). Hence, if we take 24 values as suggested, then letting $v_1$ be the value at 15°, $v_2$ at 30°, etc., we have

$$A_0 = 1/24(v_1 + v_2 + v_3 + \cdots + v_{24}), \qquad (6)$$

taking the proper algebraic signs into account.

$$A_1 = 1/12(v_1 \sin 15° + v_2 \sin 30° + v_3 \sin 45° + \cdots + v_{24} \sin 360°). \quad (6A)$$

Combining terms, so as to bring all angles under 90°,

$$A_1 = 1/12[(v_1 + v_{11} - v_{13} - v_{23}) \sin 15° + (v_2 + v_{10} - v_{14} - v_{22}) \sin 30° + (v_3 + v_9 - v_{15} - v_{21}) \sin 45° + (v_4 + v_8 - v_{16} - v_{20}) \sin 60° + (v_5 + v_7 - v_{17} - v_{19}) \sin 75° + (v_6 - v_{18})]. \quad (7)$$

Similarly,

$$A_2 = 1/12[(v_1 + v_5 - v_7 - v_{11} + v_{13} + v_{17} - v_{19} - v_{23}) \sin 30° + (v_2 + v_4 - v_8 - v_{10} + v_{14} + v_{16} - v_{20} - v_{22}) \sin 60° + (v_3 + v_{15} - v_9 - v_{21})], \quad (8)$$

$$A_3 = 1/12[(v_1 + v_3 - v_5 - v_7 + v_9 + v_{11} - v_{13} - v_{15} + v_{17} + v_{19} - v_{21} - v_{23}) \sin 45° + (v_2 - v_6 + v_{10} - v_{14} + v_{18} - v_{22})], \text{ etc.} \quad (9)$$

If the two halves of the wave are identical, as they are often found in practice, all the amplitudes with the even subscripts, starting with $A_0$, will be found to be zero; hence there is no need of testing for these. Also, in this case, half the complete wave may be used for the analysis in place of the full wave. The $B$'s are found in exactly the same way. By use of equations similar to Eq. (6A), with cosine terms substituted for sines, one obtains the expressions for the $B_n$'s.

For the actual determination of wave form, two types of apparatus are in common use. The first of these is the magnetic oscillograph, so constructed as to be utilized on frequencies throughout the audible range. It operates on the D'Arsonval principle, having two very fine conducting wires stretched between the poles of a strong permanent magnet, connected so that the current to be measured passes in opposite directions through the two wires. A very small mirror is mounted on these wires and the vibrations of a light beam reflected from this mirror are observed. For visual observation the time axis is furnished by reflecting the beam from a rapidly rotating polygon of mirrors, with its axis of rotation perpendicular to the axis of vibration of the small mirror. This causes the beam to move at right angles

to the direction of vibration with uniform velocity across the field of view, then to snap back instantaneously to its starting point, and repeat. If the speed is so adjusted that the beam keeps retracing itself, the entire wave form will show as a continuous line, and may be traced. For photographic recording a

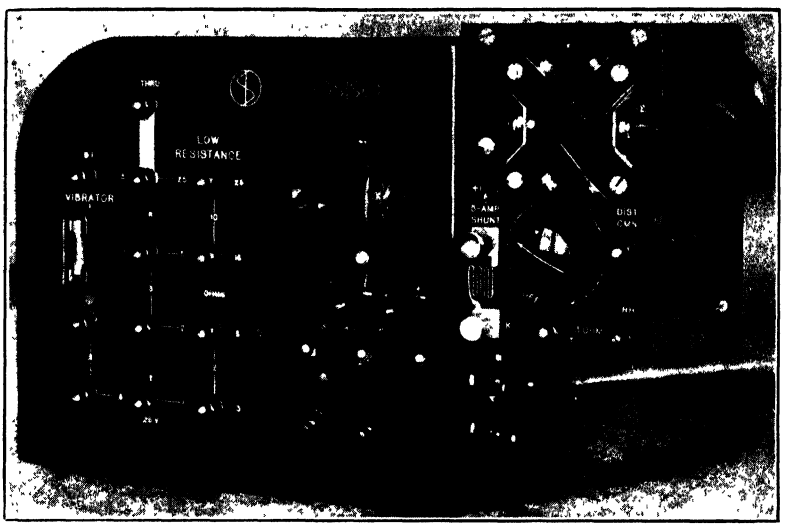


FIG. 95. The Osiso oscillograph. (Courtesy of Westinghouse Electric and Manufacturing Co.)

film is moved rapidly across the beam, so that a similar record is obtained. A portable instrument of this type, which will record frequencies up to 5000 cycles, manufactured by the Westinghouse Electrical and Manufacturing Co., is shown in Fig. 95.

**97. The Cathode Ray Oscillograph.**

The second type of instrument for determination of wave form is electrostatic in nature and dispenses with mechanical moving parts. The basic instrument of this type is the *cathode ray tube*, a simplified model of which is shown in Fig. 96. Electrons emitted from the cathode are accelerated to pass between two pairs of condenser plates which produce fields which are at right angles to each other. If there were no potential difference between the plates of either pair and the electron beam were limited by some

means to a small cross section, it would pass down the axis of the tube and produce a small spot in the center of the fluorescent screen. If, on the other hand, an alternating potential is applied across the plates of one of the elements the electron stream will be accelerated, at right angles to the original direction of motion, by the field between the plates. The spot on the fluorescent screen will oscillate, almost exactly in phase with the potential, stretching out the spot on the screen to a straight line. If

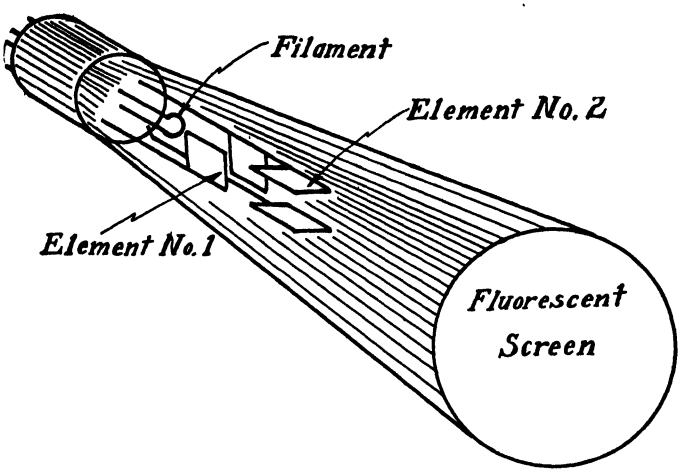


FIG. 96. Simplified cathode ray tube.

another alternating potential is applied between the plates of the second element a second oscillation at right angles to the first will be set up and the two will combine to form a figure on the screen. The nature of this figure will depend upon the frequency ratio and wave form of the two deflecting potentials, and we shall consider some of the more interesting cases.

Let us first consider that both deflecting elements have sinusoidal potentials applied. Assuming strict proportionality between potential and deflection, the position of the spot at a time $t$ will be given by

$$x = a \sin \omega_1 t,$$

and

$$y = b \sin (\omega_2 t + \phi),$$

where $x$ and $y$ are rectangular Cartesian coordinates measured from the undeflected position as origin. For general values of $\omega_1$ and $\omega_2$ the resultant image will be a confused blur. A particularly interesting case occurs, however, if $m\omega_2 = n\omega_1$, where

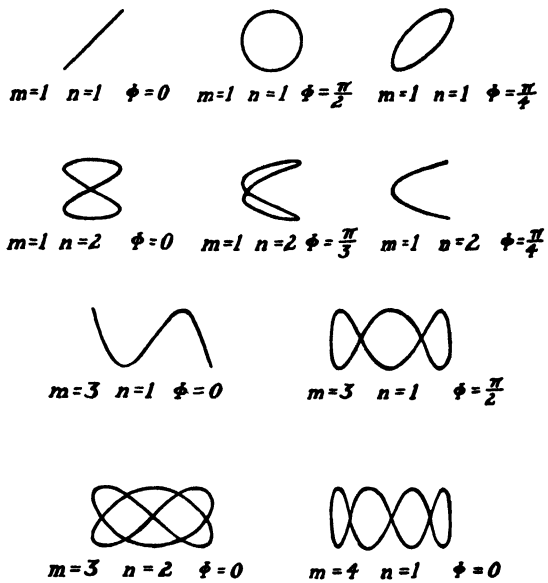


FIG. 97. Lissajou figures.

$m$ and $n$ are integers. In all such cases, since at the end of $m$ periods of the first wave and $n$ of the second, $x$ and $y$ will have returned simultaneously to their original values, a closed *Lissajou figure* will be obtained. The figure will obviously have $m$ maxima in the $x$ direction and $n$ in the $y$. A number of special cases are illustrated in Fig. 97. Applications of the oscillograph using these figures will be illustrated by experiments.

A particularly interesting set of figures are obtained if, in place of a sinusoidal potential on the horizontal deflecting plates, a linear variation, $x = at$, is used. Actually such a potential would sweep the beam off the screen in a short time, but the same effect may be obtained if a saw tooth wave, which carries the beam across the screen with uniform velocity and then returns it to its starting position in a time negligible compared with that

required for the forward trip, is used. The condenser voltage shown in Fig. 93 closely approximates such a wave if the curvature of the rising portions is made small by arranging that the maximum potential difference across the condenser is considerably

FIG. 98. The RCA Type 160 oscillograph. (Courtesy of the RCA Manufacturing Co.)

smaller than the supply voltage. If the periodic time of the saw tooth wave is commensurate with the period of the wave on the vertical deflecting plates a repeating figure will result. Particularly, if the horizontal frequency is an integral divisor of the

vertical, a single curve will be traced across the screen and this curve will reproduce the wave form of the imposed potential.

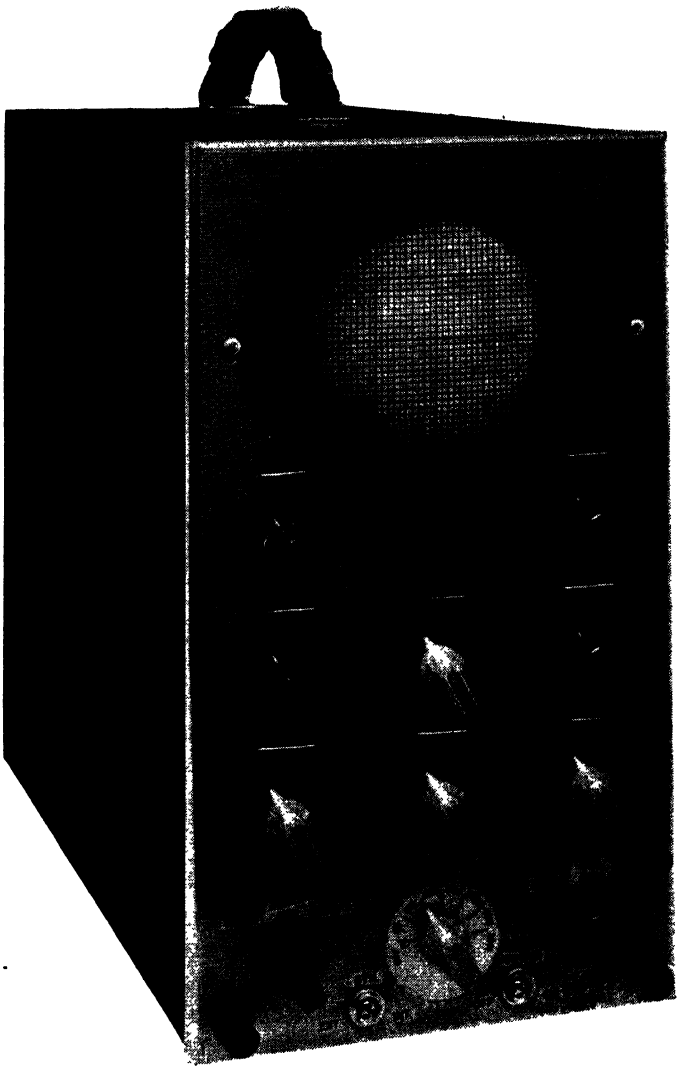

FIG. 99. The Du Mont Type 168 oscillograph. (Courtesy of Allen B. Du Mont Laboratories, Inc.)

In the commercial cathode ray oscillograph several improvements on the simple arrangement described above are usually made. In the cathode ray tube itself there are a series of electrodes between the cathode and the deflecting plates. The acceleration of the electrons is accomplished by these electrodes, and by a proper ratio of the voltages on them the electron beam may be focused to a quite sharp spot at the screen. The deflecting plates are seldom plane parallel, but are inclined to each other in such a way as to compensate for end effects and thus

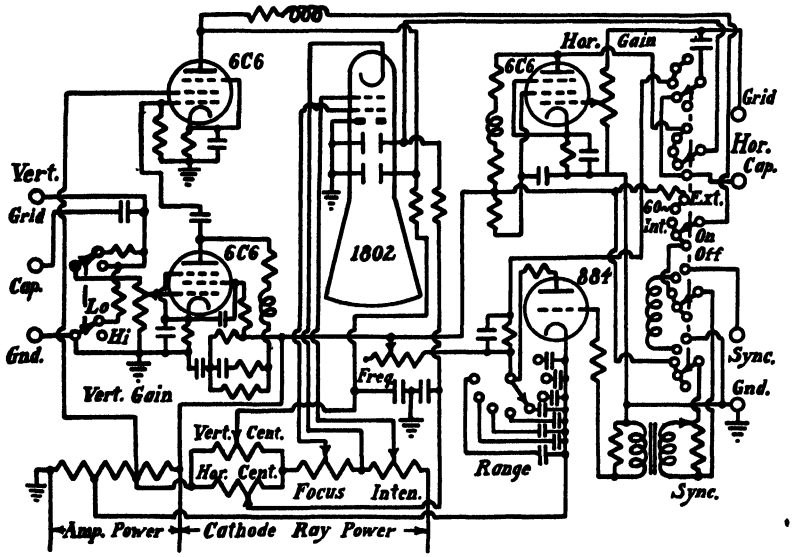


FIG. 100. Simplified circuit of the RCA 160 oscillograph.

to give a true linearity between applied voltage and deflection. In addition, auxiliary equipment for amplifying the applied voltages, obtaining the saw tooth *sweep wave* and supplying power to the cathode ray tube is usually incorporated into a single unit. Two of the most common types of oscillographs for laboratory work are shown in Figs. 98 and 99.

A simplified wiring diagram of the RCA 160 oscillograph is shown in Fig. 100. The voltage supply unit, which is constructed

integrally with the instrument, is omitted from the diagram, as are the transformers for supplying heater currents. The electron beam is controlled by two potentiometers, *Inten.* and *Focus*, which adjust the potentials of the accelerating electrodes and by which it is possible to determine the brilliance and size of the spot on the screen. Since the limiting factor in the life of a cathode ray tube is decay of the fluorescent material, the intensity should be kept as low as is consistent with convenience. For the same reason, the tube should never be left operating for any length of time with the beam concentrated at a single spot. Because stray magnetic fields and other disturbances may affect the undeviated position of the beam, provision is made through two potentiometers, *Vert. Cent.* and *Hor. Cent.*, to apply small corrective D.C. potentials to the deflecting plates. These are supplied through 1.5 megohm resistors, so as not to short circuit the deflecting potentials, and are shunted to ground through condensers, so that any A.C. component present will not cause a waver of the beam.

In order to provide a wide range of sensitivity, a two stage amplifier is connected between the vertical input terminals and the corresponding deflecting plates. The input to this amplifier may be made either between the terminals marked *Gnd.* and *Grid*, or between *Gnd.* and *Cap.*, the difference being that, if the latter pair is used, a one microfarad condenser prevents direct currents or low frequencies from affecting the deflection. The gain of the amplifier is controlled by the *Vert. Gain* potentiometer and by the *Vert. Amplifier* switch. In the *Hi* position this switch allows a maximum voltage gain of about 1200. In the *Lo* position a resistance of 6800 ohms is placed in parallel with the half megohm grid resistor and a 0.47 megohm resistor is placed in series. In this way the input resistance is maintained very nearly at its former value but the maximum voltage gain is cut down by a factor of nearly 100. The horizontal deflecting voltage is applied through another 6C6 pentode either from the horizontal input terminals or from the relaxation oscillator. This oscillator is very similar to that shown in Fig. 93, the chief difference being that the Type 884 gas filled triode is used in place of a glow tube. Ordinarily the grid of this tube is maintained

negative with respect to the cathode, and it serves to speed the extinction of the discharge, since it attracts to itself any positive ions present and thus de-ionizes the tube as the condenser potential falls. To give a wide frequency range, eight values of capacitance, varying from 0.20 to 0.00039 microfarads, are provided and any of these may be selected by the *Range* switch, so that the total variation from 4 to 22,000 cycles per second may be obtained in eight ranges, as shown on the front of the instrument. Within these ranges, fine adjustment of the frequency is achieved by changing the resistance through which the condensers charge, by means of the rheostat *Freq.* If an external supply is being used on the horizontal axis instead of the sweep wave, it may be connected either directly to the deflecting plates or through the amplifier by setting the *Hor. Amplifier* switch to the *Off* or *On* position, respectively.

The other three positions of this five pole, five throw switch provide for stabilization of the frequency of the relaxation oscillator by applying a pulse to the grid of the 884 tube. With the switch in the *Int.* position the synchronizing pulse is obtained from the plate of the vertical amplifier so that, with reasonable adjustment of *Range* and *Freq.*, the sweep circuit locks in with the potential being studied and a stable pattern is obtained. In the 60~ position the pulse is supplied from the power supply lines through a transformer, and terminals, *Sync.* and *Gnd.*, are provided so that, by throwing the switch to *Ext.*, an external synchronizing frequency may be used. Since the application of the pulse disturbs the wave form of the sweep oscillator to some extent, it is desirable to keep the setting of the *Sync.* potentiometer as low as possible, consistent with a stable figure.

Since the deflecting plates are capacitively coupled to the rest of the circuit, D.C. deflecting potentials can not be applied through the ordinary terminals. Direct connection to these plates, however, may be made through an opening in the side of the case.

The RCA 160 instrument has a screen diameter of five inches, a sensitivity of 50 volts (peak) per inch deflection without the amplifier and 0.04 volts per inch with it, and an input impedance of 0.5 or 2.2 megohms, with or without the amplifier, respectively.

A rectifying and filtering unit is included so that the only power supply needed is from the 110–120 volt, 50–60 cycle lines.

The Du Mont type 168 oscillograph shown in Fig. 99 also has a five inch screen. It uses two type 6C6 tubes as amplifiers and has, in addition, a type 76 triode which may be inserted, by means of the *Vert. Amp.* switch, between the vertical 6C6 and the cathode ray tube. With a single stage the voltage gain is 70, with two, 450. The corresponding sensitivities are 28, 0.40 and 0.065 volts (peak) per inch and the input impedance is one megohm. The relaxation oscillator may be operated from 15 to 30,000 cycles per second in seven ranges. With the exception of certain minor changes the circuit is similar to that of Fig. 100. The principal differences are that capacitive coupling is used between the synchronizing source and the grid of the relaxation oscillator tube and that the sweep signal modulates the electron beam in such a way as to eliminate the return trace from the screen.

A wide variety of cathode ray oscillographs are made by these and other manufacturers, varying from those using a one inch screen, for qualitative observations, to those with nine inch screens for detailed work. Screens with different characteristics, such as high brilliance, long or short persistence time, etc., may be obtained. In addition a wide range of auxiliary apparatus, such as pre-amplifiers, electronic switches for switching in and out two or more circuits, devices for obtaining synchronizing pulses and piezo-electric crystals for transforming vibrations into electrical waves, is available.

Certain precautions, in addition to those mentioned above, should always be observed when using the cathode ray oscillograph. Since high D.C. potentials are involved, the instrument should not be operated out of its case. If the supply is from a high impedance source it is generally beneficial to ground the case of the instrument. Notice that two of the deflecting plates are connected at ground potential, as is one of the terminals for external synchronization. If the instrument is being used in connection with a grounded source or if two supplies from the same source are used to produce Lissajou figures it is necessary that this internal interconnection be kept in mind, or short circuits may result.

## EXPERIMENT 35

### CALIBRATION OF A CATHODE RAY OSCILLOGRAPH AS A VACUUM TUBE VOLTMETER

Since the input to the conventional oscillograph is through a series condenser, the instrument must be calibrated on A.C. In order that the relation between the r.m.s. and peak voltages * be known, it is desirable that a source with fairly good sinusoidal wave form be used, and the commercial supply lines are generally satisfactory for this. The 110 volt supply may be used directly for the calibration without amplification, but it will usually be better to use a low voltage transformer for the more sensitive scales. Set up a potentiometer arrangement to vary the voltage, similar to that used in Exps. 8 and 31. As a standard use a good electrodynamometer instrument.

Connect the vertical input terminals of the oscillograph in parallel with the standard voltmeter, being careful about polarity if one side of the supply line is grounded. Since there is no advantage in a horizontal spread of the beam, the horizontal input terminals may be short circuited and the switch set to *Amp. Off*. Adjust the intensity and focus dials until a clear sharp spot is obtained and then, by means of the positioning controls, move this to approximately the center of the screen. With the vertical amplifier off apply sufficient voltage to give a vertical trace which has a length of about 3/4 of the screen diameter. Read the voltage and measure the length of this trace. Repeat at approximately 4/5, 3/5, 2/5 and 1/5 of this voltage. Now put the vertical amplifier into use and repeat the entire procedure with this amplifier set for maximum gain. Repeat for four other settings of the amplifier, equally spaced between maximum gain and zero.

Plot, on a single sheet, the observed *double deflection* against the r.m.s. voltage, for each setting of the amplifier, specifying the setting for each curve. Assuming a sinusoidal supply, calculate the peak voltage for each reading and plot, on a second sheet, the *single deflection* against peak voltage. Calculate the mean sensitivity of the instrument for each setting of the amplifier that you have used and plot a curve of sensitivity against amplifier setting.

* See Art. 73.

## EXPERIMENT 36

## ANALYSIS OF THE WAVE FORM OF AN AUDIO FREQUENCY OSCILLATOR

Connect the output terminals of the oscillator to a variable resistance in series with a thermal milliammeter and to the vertical input terminals of the oscillograph. Adjust the frequency of the oscillator to some assigned value and then adjust the variable resistance until the oscillator is carrying rated load. Having focused the cathode ray tube beam as in Exp. 35, throw the horizontal amplifier switch to "Timing" and adjust the horizontal gain until the pattern covers a horizontal distance about 2/3 of the screen diameter. (This amount of input voltage, for many cathode ray oscillographs, gives the best linearity of the sweep voltage.) Since the return of the sweep circuit may disturb the ends of the pattern somewhat, it is well to have from $1\frac{1}{2}$ to 2 complete waves showing on the screen, and to use the center portion only for analysis. With the synchronizing control set at zero adjust the range and frequency controls until this condition is satisfied. In general a moving pattern will be obtained, but it may be brought to rest by increasing the magnitude of the synchronizing pulse, with the selector switch set for internal synchronization. Do not turn the synchronizing control above the minimum point at which stable operation is obtained.

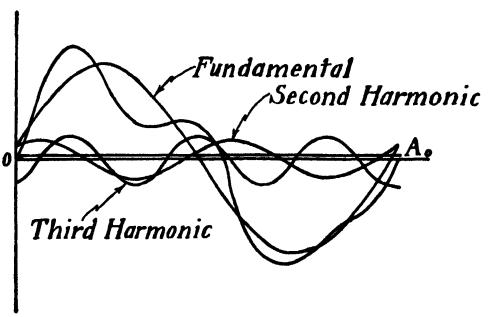


FIG. 101. Harmonic analysis of an A.C. wave.

Trace the pattern or photograph it. With the usual tube an exposure of one second, using supersensitive panchromatic plates and an F 4.5 lens, will be approximately correct, although it may

be necessary to take several trial exposures to obtain the best results. Darkening the room will aid contrast in the photograph. If the pattern is photographed it will be necessary to determine the vertical peak to peak distance at the time the exposure is made, unless the screen is provided with an etched coordinate frame. By the methods of Art. 96, find the values of $A_1$, $A_2$, $A_3$, $B_1$, $B_2$ and $B_3$, taking intercepts every fifteen degrees. In doing this, convert all vertical distances into volts by using the sensitivity of the instrument as determined in Exp. 35 and correcting the scale of the photograph by the use of the measured peak to peak distance. Plot curves, similar to Fig. 101, showing the first three harmonics and calculate the percentage amplitude of the second and third harmonics in terms of the fundamental.

## EXPERIMENT 37

### FREQUENCY CALIBRATION OF AN AUDIO FREQUENCY OSCILLATOR

The instrument will be calibrated by comparison with the commercial supply frequency which is generally sufficiently stable to furnish a good working standard. Let this frequency be $f_s$. Connect the output terminals of the oscillator to the vertical input terminals of the oscillograph and supply the horizontal input terminals by means of a transformer supplying $f_s$ at a few volts. To avoid frequency drift during the observations, it may be well to allow the oscillator to run for an hour or two in advance to enable it to come to temperature equilibrium. Adjust the oscillator output and the two oscillograph amplifiers until the pattern is approximately square and covers about half the area of the screen. Now adjust the oscillator frequency until an ellipse is obtained. If the instrument is provided with a calibrating adjustment, this should be done by the use of this control and with the main dial set for exactly $f_s$. Otherwise adjust the main dial and take $f_s - f_1$, where $f_1$ is the dial reading, as the correction to be applied. If the frequencies of the oscillator and the supply line are identical the ellipse will be stationary, but in general it will change shape and continually oscillate within a rectangular envelope. In making the frequency adjustment bring it as near to a stationary pattern as possible.

Now, leaving the calibrating adjustment untouched, adjust

the frequency of the main dial to $f_2$ for which the Lissajou figure for a 2 : 1 ratio is obtained. (This will be similar to the figures in the second row of Fig. 97, but will be rotated through 90°.) Take the correction at this point as $2f_s - f_2$. Repeat for $3f_s$, $4f_s$, $\cdots$, $10f_s$. If the patterns obtained in these cases are moving slowly, as they probably will be unless the oscillator is extremely stable, they will appear to be in three dimensions, as a closed sine curve drawn on a transparent cylinder rotating about its axis. Since this figure will rotate $n$ times per second when $f_m = m(f_s \pm n)$, an approximate determination of the minimum speed of rotation will give an estimate of the uncertainty in the calibration. After the highest of these frequencies has been observed, check the oscillator at the commercial frequency. If the percentage departure from the previous reading is appreciably more than the percentage uncertainty in the other readings, allow the oscillator more time to stabilize and repeat all the observations.

If the calibration is to be extended above $10f_s$ a different method must be used, as the Lissajou figures with more than ten peaks are very hard to distinguish from each other. The calibration may be extended, however, by using the sweep circuit and a stepwise method. Set the oscillator at some convenient multiple of $f_s$, say $5f_s$, by using the Lissajou figure. Now throw the oscillograph switch to the *60~ Timing* position. Adjust the horizontal amplitude until the extremes of the pattern show on the screen and then adjust the sweep frequency until just one complete sine wave appears. With the synchronizing voltage control bring this pattern to rest. The sweep frequency is now $5f_s$ to within the uncertainty of your calibration. Before going on, switch the oscillograph back to the sinusoidal horizontal deflection and see that the frequency has not changed during the sweep circuit adjustment. Now return to "Timing" and adjust the oscillator until two complete sine waves appear. The reading should now coincide with $f_{10}$. Go on to 3, 4, $\cdots$ waves, obtaining corrections at $15f_s$, $20f_s$, etc., occasionally returning to $5f_s$ for a check, and finally checking at $f_s$ as before.

Plot a correction curve for the frequency scale of the oscillator, specifying the estimated percentage uncertainties in the range from $f_s$ to $10f_s$ and in the range above $10f_s$.

**98. Phase Measurements with the Oscillograph.**

As was shown in the previous article, if two sine waves of the same frequency are used to give the deflections to the beam, the corresponding Lissajou figure will depend upon the phase angle, $\phi$, by which the waves are separated in time. This offers a convenient method of measuring the phase angle between any two potentials of the same frequency. Let us suppose that the two deflections at time $t$ are

$$x = a \sin \omega t,$$

and

$$y = b \sin (\omega t + \phi).$$

If we can determine the value of $y$ at the instant that $x = 0$, we can immediately determine the phase difference, since at this time $y_0 = b \sin \phi$, and, therefore

$$\sin \phi = y_0/b. \qquad (1)$$

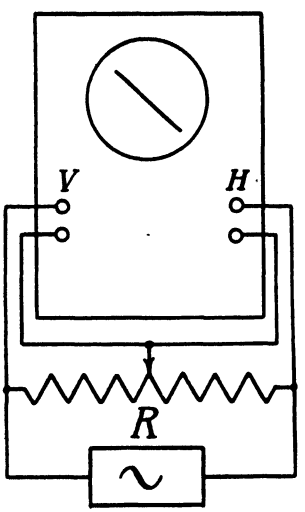


FIG. 102. Checking of amplifier phase shift.

To find the necessary quantities we may set the horizontal gain on the instrument to zero so that only a vertical line remains. If a scale is placed along this line, the total length is $2b$. Hold the scale in this position and adjust the horizontal amplitude to any convenient value. The pattern will generally change to an ellipse. The distance intercepted on the scale by this ellipse is obviously $2y_0$, so the ratio of the two distances gives $\sin \phi$.

In the use of this method it is essential that the amplifiers of the oscillograph themselves shall either introduce no phase change or shall introduce the same phase difference within the accuracy of the observation, regardless of their gain settings. To check this, the arrangement of Fig. 102 may be used. $R$ should be a good A.C. resistor. Since the two voltages in this case are 180° apart, perfect amplifiers would give a straight line

pattern as shown. If the test is first made with the contact somewhere near the middle of $R$ and with the amplifiers set for equal gain, then with the contact near to each end of $R$ and with the voltage ratio compensated for by different settings of the amplifiers, a sufficient test will have been made.

## EXPERIMENT 38

### CALIBRATION OF A PHASE SHIFTER

First test the oscillograph amplifiers for phase difference as outlined above, at several frequencies throughout the range in which the phase shifter is to be tested. If any appreciable departure from a straight line occurs, measure the apparent phase difference and take this as the residual error of the instrument.

Calibrate the phase shifter, which will be similar to that shown with the lock-in amplifier of Fig. 80, at 60, 120, 240, 480 and 960 cycles per second. Connect the output of an audio oscillator directly to the input terminals of the phase shifter and to the horizontal input terminals of the oscillograph through an audio frequency transformer * which has previously been tested and found to have a negligible phase shift. If the input impedance of the oscillograph is at least ten times that of the phase shifter, the output of the latter may be connected directly to the vertical input terminals of the former, if not, connect in series a resistor of sufficient magnitude so that the load impedance on the shifter has at least this value. This is necessary since the calibration of the phase shifter depends on load impedance unless this quantity is high. At each frequency take five or six points, equally spaced over the dial of the phase shifter, and determine the phase difference between the input and output voltages at each.

Plot, on a single sheet, five curves of phase shift versus dial reading, one for each frequency.

**99. Transient Studies with the Oscillograph.**

The applications of the oscillograph which we have discussed up to this point have all dealt with periodic phenomena. The

* If the phase shifter has a transformer input, as $T_1$ in Fig. 80, direct connection may be made to the horizontal terminals.

instrument is particularly well suited, however, to the observation of processes which take place with some change in conditions, particularly when the change takes place in a very short time. As examples we have the charging of a condenser through a low resistance or the building up of oscillations in a resonant circuit. For these short time processes, photography of the pattern is essential. When transients produced in a change from one D.C. state to another are involved we may take advantage of the fact that the instrument will not respond to D.C. potentials, due to its coupling condensers. The potential being studied is applied to the vertical plates and a sweep frequency of periodic time somewhat greater than the duration of the transient is used on the horizontal. Just before the circuit change is made the shutter of the camera is opened. The transient pattern is thus photographed on its single appearance on the screen. If the frequency of the sweep circuit is known, a horizontal time scale is available.

The chief difficulties with this method of observation are that the recurrent return trace is generally badly overexposed, as compared with the transient pattern and may cause some of the detail in the latter to be obscured, and that the phase of the saw tooth wave at which the transient starts is not subject to control. These difficulties may be removed by the use of a discharge element in parallel with the tube of the relaxation oscillator. Suppose that this element is a resistor, $r$, and that the series resistor of the oscillator is $R$. The highest potential which the condenser can reach is $V_m = Vr/(R + r)$, where $V$ is the supply voltage. If $V_m$ is slightly less than the breakdown potential of the tube, for the particular grid bias used, relaxation oscillations will not take place. If, however, a positive trip signal is applied to the grid at the same time that the transient is started, the tube can be made to break down and go through a single sweep cycle. Unless high amplification is used on the sweep wave, so that only a small part of the charging time of the condenser is effective, a sacrifice of linearity results from the arrangement.

Because the timing of transient waves is generally important, it may be desirable to mark a time scale directly on the photograph. One way of doing this is to make a second exposure, without moving the camera, of a wave of known frequency. A second method is to apply a negative bias to the first acceler-

ating grid of the cathode ray tube, sufficient to prevent any electrons from reaching the screen, and to superimpose upon this an alternating voltage of known frequency. By proper adjustment of the D.C. and A.C. potentials, the cathode tube is allowed to operate during part of the cycle of the latter. Consequently the continuous pattern on the screen will be broken up into a series of spots which have been produced at known time intervals. These devices are frequently convenient not only for transient observations but also for studies of periodic phenomena in which a time scale within the pattern may be convenient.

Mechanical phenomena, either transient or recurrent, may be studied with the cathode ray oscillograph with the help of either piezo-electric or crystal *pick-ups*, which can be arranged to produce potential differences proportional to pressure, displacement or acceleration. For details of such measurements the reader is referred to the literature dealing specifically with the cathode ray oscillograph.

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# CHAPTER 9

## ALTERNATING CURRENT IMPEDANCE MEASUREMENTS

**100. Inductance and Capacitance in A.C. Circuits.**

For the purposes of this article, we shall consider that a sinusoidal voltage wave is impressed on a pure capacitance and a pure inductance, and shall investigate the current produced.

Consider, first, a condenser of capacitance $C$, across which there is a voltage

$$v = V \sin 2\pi ft = V \sin \omega t.$$

Remembering that $v = q/C$ and that $i = dq/dt$

$$i = C\frac{dv}{dt} = \omega CV \cos \omega t = \omega CV \sin\left(\omega t + \frac{\pi}{2}\right). \tag{1}$$

That is, the sinusoidal voltage produces a current of the same form, which leads the voltage by an angle of 90°. If we wish to express the ratio of the r.m.s. voltage to the r.m.s. current (and consequently of the maxima), we have

$$X_c = \frac{1}{\omega C}, \tag{1A}$$

which is defined as the *capacitive reactance.* $X_c$ is in ohms if $C$ is in farads, and $\omega$ is in $\text{sec}^{-1}$.

Similarly, if the voltage is impressed across a pure inductance, of value $L$, we have the defining relation $v = L\frac{di}{dt}$, so that

$$i = \frac{-V}{\omega L} \cos \omega t = \frac{V}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right). \tag{2}$$

Therefore, for an inductance, the current lags the voltage by 90°, and the *inductive reactance* is defined by

$$X_L = \omega L \text{ ohms} \tag{2A}$$

when $L$ is in henries.

Now consider a circuit made up of a resistance, an inductance and a capacitance in series, and let a current

$$i = I \sin \omega t$$

be flowing. Then the voltage across the circuit is

$$v = IR \sin \omega t + I(X_L - X_C) \cos \omega t,$$

taking account of the phase relations as shown above. If we now let $v = V \sin (\omega t + a)$, expand the sine and equate coefficients, we find that

$$\frac{V}{I} \cos a = R \qquad \text{and} \qquad \frac{V}{I} \sin a = X_L - X_C.$$

The sinusoidal voltage required to produce a sinusoidal current in such a circuit therefore leads the current by an angle, $a$, defined by

$$\tan a = \frac{X_L - X_C}{R}, \tag{3}$$

while the ratio of the maximum (or r.m.s.) voltage to the maximum (or r.m.s.) current, known as the *impedance*, is

$$Z = \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2} \text{ ohms.} \tag{3A}$$

By the use of this equation, together with the known fact that for most circuits the fundamental constants $R$, $C$, and $L$ do not vary appreciably with frequency, it is possible to measure, with an A.C. ammeter and voltmeter, inductance or capacitance, or both. When only one of these quantities is present, two measurements may be made, one with direct current, the other at a known frequency; when both are appreciable, two A.C. measurements must be made, in addition to the D.C.

**101. A.C. Standards and Bridges.**

Most measurements of impedances may be made with the highest precision by means of bridge networks, essentially similar to the Wheatstone bridge. There are a number of special precautions that must be observed with A.C. measurements, how-

ever. The first of these is due to the fact that with alternating currents we have, in addition to resistance, inductive and capacitive reactance and that an element containing one of these generally possesses the others to some extent. Consider, for example, a resistance formed of a helical coil of fine wire. For D.C. purposes this would be satisfactory but on A.C., since it is a solenoid, it has a very appreciable self inductance. The inductive reactance may be minimized if the wire is bent into the form of a hairpin and then wound into a double helix, so that the two leads come out at the same end of the coil. In a coil formed in this way the magnetic fields of the two halves of the winding are very nearly equal and opposite, so that the inductance is almost zero but, since the ends of the wire, which are a maximum potential difference, are very close together, a relatively large distributed capacitance is introduced. This may be overcome, to a large extent, by using a woven coil of the form shown in Fig. 103. In this arrangement, in which the wire is used as the woof on a warp formed of silk or other insulating material, there are two windings in parallel. Adjacent turns, which are closely spaced, carry equal currents in opposite directions so that their magnetic fields nearly cancel, and, since they are at almost the same potential, capacitive effects are small. Because, as shown by Eqs. (1) and (2) of the preceding article, the effects of series inductive or parallel capacitive reactance decrease with decreasing frequency, the hair pin non-inductive winding is usually satisfactory for commercial frequencies. For moderate audio frequencies, say up to about 1000 per second, the woven resistors, without any special precautions, are satisfactory.

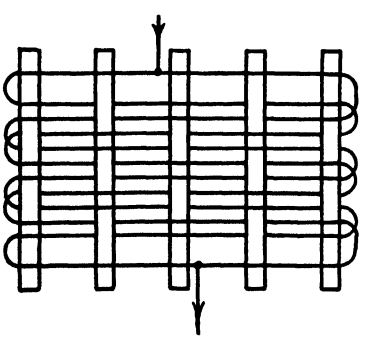

FIG. 103. Arrangement of a woven resistor.

At high audio frequencies or in the radio frequency range even very small inductances or capacitances become very troublesome. To a first order, careful design may remove their disturbing effects. A resistance element may be represented approxi-

mately by the circuit shown on the left in Fig. 104, where $L$ is the inductance of the resistor and $C$ is its distributed capacitance. Let the common voltage across $R$ and $C$ be $V_0 \sin \omega t$, then by adding the currents in $R$ and $C$ we find the current through the element to be

$$I = \frac{V_0}{R}(1 + \omega^2 C^2 R^2)^{1/2} \sin (\omega t + \tan^{-1} \omega CR).$$

Now consider the circuit on the right in the figure. If the voltage between the points $A$ and $B$ is $V_0 \sin \omega t$, the current flowing is

$$I' = V_0 \left\{ \frac{1}{(R^2 + \omega^2 L^2)^{1/2}} \right\} \sin \left( \omega t - \tan^{-1} \frac{\omega L'}{R} \right).$$

If we now make the substitution

$$L' = - R^2 C \tag{1}$$

we find that the phase angles in the two cases become identical and that the amplitudes are the same except for terms in $(\omega CR)^2$ or terms of higher order. Thus, if $\omega CR \ll 1$, to a first order in

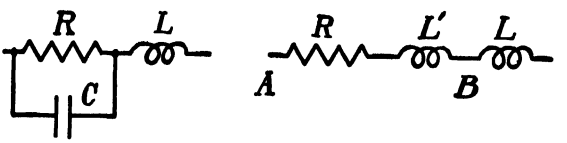


FIG. 104. Reactance compensation in resistors.

this quantity a capacitance in parallel with a resistance has the effect of a negative inductance. If we make $L = - L' = R^2 C$, the capacitance and inductance will balance to terms of the order of $(\omega CR)$ and we shall have, very closely, a pure resistance. Thus, by careful design and adjustment of resistance elements, reactive effects may be eliminated up to quite high frequencies.

Unless a second precaution is observed the compensation of A.C. standards loses its value. In any complex network the capacitances between various elements, or between elements of the circuit and their surroundings, including the observer, become increasingly important with increasing frequency. A correction

might be made for this were it not for the fact that relative motion of the parts of the circuit and their surroundings changes the capacitance. To avoid this it is essential that each capacitance be strictly localized at some fixed point. This may be accomplished by surrounding each element of the circuit with a shield of thin conducting material which is electrically connected to one of the terminals of the element or to a point at fixed potential. To prevent variable capacitance between these shields and the environment, the entire circuit must be enclosed in a larger shield, which is usually maintained at ground potential. In the better A.C. standards and bridge circuits, this *shielded construction* is used not only for resistors, but also for inductances and condensers, both of which may have stray capacitance. It may frequently be necessary as well to shield any lead wires used in connecting the unknown to the bridge circuit. If this is done, correction may be needed for the capacitance of the leads. Whenever bridge circuits are mentioned in this chapter it must be understood that they are composed of elements in which these precautions have been taken.

The inductance of condensers is usually sufficiently low so that no precautions need be taken to eliminate it. The stray magnetic field of inductances may cause trouble, but the better standards are constructed in a toroidal form so that the external field is negligible and no magnetic shielding is needed.

In the following sections there will be discussed a number of basic A.C. bridge circuits. These will be followed by brief descriptions of some commercial self contained bridges, all of which are variants on the basic circuits.

**102. The Capacitance Bridge.**

Since the equations of balance of the capacitance bridge described in Art. 51 were derived on the basis of a changing current, they apply equally well when the bridge is excited with A.C., and the condition of balance given in Eq. (4) of that article applies. In this case, of course, the ballistic galvanometer is replaced by an A.C. null indicating instrument.

The same difficulty that was mentioned in connection with the D.C. capacitance bridge applies here; namely, that the condensers are not perfect capacitances, but possess resistance as well. To

overcome this difficulty, Wien developed the bridge shown in Fig. 105, where $R_s$ and $R_x$ are variable auxiliary resistances used to balance out this resistance current. The balance is obtained by first varying $R_1/R_2$, with $R_s$ and $R_x$ equal to zero, in the usual way until a minimum current through the detecting device is obtained, then reducing this minimum as much as possible by increasing $R_s$ or $R_x$, then alternately readjusting $R_1/R_2$ and $R_s$ or $R_x$ until the balance is perfect.

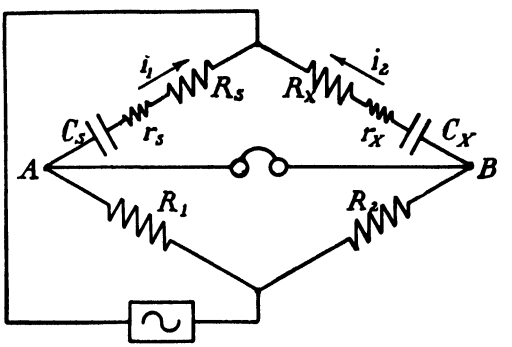


FIG. 105. The Wien capacitance bridge.

Now, if the condensers have inherent resistance, a certain amount of energy will be consumed in these resistances, and they may be replaced, for A.C., by fictitious series resistances in deriving the conditions of balance.

In order that the bridge may balance, the conditions which must hold at any instant are

$$i_1 R_1 = i_2 R_2, \tag{1}$$

$$\frac{1}{C_s}\int i_1 dt + (R_s + r_s) i_1 = \frac{1}{C_x}\int i_2 dt + (R_x + r_x) i_2. \tag{2}$$

Eliminating $i_2$ between these two equations, we find

$$\left(\frac{1}{C_s} - \frac{R_1}{R_2 C_x}\right) i_1 + \left[(R_s + r_s) - \frac{R_1}{R_2}(R_x + r_x)\right]\frac{di_1}{dt} = 0.$$

Here the coefficients of $i_1$ and $di_1/dt$ are constant with time. But for any form of pulsating current $i_1$ and $di_1/dt$ are sometimes of the same sign and sometimes of opposite sign. Hence this

equation can be satisfied at all times only by setting the coefficients separately equal to zero, so that

$$\frac{C_x}{C_s} = \frac{R_1}{R_2}, \tag{3}$$

$$\frac{R_s + r_s}{R_x + r_x} = \frac{R_1}{R_2}, \tag{4}$$

which are the equations of the Wien bridge.

If a very good standard air condenser is used for $C_s$ its resistance is infinite, so that $r_s$ is zero, and $r_x$, and consequently the phase angle of $C_x$, can be found.

## EXPERIMENT 39

### MEASUREMENT OF CAPACITANCE WITH THE WIEN BRIDGE

Set up the apparatus as shown in Fig. 105, using phones for detection if an audio frequency oscillator is used as a source, and a vibration galvanometer or A.C. galvanometer if commercial frequency is used. For $C_s$ use a standardized mica condenser of good characteristics, and for $C_x$, a subdivided 1 mf. condenser. Measure the capacitance of each section and check against all in parallel, as in Exp. 17.

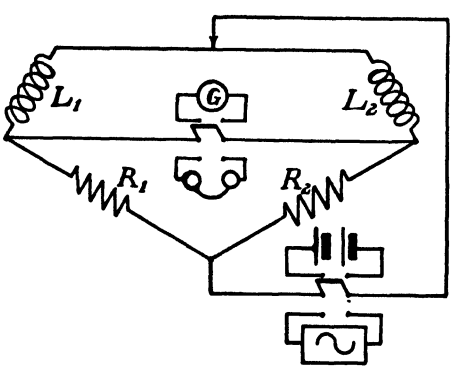


FIG. 106. The self inductance bridge.

**103. The Self Inductance Bridge.**

Fig. 106 shows the wiring diagram of the simple inductance bridge, which is identical with that shown in Fig. 41, except that both an A.C. and D.C. source are used, as are the two types of detecting devices. The derivation of Art. 52 holds for this bridge, so that the equation of

balance is

$$\frac{L_1}{L_2} = \frac{R_1}{R_2}. \tag{1}$$

**104. The Wagner Earth Connection.**

The performance of either the inductance or capacitance bridge is often impaired by the capacitance between the detecting device and ground. This trouble may be removed by means of the Wagner earth connection. This connection really consists of a duplication of half of the bridge circuit, with a central point grounded. Two arrangements are shown in Fig. 107. Balance is obtained by balancing the bridge in the ordinary way until a minimum current is obtained in the detector, then varying the

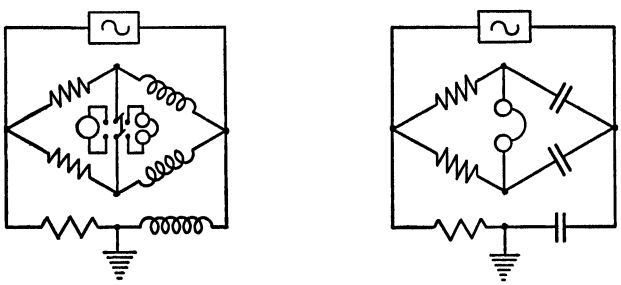

FIG. 107. The Wagner earth connection.

earth connection until a still further minimum is obtained, and so on, by successive approximations, to a perfect balance. When final balance is obtained, we have, in reality, two balanced bridges with two common arms, and have the detecting device at ground potential.

**105. Choice and Use of Null Detectors.**

With the two bridges just discussed, or with other A.C. bridges, there is a wide choice of null detectors available. Most of these have been described in Arts. 82–85. For our present purpose we may divide them into two classes, the *phase-selective detectors*, such as the lock-in amplifier, and the *non-phase-selective*, such as the telephone receiver or the vibration galvanometer. As was indicated by Eqs. (1) and (2) of Art. 102, there are

generally two balance conditions to be satisfied in using an A.C. bridge, one involving steady, the other varying currents. There are three ways of simultaneously satisfying these conditions:

1. Two detectors, one suited to D.C. and one to A.C., and two corresponding sources, may be used as shown in Fig. 106. This method is particularly suitable if the A.C. balance may be changed without disturbing the D.C. The bridge is first balanced for steady currents, then switched to A.C. and balanced again.

2. A single A.C. detector and a method of successive minima are used. The reactive elements are adjusted until a minimal response is obtained, then the resistors are changed until the detector is again at minimum. A return is then made to the reactive impedances and the process is continued until zero response is obtained. This method is very commonly used, as it is suitable to a single non-phase-selective detector, but is difficult if a change in one balance disturbs the other.

3. A single detector of the phase-selective type is used. Since the steady state balance only involves potential drops across resistances, the potential difference to be made zero in this case is in phase with the supply to the bridge. The drops across the reactances, on the other hand, are 90° out of phase with this supply. If the lock-in amplifier is used, the phase shifter may be alternately set for 0° and 90° phase shift, and balances obtained. In the final state the response should be zero for any setting of the shifter. In this case, of course, the locking-in voltage is supplied from the same source as the bridge. Another phase-selective detector which is very satisfactory is the cathode ray tube. If the bridge supply voltage is fed to the horizontal input and the vertical input is used as the detector, the general pattern obtained on the screen will be an ellipse with its major axis inclined to the horizontal. Decrease of the in phase component rotates the axis of this ellipse toward the horizontal, while decrease of the out of phase component decreases its minor axis. Consequently with this detector the two balances may be made simultaneously. A third phase-selective instrument is the dynamometer, with its field coils supplied from the bridge source. This may be used in a manner similar to the lock-in amplifier.

In those A.C. bridge circuits for which the balance condition

does not involve the frequency, frequency selection in the detector is not essential, although it may help in the elimination of disturbances. For some bridges, however, the frequency enters explicitly and for these frequency selective amplifiers are needed.

In the diagrams of this chapter either a pair of telephone headpieces or a combination of these with a D.C. galvanometer will be shown as detectors. Either of these must be understood to indicate any satisfactory detector or pair of detectors, as outlined above.

## EXPERIMENT 40

## MEASUREMENT OF SELF AND MUTUAL INDUCTANCE OF TWO COILS

Use the same coils as were used in Experiment 18 and determine the same quantities as were measured there. Use the circuit of Fig. 106, or an equivalent commercial bridge. Use a Wagner earth connection if difficulty is found in obtaining a reproducible balance.

### 106. The Stroude and Oates Bridge.

As standards of capacitance and resistance are usually available, it is convenient to have a bridge which will measure self inductance as a function of capacitance. The first of such bridges was suggested by Maxwell. Several modifications of this bridge have been suggested, the most sensitive of which is probably that of Stroude and Oates, shown in Fig. 108.

The conditions that no current flow through the detector are

$$\frac{1}{C}\int i_c dt = i_5 R_5 + i_3 R_3, \tag{1}$$

$$i_3 R_3 = i_1 R_1, \tag{2}$$

$$R_4(i_c + i_3) = R_2 i_1 + L\frac{di_1}{dt}, \tag{3}$$

and

$$i_5 = i_3 + i_1. \tag{4}$$

If (1) is differentiated and solved for $i_c$, the value thus obtained

may be substituted in (3). $i_5$ and $i_3$ are then eliminated by (2) and (4), giving

$$\frac{R_4R_1}{R_3}i_1 + CR_4\left(R_5\frac{R_1}{R_3} + R_5 + R_1\right)\frac{di_1}{dt} = R_2i_1 + L\frac{di_1}{dt}. \quad (5)$$

By the reasoning of Art. 102 the coefficients of $i_1$ and $di_1/dt$ must be separately equal to zero. Setting the coefficient of $i_1$ equal to zero, we have

$$R_3R_2 = R_1R_4. \quad (6)$$

This condition can be satisfied by making a D.C. balance.

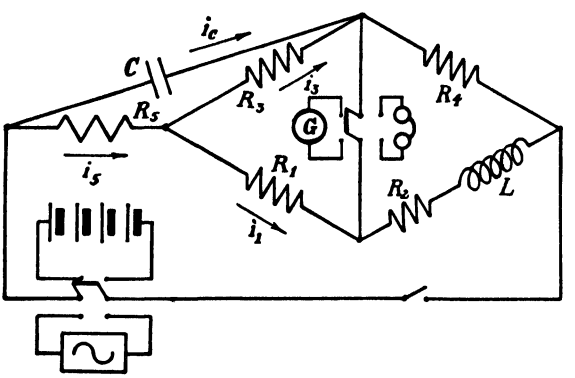


FIG. 108. The Stroude and Oates bridge.

Setting the coefficient of $di_1/dt$ equal to zero, we have, for the additional condition for an A.C. balance,

$$L = CR_4\left[R_5\left\{\frac{R_1}{R_3} + 1\right\} + R_1\right]. \quad (7)$$

The bridge may be used, of course, to determine $C$ in terms of $L$, with equal facility.

## EXPERIMENT 41

### CALIBRATION OF AN INDUCTOMETER WITH THE STROUDE AND OATES BRIDGE

Connect the apparatus as shown in Fig. 108, using a subdivided standard condenser for $C$, and dial boxes or plug resistances

for $R_1$, $R_3$, etc. $R_2$, of course, is the resistance of the inductometer.

First connect to the D.C. supply and balance the bridge by varying $R_1/R_3$, using any value of $R_4$. Calculate the approximate value of $R_2$ and reset $R_4$ and $R_3$ nearly to this value, then readjust $R_1/R_3$ for a balance. This will give the most sensitive condition for the Wheatstone bridge, with all arms of approximately the same value.

For the A.C. balance, the two terms in the brackets should be of about the same order, for greatest accuracy, so $R_5$ may be given a value of about one-half of $R_1$ for a first trial. With this value of $R_5$, adjust $C$ until an approximate balance is obtained, then readjust $R_5$ for an exact balance.

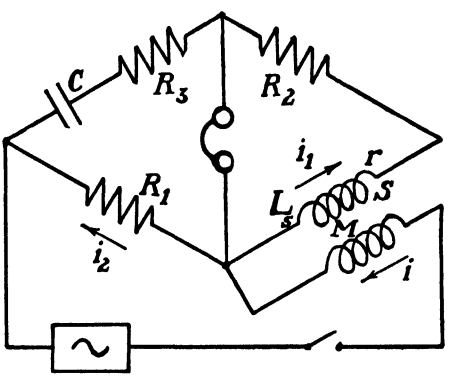


FIG. 109. The Heydweiller method for mutual inductance.

Perform the A.C. balance for about ten settings of the inductometer, equally spaced over the scale. Check the D.C. balance at frequent intervals. Plot a curve of correction in millihenries against reading of the inductometer.

**107. Heydweiller's Method for Mutual Inductance.**

The method for the measurement of mutual inductance in terms of capacitance, as outlined in Art. 55, will not work satisfactorily with alternating current excitation, due to the fact that the phase angles of the potentials will not balance. The circuit, as modified by Heydweiller for A.C. excitation, is shown in Fig. 109.

For the bridge to balance, we have

$$i = i_1 + i_2, \tag{1}$$

$$R_1 i_2 = \frac{1}{C}\int i_1 dt + R_3 i_1, \tag{2}$$

$$L_s \frac{di_1}{dt} + (R_2 + r) i_1 - M\frac{di}{dt} = 0. \tag{3}$$

Eliminate $i_2$ from (1) by substituting the value given by (2), differentiate the result, substitute for $di/dt$ in (3) and obtain:

$$\left(R_2 + r - \frac{M}{CR_1}\right) i_1 + \left(L_s - M - \frac{MR_3}{R_1}\right)\frac{di_1}{dt} = 0.$$

Since this must hold for all values of $i_1$ and $di_1/dt$, the coefficient of each term must be zero by the reasoning of Art. 102, so that

$$M = CR_1(R_2 + r), \tag{4}$$

$$L_s = M\left(1 + \frac{R_3}{R_1}\right). \tag{5}$$

## EXPERIMENT 42

### CALIBRATION OF A VARIABLE MUTUAL INDUCTANCE BY HEYDWEILLER'S METHOD

Calibrate the same laboratory standard as was used in Experiment 20, using the same precautions in regard to the relative values of $C$, $R_1$ and $R_2$ as were recommended there. With any convenient value of $R_3$, obtain as close a balance as possible, then balance by changing $R_3$. The changes then required in $R_1$ and $R_2$ should be small.

As with the Carey-Foster method, the polarity of the coils must be correct, and, if no balance can be obtained, it will be necessary to interchange the terminals of either the secondary or the primary. Also, from Eq. (5) $L_s$ must be equal to or greater than $M$, so it may be necessary to add an inductance to the arm containing the secondary.

Plot a curve of mutual inductance in millihenries against the position of the coil.

**108. The Heaviside Bridge.**

A bridge for the measurement of mutual inductance against self inductance, due to Heaviside, is shown in Fig. 110. The

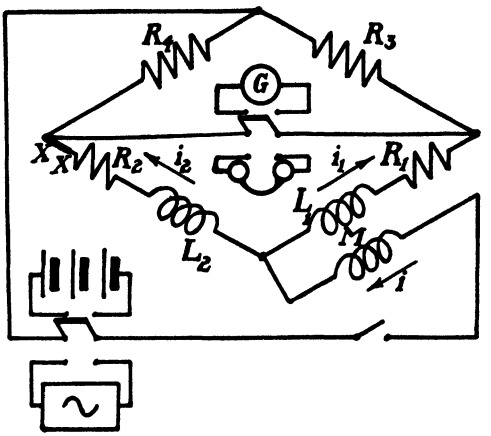


FIG. 110. The Heaviside bridge.

conditions that no current flow through the detector are:

$$i = i_1 + i_2, \tag{1}$$

$$L_1 \frac{di_1}{dt} + R_1 i_1 - M \frac{di}{dt} = L_2 \frac{di_2}{dt} + R_2 i_2, \tag{2}$$

and

$$R_4 i_2 = R_3 i_1. \tag{3}$$

Eliminating $i$ and $i_2$ from Eq. (2) with the help of (1) and (3), we find that

$$\left[L_1 - M\left(1 + \frac{R_3}{R_4}\right) - L_2 \frac{R_3}{R_4}\right] \frac{di_1}{dt} + \left(R_1 - \frac{R_2 R_3}{R_4}\right) i_1 = 0. \tag{4}$$

By the reasoning of Art. 102, the coefficients of $i_1$ and $di_1/dt$ must be equal to zero separately. Setting the coefficient of $i_1$ equal to zero, we have

$$R_1 R_4 = R_2 R_3. \tag{5}$$

This condition can be satisfied by making a D.C. balance. Setting the other coefficient equal to zero gives the other condition for A.C. balance to be

$$M = \frac{R_4L_1 - R_3L_2}{R_4 + R_3}. \tag{6}$$

For ease of computation, it is usual to keep $R_3$ equal to $R_4$ and to obtain the D.C. balance by varying $R_2$ or $R_1$. Under these conditions

$$M = \frac{L_2 - L_1}{2}. \tag{7}$$

Notice that $M$ comes out either plus or minus, depending upon the relation of the two coils.

As Campbell has shown, this bridge may be adapted to the accurate measurement of small values of self inductance by means of a differential method. Suppose that the bridge is balanced as above, and then the inductance to be tested, $L_x$, is inserted in place of the short circuiting link $X$–$X$. The D.C. balance may be brought about by varying $R_2$ or $R_1$, then the A.C. by varying $M$. If $M_1$ be the original value of $M$, and $M_2$, the new value, then

$$M_1 = \frac{L_2 - L_1}{2}$$

and

$$M_2 = \frac{L_2 + L_x - L_1}{2},$$

so that

$$L_x = 2(M_2 - M_1). \tag{8}$$

## EXPERIMENT 43

### MEASUREMENT OF SELF AND MUTUAL INDUCTANCE WITH THE HEAVISIDE BRIDGE

Two coils, the self inductances of which have been previously measured, will be measured for mutual inductance, when one is inserted inside the other. Connect the apparatus as shown in Fig. 110 and measure their mutual inductance, using an inductom-

eter for $L_2$. Then interchange the two and again make the measurement. Keep $R_3 = R_4$ throughout the work.

Next replace the coils by a mutual inductance standard, previously calibrated, and measure the inductance of a small radio frequency inductance by Campbell's modification.

**109. The Mutual Inductance Bridge.**

The bridge shown in Fig. 111 is generally used for comparing a mutual inductance with a second mutual inductance, the latter being a variable standard.

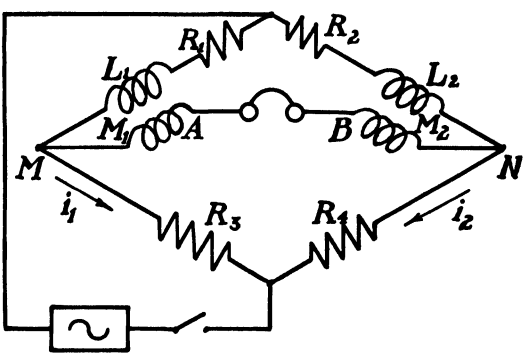


FIG. 111. The mutual inductance bridge.

With the secondaries disconnected, and the detector connected to the points $M$–$N$, the bridge is balanced as a self inductance bridge, according to the methods outlined in Art. 105. The following conditions are then satisfied:

$$R_1R_4 = R_2R_3, \tag{1}$$

and

$$L_1R_4 = L_2R_3. \tag{2}$$

Then the secondaries are inserted and the bridge is balanced again by changing the variable standard mutual inductance. Under these conditions, the potentials at $A$ and $B$ must be equal, so that

$$R_1i_1 + L_1\frac{di_1}{dt} + M_1\frac{di_1}{dt} = R_2i_2 + L_2\frac{di_2}{dt} + M_2\frac{di_2}{dt}, \tag{3}$$

$$R_3 i_1 - M_1 \frac{di_1}{dt} = R_4 i_2 - M_2 \frac{di_2}{dt}. \tag{4}$$

Adding these two equations, we see that the mutual inductance does not affect the ratio of the currents $i_1$ and $i_2$, so that

$$R_3 i_1 = R_4 i_2. \tag{5}$$

Hence

$$M_1 \frac{di_1}{dt} = M_2 \frac{di_2}{dt}.$$

and, substituting from (5), we find that

$$\frac{M_1}{M_2} = \frac{R_3}{R_4}, \tag{6}$$

which is the equation of the mutual inductance bridge.

**110. The Frequency Bridge.**

In Fig. 112, there is shown a bridge which may be used, at known frequency, to measure either inductance against capacitance or vice-versa, or, more commonly, to measure frequency

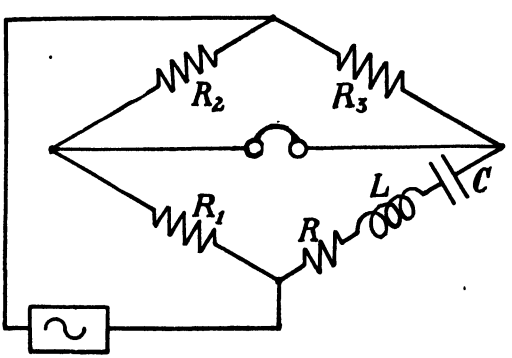


FIG. 112. The frequency bridge.

with known capacitance and inductance. The principle of the bridge is simple. The bridge, if balanced for steady currents, satisfies the equation

$$RR_2 = R_1 R_3,$$

where $R$ includes the resistance of the inductance coil and the equivalent resistance of the condenser. If an alternating potential is impressed on the bridge, since three arms possess no reactance, no balance is possible unless the fourth arm has a total reactance of zero, as any other condition would cause a phase angle between current and voltage which could not be compensated. But this condition is satisfied, according to Eq. (3) of Art. 100, only when

$$X_L - X_C = 0$$

or

$$2\pi fL - \frac{1}{2\pi fC} = 0.$$

This gives, then, for the equation of balance

$$f = \frac{1}{2\pi\sqrt{LC}}. \tag{1}$$

## EXPERIMENT 44

### FREQUENCY CALIBRATION OF AN AUDIO FREQUENCY OSCILLATOR

Connect the apparatus as shown in Fig. 112, including, in addition, a pair of double-pole double-throw switches with D.C. galvanometer and source. For $C$ use a good subdivided standard condenser, whose resistance loss is negligible, and for $L$, a continuously variable standardized inductance. Short circuit $C$ and balance on direct current, then remove the short. Take a number of readings over as large a range as your inductance and capacitance will cover, balancing the bridge at each setting of the oscillator. Check the D.C. balance occasionally.

Plot a curve of correction in cycles per second against the frequency reading of the oscillator.

**111. The Leeds and Northrup Shielded Bridge.**

An audio frequency bridge of high precision, which may be used as a capacitance or inductance measuring instrument, is manufactured by Leeds and Northrup Co. The essential element of this bridge is a *Campbell-Shackelton ratio box*, the wiring

diagram of which is shown in Fig. 113. This is an equal arm bridge, that is $M$ and $N$ are equal resistances, so that it is primarily intended for comparing two equal unknowns. The bridge elements are completely shielded, as indicated by the dotted lines of the figure, and the entire assembly, with the exception of the twelve terminals, is enclosed in a metal box which may be grounded. It will be noticed that the shielding is quite symmetrical, so that capacitances to ground balance out. In order

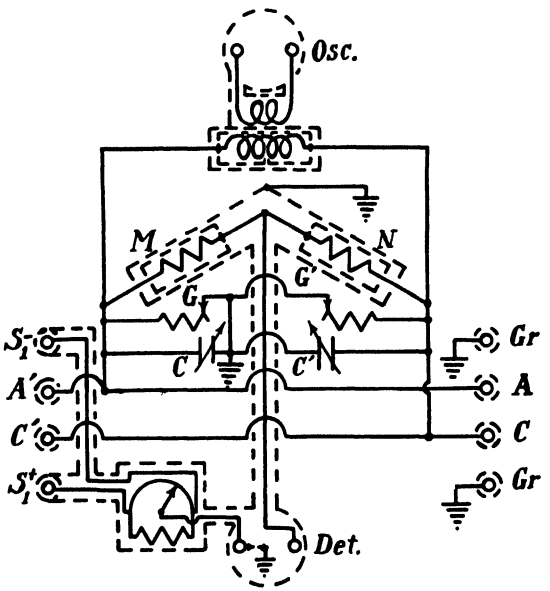


FIG. 113. Campbell-Shackelton shielded ratio box.
(Leeds and Northrup Co.—No. 1553)

to avoid disturbing effects, the bridge should be operated with metal shields surrounding the leads to the external standard and unknown impedances. Metal caps are provided for all the terminals to facilitate connection to these shields. Since complete symmetry can seldom be obtained in the external leads, the resistors $G$–$G'$, controlled by a single external knob and the condensers $C$–$C'$, also with a single control, are provided for making a preliminary balance, with the shields and leads in place but without the standard and unknown connected. A small vernier

condenser in parallel with $C'$ enables the final adjustment to be made with extreme nicety.

For use as a capacitance bridge, the unknown and standard are connected between $A$ and $Gr$ and $C$ and $Gr$, respectively, and the left hand detector terminal is grounded by means of the

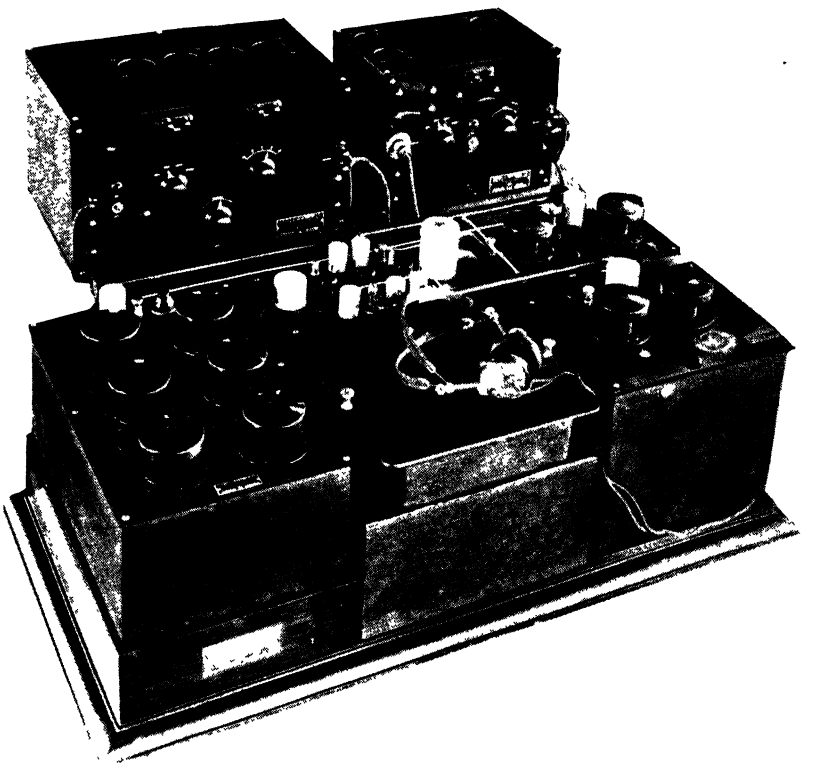

FIG. 114. Shielded capacitance bridge assembly.
(Courtesy of Leeds and Northrup Co.)

switch. For measurement of the resistance of the unknown condenser, a resistance may be placed in series or in parallel with the standard, by means of the terminals $S_1$–$C'$. As a standard, a good air condenser is to be preferred, due to its negligible leakage. The instrument set up for this measurement, with the necessary auxiliary equipment, is shown in Fig. 114.

If an unknown and a standard variable inductance are connected to the terminals $S_1^- - A'$ and $S_1^+ - C'$, the bridge becomes equivalent to the circuit of Fig. 106, with the resistance $R$ offering a means of a delicate steady state balance. This element has a resistance of 0.6 ohm and an inductance of less than $10^{-7}$ henry. If necessary a non-inductive resistance may be placed in series with either inductance. The ratio box may also be used in the circuits of Figs. 110 or 112.

**112. The General Radio Audio Frequency Bridges.**

The General Radio Company manufacture convenient precision A.C. bridges, among which are the Type 716A capacitance bridge and the Type 667A inductance bridge, both for use in the frequency range from 50 cycles to 10,000 cycles. The first of these is shown in Fig. 115, and its wiring diagram, which is that

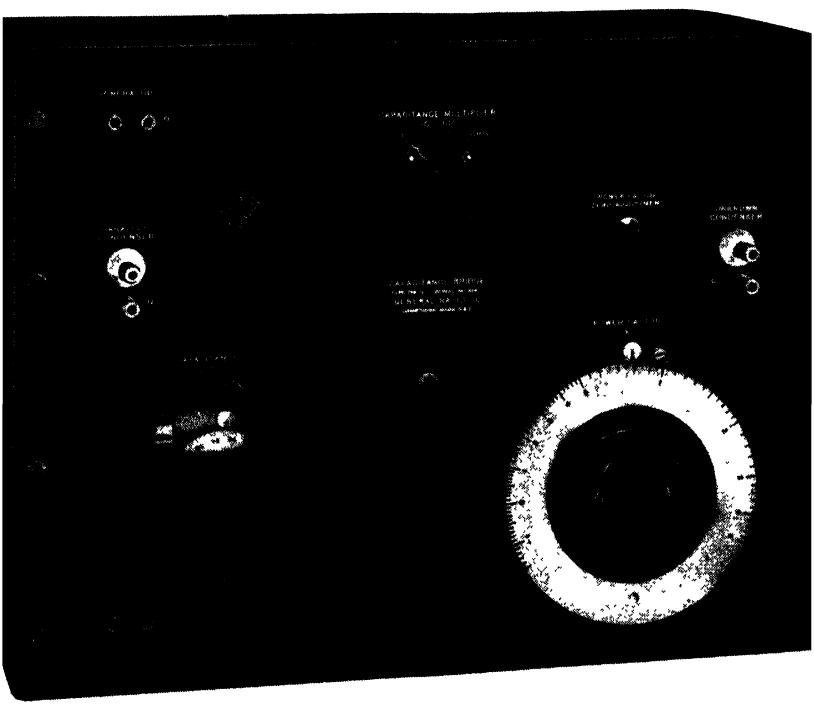

FIG. 115. Type 716A capacitance bridge. (Courtesy of General Radio Co.)

of the *Schering bridge*, is shown in Fig. 116. This differs from the Wien bridge in that the ratio arms contain capacitance in parallel with resistance so that compensation for resistance in the

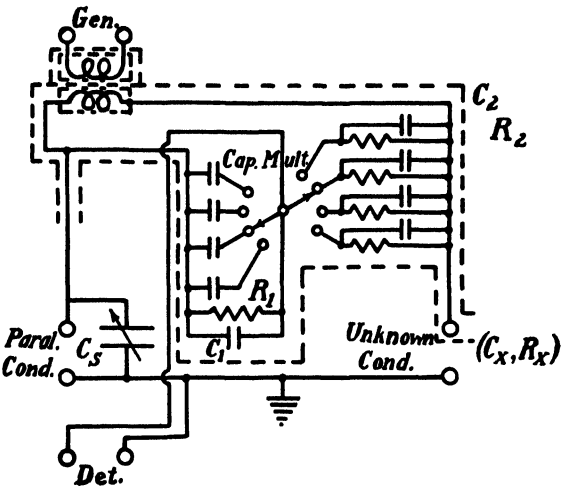


FIG. 116. Arrangement of the Type 716-A bridge.

condensers may be made in these arms. If we neglect the small compensating condensers, $C_2$, and consider that $C_s$ is a perfect condenser (i.e. without leakage) and that a condenser of capacitance $C_x$ and a parallel leakage resistance $R_x$ is connected across the unknown condenser terminals, the conditions for balance are

$$R_1 i_1 = \frac{1}{C_1}\int i_2 dt, \tag{1}$$

$$R_x i_6 = \frac{1}{C_x}\int i_5 dt, \tag{2}$$

$$i_2/C_1 = i_4/C_s, \tag{3}$$

$$R_2 i_3 = R_x i_6, \tag{4}$$

$$i_1 + i_2 = i_3, \tag{5}$$

$$i_5 + i_6 = i_4, \tag{6}$$

where $i_1$, $i_2$, $i_3$, $i_4$, $i_5$ and $i_6$ are the currents in $R_1$, $C_1$, $R_2$, $C_s$, $C_x$

and $R_x$ respectively. If we use Eqs. (1), (2), (5) and (6) to eliminate $i_1$, $i_3$, $i_4$ and $i_5$ from Eqs. (3) and (4), then differentiate and combine the resultant equations, we obtain

$$(R_1C_1R_xC_x/C_s)\frac{d^2i_6}{dt^2} + (C_1R_1/C_s - R_1R_x/R_2 + R_xC_x/C_s)\frac{di_6}{dt} + (1/C_s)i_6 = 0.$$

The second derivative term may be combined with the term in $i_6$ by use of the relation

$$\frac{d^2i}{dt^2} = -\omega^2 i,$$

leading to the final equations, if we assume $(\omega R_1C_1)^2 \ll 1$,

$$C_x = (R_1/R_2)C_s, \tag{7}$$

and

$$R_x = R_2/(\omega^2 R_1^2 C_1 C_s). \tag{8}$$

The Schering bridge is particularly convenient for the testing of condensers, since it can be arranged to read the *power factor*, or the cosine of the phase angle between the current and voltage, directly. For the unknown condenser, this angle is

$$\theta = \tan^{-1}\omega R_xC_x,$$

and if we substitute in this equation from Eqs. (7) and (8), and remember that for a good condenser this angle is always very nearly 90° ($R_x \gg 1/\omega C_x$), we obtain for the power factor

$$\text{P. F.} = \omega R_1C_1.$$

If, as is true in the Type 716A bridge, $R_1$ is constant for all settings of the ratio arms, $C_1$ may be made to read directly in power factor for a given frequency, in this case 1000 cycles.

In this bridge $C_s$ is an air condenser of $1100 \times 10^{-12}$ farads capacitance, and the main condenser of $C_1$ has a capacitance of $500 \times 10^{-12}$ farads. The other condensers in the ratio arms are small compensating condensers. The ratio arms have settings of 1000 : 1, 100 : 1, 10 : 1 and 1 : 1, with $R_1$ constant at 20,000 ohms and $R_2$ at 20, 200, 2000 or 20,000 ohms, respectively. The shielding system is indicated by the dashed lines in the figure.

A second bridge by this manufacturer is the Type 667A in-

ductance bridge. This bridge has a circuit which is essentially that of Fig. 106, but differs from it in that a small variable inductor is included in series with the unknown. When the resistance balance and an approximate inductance balance have been obtained, this inductor may be used to achieve the final balance without disturbing the resistance. The bridge is furnished with an internal standard of one millihenry, has a range from 0.1 microhenry to 1 henry, and is arranged so that an external standard may be substituted for the internal. The arms are shielded

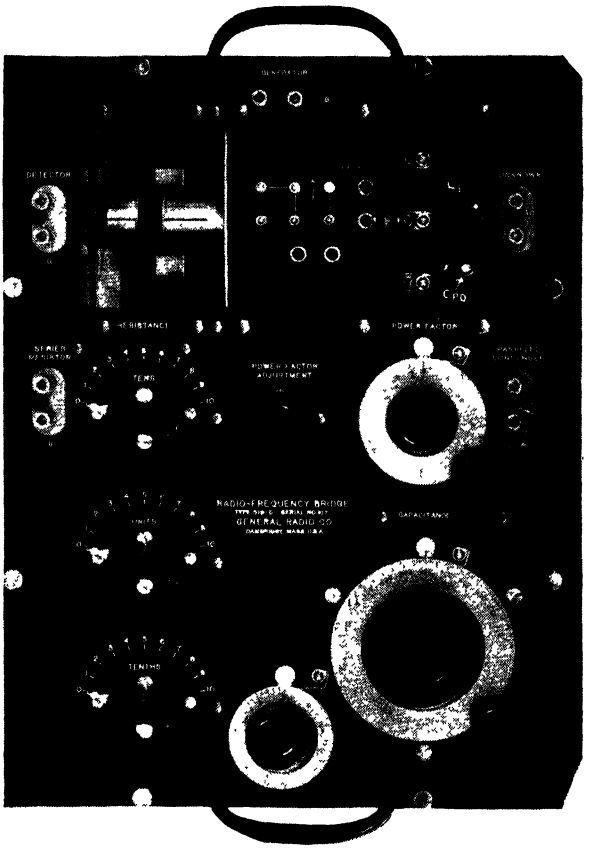


FIG. 117. Type 516-C bridge. (The plate which ordinarily covers the upper center has been removed to show the detector transformer, the jack board and the correcting adjustments.) (Courtesy of General Radio Co.)

from each other and the elements are compensated for resistance and inductance changes at different positions of the switches.

**113. The Type 516C Radio Frequency Bridge.**

While all the A.C. bridges thus far described are applicable, in principle, to measurements at radio frequencies, their extension into this range offers difficulty due to the great importance

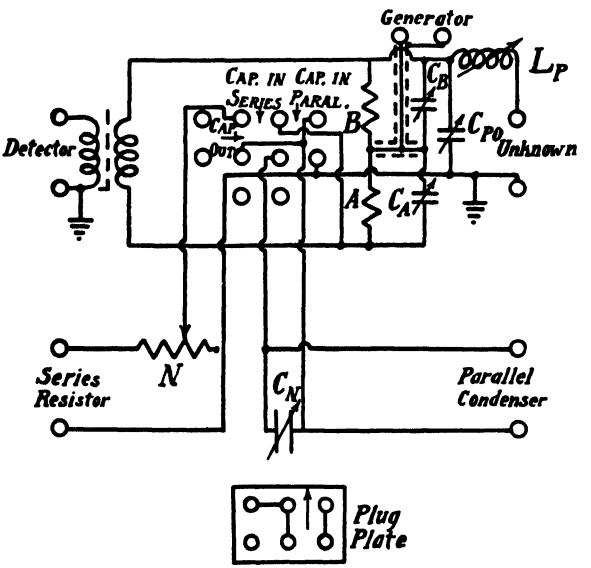


FIG. 118. Wiring diagram of the Type 516-C bridge.

of stray series inductances and distributed parallel capacitances. A bridge specially designed for measurements up to 1.5 megacycles with reasonably high accuracy and at higher frequencies with a sacrifice in precision, is the Type 516C bridge of the General Radio Company, shown in Fig. 117. The wiring diagram is shown in Fig. 118, and it will be found to be identical with that of a Schering bridge except that the variable inductance $L_P$ and the variable condenser $C_{PO}$ have been added. In following the diagram, consider that the plug plate has been inserted in the jack board at the center in such a way that the arrow points to *Cap. in Series*, *Cap. in Parallel* or *Cap. Out.* The bridge

is an equal arm instrument, since it is only with this type that easy compensation can be made. The resistance $N$ has an inductance of about one microhenry, and this is constant to about ten per cent for all settings of the resistor from 0.1 to 111 ohms. This may be compensated for by adjusting the inductance of $L_P$ by means of an adjusting screw beneath the cover. The $20 \times 10^{-12}$ farad adjustable condenser, $C_{PO}$, provides a means of introducing into the unknown arm a capacitance balancing the distributed capacitance of $N$ and the leads of the standard arm, as well as the capacitance of $C_N$ to ground. The control of the $50 \times 10^{-12}$ farad $C_A$ is labeled *Power Factor* and the dial on this is calibrated to read this quantity directly at one megacycle. $C_B$ is a $25 \times 10^{-12}$ farad condenser which is labeled *Power Factor Adjustment* and which is used to balance the capacitance of $C_A$ when the latter is set to zero. At the lower right hand corner of Fig. 117 there are shown two condenser dials. The larger of these controls the main condenser of $C_N$ and reads from 50 to $1170 \times 10^{-12}$ farads in 112 divisions. The other controls a small condenser in parallel with this and allows a fine adjustment to be made to an estimated $10^{-13}$ farads.

In using the radio frequency bridge there is some choice regarding the oscillator and the detector to be used. The oscillating unit must be shielded to prevent its field from interfering with the measurement. If a simple radio frequency oscillator is used, the detector must be capable of indicating radio frequencies. At low frequencies (say up to 100,000 cycles) amplifiers and vacuum tube voltmeters of any type or the cathode ray oscillograph are satisfactory, but at high radio frequencies it may be necessary to beat the output with a second oscillator of almost the same frequency and then to detect the audio frequency or supersonic beat note. The other possibility is to modulate the supply oscillator with an audio frequency note. If this is done, the null instrument may be a radio receiving set including a demodulator, so that an audio frequency signal is obtained in the output.

In order that the bridge may be used with accuracy at radio frequencies it is necessary that the adjustments of $L_P$, $C_{PO}$ and $C_B$ be checked. To do this a set of standards with calibrations independent of frequency must be available. These should first

be measured on an audio frequency bridge, then inserted in the unknown arm of the radio frequency bridge. The standards in the *N* and *A* arms of the bridge are set to the correct values and the adjustment made with the three correction elements. The standards recommended by the manufacturer are a Type 505A and a Type 505F condenser (100 and 1000 $\times$ $10^{-12}$ farad, and 0.01 per cent and 0.05 per cent power factor, respectively) and a Type 500D (100 ohm) resistor. It is necessary to measure or calculate the impedance of the leads to the unknown and correct for these whenever small capacitances or inductances are being measured. For measurements of resistance or capacitance within the range of the instrument the bridge is used as a Wheatstone or Scherer bridge, by inserting the plug plate in the proper position. For measurements of inductance, or for values of capacitance above the range of the bridge, a condenser may be inserted in series with the unknown and the impedance of the latter computed from the measured values for the condenser and the combination.

**114. Substitution Measurements with A.C. Bridges.**

Due to the disturbing effects of distributed inductance and capacitance, substitution measurements, in which a standard and unknown impedance are made to change roles in a circuit, are particularly valuable in A.C. measurements. In such measurements, since the other three arms of the bridge remain untouched, residual uncompensated errors in these arms lose their importance. While the range of substitution methods is limited only by the ingenuity of the experimenter, only one experiment will be given as an illustration.

## EXPERIMENT 45

### CALIBRATION OF AN AIR CONDENSER BY SUBSTITUTION

Set up a Schering bridge, with appropriate audio or radio frequency source and detector. Connect the variable air condenser, which is to be calibrated, in parallel with the standard condenser. As a dummy in the other arm place any shielded condenser of about the same capacitance as the standard. Now disconnect the unknown, leaving the leads to it mechanically

undisturbed and connected to the bridge. Balance the bridge by adjustment of the standard and the power factor condensers, read the capacitance of the former and call this the original reading. Now reconnect the unknown, set its dial at zero and check the balance. If the capacitance of the condenser is zero and its parallel resistance infinite no change will have taken place, but probably neither of these conditions will be satisfied. Since the value of the power factor condenser does not enter into the capacitance reading (cf. Eq. (7), Art. 112) it may be changed to balance the resistive component. If balance cannot be restored by this means alone, decrease the capacitance of the standard condenser. The difference between its reading at this balance and the original reading is the zero capacitance of the unknown. Now, leaving all bridge adjustments untouched, set the unknown dial to read 1/5 of its full scale. Rebalance by means of the standard and take the difference from the original reading as the capacitance at this setting. Repeat for 2/5, 3/5, 4/5 and 1 times full scale, using the power factor condenser if necessary to bring about the exact balance in each case.

If the capacitance of the unknown is greater than the original reading it will be impossible to reach full scale by this method. In case it is impossible to achieve a balance on a particular setting return the unknown and standard condensers to their values on the last setting, so that the bridge is again balanced. Now replace the dummy condenser in the unknown arm by one of approximately twice its capacitance. Leaving the unknown condenser untouched rebalance the bridge by means of the standard and the power factor condensers. The difference between the reading of the standard and its last reading must be added to all future capacitances, but otherwise the procedure may be continued as before.

Plot a curve of capacitance against dial reading of the unknown condenser.

**115. Vacuum Tube Voltmeter Methods.**

When the precision of bridge methods is not needed, comparisons of impedances with the vacuum tube voltmeter offer convenient methods of comparing impedances. A simple circuit for this purpose, due to Dowling, is shown in Fig. 119 and is suitable

for either audio or radio frequency use for any impedances small compared with the input impedance of the vacuum tube. $R$ is an accurately known adjustable pure resistance and $Z_x$ the impedance to be measured. Since the same current flows through both, under the condition given above, it follows that the two have equal magnitudes of impedance if the same voltage drop occurs across both. $R$ is adjusted until reversal of the switch does not change the reading of the galvanometer in the plate circuit of the rectifying meter.

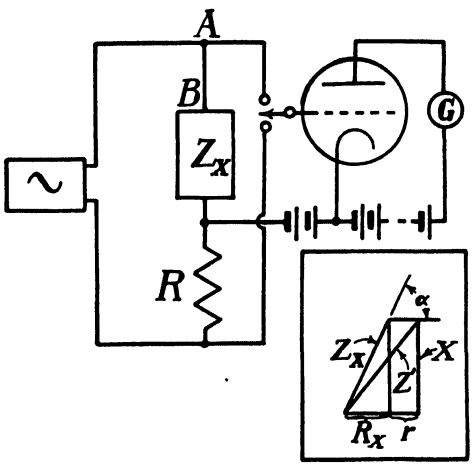


FIG. 119. Vacuum tube voltmeter comparison of impedances.

In general the condition $Z_x = R$ is not sufficient to determine the characteristics of the unknown, since this does not separate capacitance, inductance and resistance. The combined effect of the first two may be separated from the last by a simple device. Suppose that another pure resistance is inserted between the points $A$ and $B$, and the impedance of this in series with $Z_x$ is measured and found to be $Z'$. The resistance itself is measured by removing $Z_x$ and is found to be $r$. We now have, from Eq. (3A) of Art. 100

$$Z_x^2 = R_x^2 + X^2, \tag{1}$$

and

$$Z'^2 = (R_x + r)^2 + X^2, \tag{2}$$

where $X$ is the total reactance of $Z_x$. Since these expressions represent the relations between the sides of right triangles they may be represented geometrically as in the insert of Fig. 119. It is then obvious that

$$\cos \alpha = (Z'^2 - Z_x^2 - r^2)/2rZ_x, \tag{3}$$

and since, from Eq. (3) of Art. 100,

$$\tan \alpha = X/R_x, \tag{4}$$

we can determine $X$ and $R_x$ from the two simultaneous equations (1) and (4). If another set of measurements are made at a new frequency, the capacitance and inductance of the unknown may be found from the relation

$$X = \omega L - 1/\omega C.$$

Another ingenious application of the vacuum tube voltmeter is illustrated in Fig. 120. If an oscillating circuit is set up with a nearly perfect air condenser, practically the entire resistance $R$ will be associated with the inductance $L$. If the circuit is tuned to resonance, by adjustment either of the capacitance or of the oscillator frequency, the ratio of the oscillator peak voltage, $V_0$, to the peak voltage across the inductance, $V_L$, is, by Eqs. (7) and (9) of Art. 88 and Eq. (3A) of Art. 100,

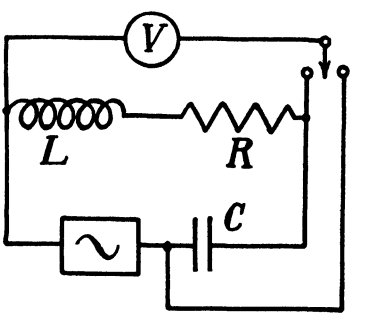

FIG. 120. Basic circuit of the "Q" meter.

$$V_L/V_0 = \sqrt{1 + \omega^2 L^2/R^2} = \sqrt{1 + Q^2} \approx Q,$$

the last relation being almost exact if $Q \gg 1$. Since both of these potentials may be measured by the vacuum tube voltmeter $V$, the $Q$ of the coil may be directly determined. If the inductance and resistance are needed an auxiliary resistance $R'$ may be inserted in series with the coil and the measurement repeated.

If the new value is $Q'$ it follows that

$$\omega L = \frac{QQ'}{Q - Q'} R',$$

and

$$R = \frac{Q'}{Q - Q'} R'.$$

**116. The Boonton "Q" Meter.**

An instrument using the resonant circuit of the last article is manufactured by the Boonton Radio Corporation and is shown in Fig. 121. This instrument contains an oscillator, adjustable

FIG. 121. The Boonton "Q" meter, Type 160-A. (Courtesy of Boonton Radio Corp.)

in frequency from 50 kilocycles to 75 megacycles, in eight ranges, with output voltage control, an air condenser with a vernier adjustment and a vacuum tube voltmeter, as well as appropriate power supplies. The terminals at the right hand top of the meter allow unknown impedances to be inserted in the circuit. Inductances may be measured by the method of the last article, the value of the frequency being determined directly from the

tuning dial of the oscillator condenser, which is calibrated to about 1 per cent. Arrangement is made to hold the oscillator output voltage at a constant value, so that the vacuum tube voltmeter is calibrated directly in $Q$. If a capacitance is to be measured it may be inserted in parallel with $C$ and the change in the latter necessary to restore the circuit to resonance at the same frequency gives a measure of the capacitance. Resistances may be measured by inserting them in series or parallel with a previously measured inductance. Their resistances may be measured by the change in $Q$ and their residual inductances or distributed capacitances by the change in the resonant frequency. Provision is made for the use of an external oscillator for measurements at frequencies from one to fifty kilocycles.

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# CHAPTER 10

## ELECTRICITY IN GASES—ELECTRICAL VACUUM MEASUREMENTS

### 117. Normal Conductivity of Gases.

In Art. 60 it was explained that an electric current consists of the movement of electrons or of charged atoms or molecules. Moreover, it was pointed out that, in their normal state, the atoms or molecules of a gas may be considered to be electrically neutral. From this we should expect gases to be nonconductors. If a gas were completely undisturbed, this would probably be rigorously true, and, in actual practice, when suitable precautions are taken, gases make excellent insulators. However, if sufficiently sensitive current measuring devices are used, a current is always found to flow when a potential gradient exists in the gas. This phenomenon can be accounted for by the presence of a small number of free electrons and ions in the gas, the latter being charged atoms or molecules. The existence of these ions and free electrons may be due to several causes, among which are small amounts of radio-active matter in the vicinity, chemical action, visible or ultraviolet light, x-rays and cosmic rays. The presence of charged particles may be demonstrated by establishing in the gas a small potential gradient, insufficient to produce appreciable ionization by the process described in the next article. At first, as the field is increased, the minute current flowing will increase linearly with it, but will eventually fall off to a constant saturation value, when all the ions and electrons produced in the gas are being used to carry current.

### 118. Discharges in Gases.

Now suppose that a high potential is applied between two electrodes immersed in a gas which contains the ions and electrons which we have just discussed. Due to the electrostatic field set up, the electrons and negative ions will move toward the positive electrode or *anode*, while the positively charged ions will

move toward the negative electrode or *cathode*. Both will acquire velocities in accordance with Eq. (2) of Art. 60, where $m$ is the mass of the particle under consideration, and $e$ its charge. They will move freely in this manner for a short time, depending upon the density of the gas, then will collide with a gas atom or molecule. If, at the time of this collision, they are moving with a high enough velocity, the impact may be sufficient to remove one or more electrons from the particle which they strike, thus furnishing new carriers of electricity, which may repeat the process. Thus, after a very short time, the discharge consists of an atmosphere of positively and negatively charged particles migrating toward the cathode and anode respectively. Certain of the ions and electrons will *recombine* to form neutral atoms or molecules and in this process of recombination light is emitted, accounting for the color of the discharge. This light, by means of the photoelectric effect,* may bring about the emission of electrons from other atoms and from the electrodes, so that the condition of equilibrium involves many factors.

Now let us consider a discharge tube across which a constant potential is maintained. Initially, let the pressure of the gas in the tube be high. Then, since the electrons (which seem to be the principal ionizing agents) will experience collisions before they have acquired sufficient energy to disrupt an atom, no discharge can take place. As the pressure is decreased, however, the length of path between collisions becomes greater and greater, until, finally, the tube lights up with a brilliant glow and a very appreciable current flows between the electrodes. As the pressure is decreased still fur-

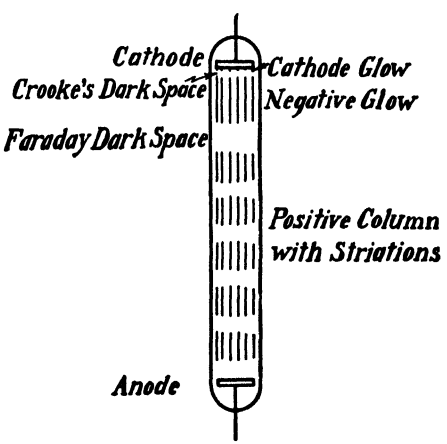


FIG. 122. The gaseous discharge.

* See Chapter XII.

ther, the conductivity keeps increasing, and soon the discharge takes on a characteristic appearance, as shown in Fig. 122. On the surface of the cathode, a luminosity, the *cathode glow*, can be observed. Beyond this is a dark region, known as the *Crooke's dark space* or the *cathode dark space*. This is followed by a second region of luminosity, the *negative glow*, by another dark region, the *Faraday dark space*, and, finally, by the brilliant *positive column*. This column may be either a continuous stream of light, or, more commonly, a series of alternate bright and dark strips, known as *striations*.

Now, as the pressure is further decreased, the long path between collisions of the electrons is compensated for by the decreasing number of collisions, and the current, after passing through a maximum value, begins to decrease. At the same time, the cathode dark space extends farther and farther into the tube, until, eventually, it occupies the entire tube, the walls of which now fluoresce under the impact of the *cathode rays*. If the vacuum is increased still further, the current finally ceases.

Not only are these phenomena of extreme interest in the study of gases, but, also, they may be used to measure approximately the pressure existing in a closed system. Both the conductivity of the gas and the size of the cathode dark space are used for this purpose.

### 119. Primary Pressure Measurements.

Before we can enter into the experimental study of gaseous discharges, since these depend so essentially on the pressure of the gas, it will be necessary for us to discuss some of the usual means of measuring this pressure.

The most common unit of gas pressure is the millimeter of mercury. The origin of this unit may be easily understood by reference to the closed end manometer shown in Fig. 123 *A*. The region *M*, above the mercury, is thoroughly evacuated and sealed off, and the other end of the tube is connected to the chamber in which the gas pressure is to be measured. Now, for equilibrium, we must have the forces in two directions at *N* equal; hence, if the pressure of the gas is *P*, the cross sectional area of the tube is *A*, and the difference in height of the two mercury

columns is $p$, as shown, we have

$$PA = pdA, \tag{1}$$

where $d$ is the density of the mercury. Hence,

$$P = pd. \tag{2}$$

But, since $d$ is a constant, it is equally satisfactory to denote the pressure by $p$, the height of the mercury column. Since $d = 13.6$

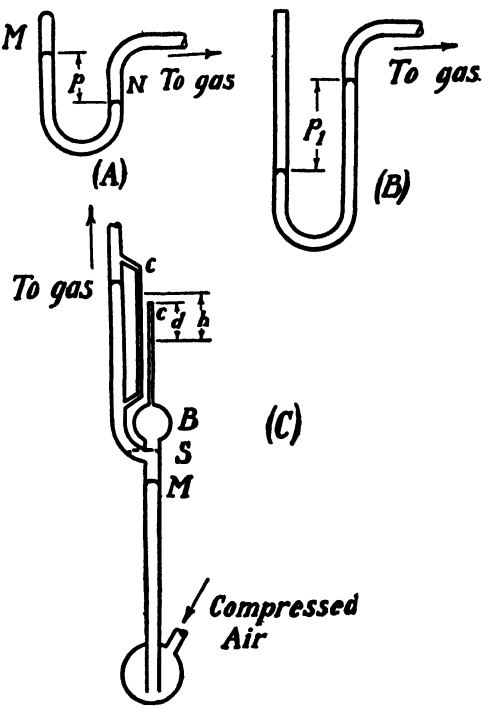


FIG. 123. Primary pressure measuring devices. *A*, the closed end manometer. *B*, the open end manometer. *C*, the McLeod gauge.

grams per cc., one millimeter of mercury is equal to 133 newtons per square meter. In Fig. 123 *B*, the *open end manometer* is shown. This instrument, having the pressure of the atmosphere added to that of the left hand mercury column, reads the differ-

ence between the pressure of the gas and that of the atmosphere, and consequently is often used for pressures near atmospheric.

For low pressures, say from one or two millimeters downward, the manometer, of course, is practically useless, since the mercury columns cannot be read with any degree of accuracy. For this range, the *McLeod Gauge* shown in Fig. 123 *C* is very generally used. This gauge in the usual form consists of a barometric column, with the reservoir arranged for connection to a compressed air system, or other means of raising and lowering the mercury, a bulb, *B*, and two capillary tubes, *C–C*, selected for uniformity of diameter throughout. Now suppose that the upper part of the gauge, connected to the vacuum system, has been evacuated to a low pressure of $p$ millimeters of mercury. The barometric column, with the reservoir open to the atmosphere, will stand at some level such as *M*, behaving as an open end manometer. Now let the column be raised. As it reaches the height *S* it closes the connection between the two parts of the gauge. Now, as it is raised still farther, the gas in both parts is compressed. The left hand side, however, is connected to a vacuum system, the volume of which is usually large compared with that of the gauge, so that the compression here can be considered negligible. On the right, however, the gas is compressed quite appreciably, and, particularly after the mercury has reached the capillary, the right hand column will lag behind that on the left. Eventually, some condition such as that shown in the figure will be attained, with the right column $d$ millimeters below the top of the tube, the left capillary column $h$ millimeters above this level and the column in the connecting tube somewhat above that in the capillary. The last difference is due to the capillary depression of the mercury in the small tube, and shows why it is necessary to have the two capillaries, used in the measurement, of the same diameter, in order that this depression may be balanced out.

We shall now assume that the gas in the gauge is an ideal gas, that is, that it obeys Boyle's law

$$p_1V_1 = p_2V_2.$$

Then, if $V$ is the combined volume of the bulb and capillary and $A$ is the cross sectional area of the latter, the new pressure in the tube will be

$$P = \frac{V}{Ad}p.$$

This pressure must be balanced by the pressure on the left, so

$$h + p = \frac{V}{Ad}p$$

or

$$p = \frac{Ahd}{V - Ad}. \tag{3}$$

In the usual construction, the volume $Ad$ is from 0.01 to 0.0001 times $V$, and since the errors in the gauge due to frictional charges on the glass, friction, etc., are greater than this correction, this term may safely be neglected, giving the formula of the McLeod gauge as

$$p = \frac{Ahd}{V}. \tag{4}$$

During the construction of the gauge, the factor $A/V$ is measured, and is given as the constant of the gauge. Since $A$ may not be a strict constant, due to irregularities at the end of the capillary, it is customary to measure the factor $Ad/V$ for various points of the capillary, which are then marked off on the gauge. When the instrument has been calibrated in this way the right hand column is brought up to the point marked, and only $h$ is read. Another system sometimes used is to bring the left hand column to a predetermined position and to read $d$ only.

Notice that, due to the fact that the pressure of the gas over the left column is small compared with that of the mercury, any changes in this pressure do not affect the reading, and consequently the pressure read is that existing at the time that the mercury passes the seal off point, $S$. McLeod gauges are used commonly for measurement of pressures from several millimeters down to about $10^{-5}$ millimeters. To increase the range, two bulbs are often used on the same gauge, the small bulb and the straight capillary being used for high pressures while the large bulb and the same capillary are used for low.

## EXPERIMENT 46

## CHARACTERISTICS OF A DISCHARGE TUBE

An arrangement for determining the characteristics of a discharge in air is shown in Fig. 124. The discharge tube $D$ is connected to a McLeod gauge, a manometer, $P$, and, through a two-way stopcock, either to the vacuum pump (which may be

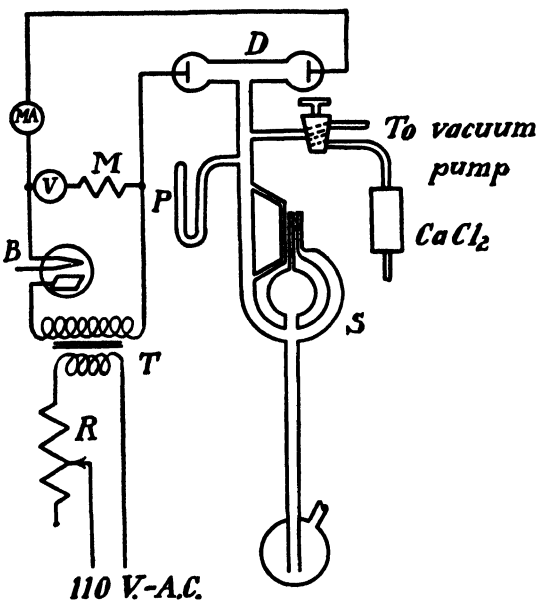


FIG. 124. Characteristics of a discharge tube.

an oil pump, and should evacuate to a pressure of about $10^{-3}$ mm.) or to the atmosphere. The latter connection is made through a drying tube containing calcium chloride. The current is supplied by a transformer having a secondary potential of about 2000 volts, and may be rectified either by the simple half wave rectifier shown or by a two or four tube full wave rectifier. $V$ is a voltmeter with a series multiplier $M$, and $MA$ is a milliammeter, preferably one with a multiple range.

The test may be operated either at constant current or at constant potential. While the latter is of greater theoretical

interest, the former will usually be found simpler to run with the apparatus shown and will cover a wider range of pressures, so it is better adapted to the measurement of pressure in the system. With the voltage on and the rheostat $R$ all the way in, start evacuating the tube. As soon as the discharge starts, close the stopcock to the pump and adjust $R$ until the milliammeter reads the predetermined value. (For a tube of about one inch diameter and seven or eight inches long between electrodes, ten to twenty milliamperes will be a satisfactory value; for smaller tubes less should be used to avoid excessive heating.) Throughout the rest of the test hold the current to this value. Read the pressure on the McLeod if it is within the range of the gauge; if not, read it on the manometer. Record the current and voltage. Then reduce the pressure and take another set of readings. When reading on the McLeod, be certain to read the voltmeter at the time of sealing off with the mercury column.

Take about fifteen readings between the maximum pressure at which discharge takes place and the minimum. As soon as you have reached pressures low enough for the cathode dark space to become appreciable, measure the length of this space with a millimeter scale for each reading you take. It may be necessary to let air into the apparatus and to run over the range of pressures several times in order to get sufficient points. Never open the system to atmospheric pressure *when the mercury stands above the seal off level, S.* To do so may drive the mercury upward and break the gauge. Also, *never let air into the system rapidly* or the mercury in the manometer may be driven upward with sufficient force to break the glass at the top.

Repeat your observations at a current of one-half or one-third of that used in the first run. Plot a curve of resistance in ohms against pressure in millimeters for each current used. On a second sheet, plot the length of the cathode dark space against pressure.

**120. Vacuum Measurements by Electrical Methods.**

In recent years, the use of vacuum tubes and of electrical discharges in gases at low pressures has become so widespread that the methods of measuring vacua have become important. The McLeod Gauge already described provides an absolute

method of determining the pressure but has several limitations. For one thing, the use of liquid air traps or equivalent devices is necessary to prevent the introduction of mercury vapor into the system. Also, in cases where the vacuum system remains connected to the pump, the pressure in different parts may differ by a large factor, and it is difficult to get a McLeod Gauge to measure the pressure at the point where it is desired, due to the long connections required. Again, if the pressure is rapidly changing, the McLeod Gauge may be too slow to measure it. Finally, at exceedingly low pressures, the McLeod Gauge is impracticable. To meet these situations, various other methods of measuring low pressures have been devised. Most of these are "secondary methods" which require calibration against some primary standard such as the McLeod Gauge.

**121. The Hot Wire Manometer.**

If a hot wire is surrounded by a gas it will lose heat by conduction through the supports, by radiation and by increasing the kinetic energy of the gas molecules which strike it. The first of these losses is proportional to the thermal conductivity of the wire, the cross section and the temperature gradient at the ends, and may be made very small by using a long, fine wire. The second is proportional to the difference of the fourth powers of the absolute temperature of the wire and its surroundings. If the wire is at a temperature $T_2'$ and the walls of the containing vessel are at $T_1'$, this loss per unit area per second may be written as

$$E_R = Q(T_2'^4 - T_1'^4)$$

where $Q$ is a constant characteristic of the wire. If the temperature difference is small compared with the absolute temperature of the walls, this expression may be expanded to give

$$E_R = Q\{(T_1' + T_2' - T_1')^4 - T_1'^4\} \approx 4T_1'^3(T_2' - T_1'), \quad (1)$$

so that the radiation loss is proportional to the temperature difference to terms in the first order of $(T_2' - T_1')/T_1'$.

Now consider that the wire is surrounded by a gas at pressure $p$ and temperature $T_1$. Since this pressure is due to the change of momentum at the wire it will be, by simple kinetic theory

considerations

$$p = 2nmv \approx 2n(3mkT_1)^{1/2}, \tag{2}$$

where $v$ represents the mean molecular velocity, $m$ is the mass of the molecule, $k$ is the Boltzmann constant and the last equation is obtained from the relation

$$3kT/2 = mv^2/2.$$

Each molecule will carry away an amount of energy equal to its specific heat, $c$, times its change in temperature so, solving Eq. (2) for $n$, we find that the amount of energy conducted away from each square centimeter of the wire per second by the gas, if its temperature on leaving the wire is $T_2$, is

$$E_G = \frac{pc(T_2 - T_1)}{2\sqrt{3mkT_1}}. \tag{3}$$

This would be the equation of a hot wire manometer if $T_2$ and $T_1$ were equal to $T_2'$ and $T_1'$. It was experimentally found by Knudsen, however, that a molecule does not come to equilibrium with a solid on a single collision with the surface but that

$$T_2 - T_1 = a(T_2' - T_1') \tag{4}$$

where $a$ is a constant, depending upon the nature of the gas, the surface and the temperature, known as the *accommodation coefficient.*

In the particular apparatus which we have postulated the surface of the glass cylinder is large compared with that of the wire. We shall further assume that the pressure is sufficiently low so that the mean free path of the molecules is large compared with the linear dimensions of the tube. Hence the molecules experience many collisions with the wall between successive collisions with the wire, but in general they reach the wire immediately after colliding with the wall and before they have collided with any other molecule. It follows that they have a velocity corresponding to $T_1'$ on reaching the wire but that their velocity on leaving it is determined by Eq. (4). Consequently

$$E_G = \frac{apc(T_2' - T_1')}{2\sqrt{3mkT_1'}}. \tag{5}$$

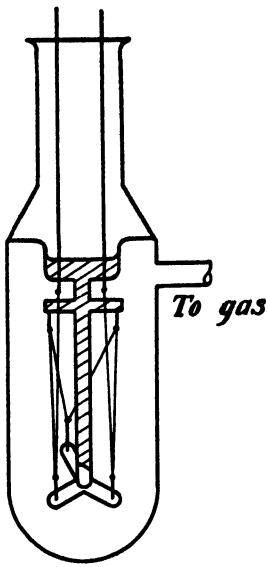


FIG. 125. The hot wire manometer.

A successful type of hot wire gauge is shown in Fig. 125. The heated wire is platinum of about 0.03 mm. diameter and this is welded to supports which are about twice as heavy, and which are attached to a glass supporting frame. This construction insures uniform conduction losses at the points of contact. The tube has thin walls and is immersed in a constant temperature bath. It has a long stem protruding from the bath so that moisture condensation will not spoil the insulation of the leads. While construction of this type is essential for the measurement of very low pressures, for a great deal of ordinary vacuum work the extreme care taken is not necessary. Unless corrosive gases are to be used with the gauge, other materials than platinum may be used, and tungsten has a particular advantage in that very fine wires may easily be obtained.

One of the most convenient circuits for using the hot wire manometer, or the *Pirani gauge*, as it is commonly known, is shown in Fig. 126. One arm of the Wheatstone bridge contains the gauge, whose resistance is $R$ at the temperature used, the other three arms being composed of good manganin resistances of negligible temperature coefficients. The bridge is first brought to a balance with a voltage $V_0$ across it, with zero gas pressure in the tube, this voltage being sufficient to maintain the wire at a desired temperature. As the pressure in the gauge is changed, the voltage across the bridge is varied so as to maintain a balance. Under these conditions the voltage across the gauge is obviously proportional to the voltage across the bridge, and the power consumed in the gauge equals $V^2R/(R + R_1)^2$. With zero gas pressure the energy loss by conduction and radiation will equal $V_0^2R/(R + R_1)^2$. At any other pressure the conduction loss will change somewhat, due to a change of temperature distribution along the wire, but if this loss is small the energy consumption

in the wire will approximate closely to

$$\frac{V^2R}{(R+R_1)^2} = \frac{pac(T_2' - T_1')}{2\sqrt{3mkT_1'}} + \frac{V_0^2R}{(R+R_1)^2}, \tag{5}$$

which may be put into the simpler form

$$V^2 - V_0^2 = Kp \tag{6}$$

where $K$ is the constant of the gauge. It is obvious that this formula will hold only up to pressures such that the mean free path is of the order of the linear dimensions of the tube. Due

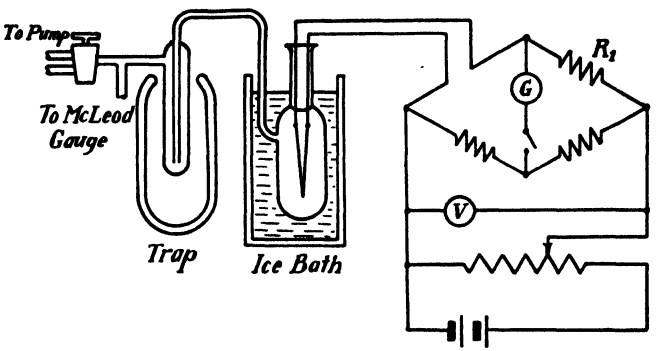


FIG. 126. Calibration of a resistance manometer.

to the fact that the accommodation coefficient is extremely sensitive to the cleanliness of the surface, the hot wire manometer cannot be used as a precision instrument unless great care is taken to maintain a clean, reproducible surface on the wire, and this can seldom be done in the course of the pressure measurements. For the same reason it is necessary that the gauge be calibrated with the same gases as those with which it is to be used, if more than a qualitative indication of pressure is needed.

A discussion of the conditions under which the gauge is best used may not be out of place, and the essential points are all indicated by Eqs. (5) and (6). It is desirable to have the sensitivity, $K$, as large as possible. Unless the voltmeter is shunted by a potentiometer to reduce the initial reading to zero, $V_0$ should be kept small in order that a large part of the scale of the indi-

cating instrument shall not be used for reading this part of the total voltage. To keep this term small the radiation and conduction losses must be minimized. Since the cross sectional area of the wire, which determines the conduction loss, varies as the square of the diameter, and the surface, which enters linearly into the loss through the gas, varies only as the first power, the ratio of $V$ to $V_0$ for a given pressure will increase with decreasing size of wire. Extreme length of wire also decreases the relative value of the conduction loss, as conduction varies inversely with the length, and loss through the gas directly. The ratio

$$\frac{R}{R + R_1}$$

is limited to values near one-half by the conditions for sensitive balance of the Wheatstone bridge. The only other variables which are under control of the worker are the temperatures of the wire and the walls. No definite rule can be provided for these, as two opposing factors enter. The radiation loss, which varies as the difference of the fourth powers of the temperatures, increases $V_0$, while the loss through the gas, which varies as the difference, increases $K$. The best values will vary with the length of the wire, the gas used, etc., but in general little is gained by using an excessive difference of temperature, since the sensitivity increases so slowly. Temperature differences of the order of 100° C. have been found quite satisfactory for most cases. The tube itself, since $T_1'$ enters into $K$, should be held at constant temperature, either by an ice bath or by running water or some similar scheme.

## EXPERIMENT 47

### CALIBRATION OF A RESISTANCE MANOMETER

The set up for the calibration and use of a resistance manometer is shown in Fig. 126. The manometer will be kept at constant temperature by means of a water bath containing melting ice. The exact voltage used will depend upon the construction of the manometer. If it is not specified, the resistance of the manometer may be measured at small currents and the tempera-

ture computed from the known temperature coefficient of the metal. Use a temperature of about 100° C.

The pressures measured on a hot wire manometer go below those to which an oil pump will ordinarily evacuate, so it will be necessary to use a high vacuum pump, such as a mercury or oil diffusion pump, in conjunction with the mechanical oil pump. A McLeod Gauge will be used as the primary pressure standard. Mercury vapor from the McLeod or the pumps will seriously affect the manometer, and must be kept from it by freezing out the mercury in a trap, the construction of which is shown. The Dewar flask may contain liquid air, a slush formed by mixing alcohol and carbon dioxide snow (which is quite commonly available) or even a freezing mixture of rock salt and cracked ice.

Pump out the apparatus to about 0.1 millimeter, then take readings of the voltage necessary to maintain the initial resistance and of the pressure in the system. Take ten or twelve readings between this pressure and the lowest which the McLeod will accurately read; Eq. (6) may be put in a little more convenient form for plotting if the constant term is transferred, giving

$$V^2 = Kp + K_1.$$

Plot the voltage squared against the pressure in millimeters. If deviations of more than about 5 per cent from the linear law are observed, plot, of course, the best smooth curve through the points. Extrapolate your curve to zero pressure. On the curve, specify the temperature of the cold bath.

**122. The Ionization Gauge.**

When electrons travel through a gas with sufficient velocity, they may disrupt the gas molecules which they strike, forming positive ions as described in Art. 118. If an electric field is present, these ions will be accelerated in the opposite direction from the electrons and transfer a positive charge to any electrode which they strike. If the mean free path of the electrons is of the same order of magnitude as or greater than the distance to the walls, the number of collisions and hence the positive current will increase as the number of molecules increases.

The device which makes use of this phenomenon to measure gas pressures is known as the ionization gauge. It is necessary to have three electrodes, one to supply the electrons, one to collect them and one to collect the positive ions. Certain limitations are imposed on the electron source. If tungsten filaments are used, water vapor and appreciable amounts of oxygen must be excluded from the gauge. If thoriated tungsten or oxide coated filaments are used, their surfaces will eventually disintegrate under the positive ion bombardment and they will have to be replaced. Although any three element vacuum tube may be used, they are not usually designed for this purpose and the geometrical configuration of the electrodes is rarely the most efficient for the production and collection of the positive ions. One of the best arrangements for an ionization gauge is shown, with its electrical connections, in Fig. 127.

The filament is a hairpin loop of tungsten wire, of 0.005″ to 0.010″ diameter. Surrounding this is a double spiral of tungsten wire of somewhat greater diameter, which acts as a grid. The plate is a cylinder of nickel or tantalum, coaxial with the grid, or, in some cases, a metal deposit on the inside of the glass tube. The filament supplies the electrons and the grid, which is maintained at 125 to 250 volts positive with respect to it, collects these electrons. The plate is operated at a potential of 10 to 50 volts negative to the filament and serves as the positive ion collector. If the radius of the grid is kept reasonably small with respect to that of the plate, few positive ions are formed between it and the filament, and so there is little bombardment of the latter. The relay in the grid circuit serves to maintain a steady average electron current, and should be so chosen and adjusted that its contact closes at grid currents slightly above that at which the gauge is to operate and opens at an equal amount below it. The relay armature should oscillate with a period considerably less than that of the galvanometer in the plate circuit. If the supply voltages are sufficiently steady, this relay may be omitted. It is sometimes replaced by a vacuum tube or three electrode gas tube control.

The readings on this gauge, like those on the resistance manometer, depend upon the nature of the gas used. Dushman has found that, except in the case of hydrogen and helium, the

positive ion current, at a given pressure and electron current, is roughly proportional to the number of electrons in the molecule.

## EXPERIMENT 48

### CALIBRATION OF AN IONIZATION GAUGE

The electrical connections will be those shown in Fig. 127, while the vacuum apparatus will be identical with that used for Experiment 47. The gauge will be calibrated for pressures of $10^{-3}$ millimeters of mercury and below.

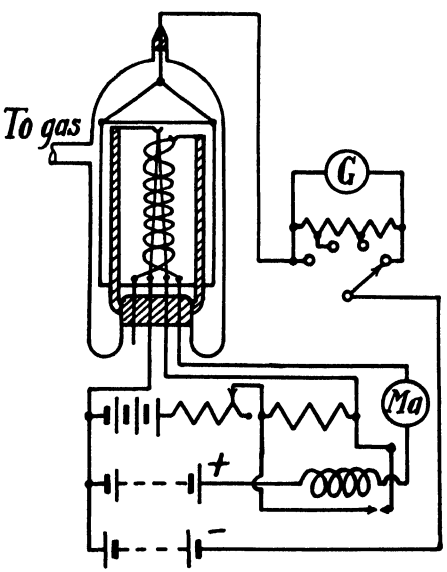


FIG. 127. The ionization gauge.

It is essential that the metal parts of the gauge be thoroughly outgassed. To accomplish this an electric furnace will be used. First pump the apparatus until the limit of the pumps has been reached, then, by means of the furnace, completely surrounding the gauge, heat to about 450–500° C. Keep pumping at this temperature until the limit of the pumps has again been reached then, with the furnace and pumps still on, begin raising the temperature of the filament with the plate and grid at a positive

potential of several hundred volts with respect to the filament. (During this process the galvanometer should either be removed from the plate circuit or shunted to zero sensitivity.) Do this slowly enough so that the evolved gas never reaches a pressure of more than $10^{-3}$ mm., and continue until the limit of the pumps is obtained with the filament running at the temperature which will be used in operation. Under these conditions the electron bombardment will also heat the plate to a higher temperature than that at which it will ordinarily run. If the gauge has a double spiral grid pass a current through this until it is brought to a red heat. This procedure must be repeated whenever air is let into the apparatus, before new measurements are made.

After the furnace has cooled and been removed, start the gauge operating with the relay controlling the electron current. With the electron current constant at the rated value (generally between 5 and 20 ma.) take about six readings of the ion current, at pressures varying between $10^{-3}$ mm. and the lowest at which the McLeod gauge can be read with 5 per cent accuracy. At each pressure read the McLeod before and after the ionization gauge reading and retain your readings only if the pressure has remained unchanged. Some time may be required for the system to reach equilibrium after the stopcock has been closed. Use the Ayrton shunt to cut down deflections at the higher pressures. Plot a curve of galvanometer deflection, reduced to the 1.0 shunt, against pressure, specifying on your curve the electrode potentials, the electron current and the sensitivity of the shunted galvanometer.

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# CHAPTER 11

## ELECTRICAL THERMOMETRY

**123. Thermo-electricity.**

When two dissimilar metals are placed in contact, a difference of potential usually exists between them. If they form part of a closed circuit no permanent current will flow if the temperature be uniform throughout, because the contact electromotive forces balance each other. If one of the junctions be heated or cooled, the E.M.F. at this junction is changed, destroying the balance and setting up a current, the magnitude and direction of which depend on the temperatures of the junctions and the materials comprising them. Two metals joined in this way constitute a *thermo-couple* and may be used for measuring temperatures. Fig. 128 shows such a circuit, which includes a millivoltmeter and the two metals $X$ and $Y$ which are joined at $A$ and $B$. If the instrument and its contacts are at a uniform temperature the resultant E.M.F. in the circuit will depend only on the temperatures of $A$ and $B$. If that of $B$ is kept constant, that of $A$ may be measured by the meter reading.

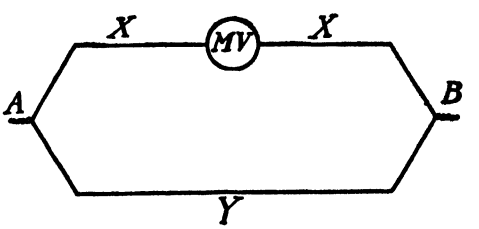

FIG. 128. Simple thermocouple circuit.

If the resistance of the external circuit were negligible compared with that of the millivoltmeter, it would read directly the resultant thermal E.M.F. in the circuit. In any case, if $R$ is the resistance of the external circuit, $r$ that of the meter and $V_1$ and $V_0$ the voltages at the junctions $A$ and $B$, then the measured E.M.F. is

$$V = \frac{V_1 - V_0}{R + r} r. \tag{1}$$

Consequently, the calibration of the couple is a function of the resistance of the meter and of the leads. From Eq. (1), we see

that $V$ approaches $V_1 - V_0$, however, as $r$ approaches infinity. This shows that the ideal instrument for the measurement is the potentiometer, which has an effectively infinite resistance at the balance condition. Since the voltages measured are very

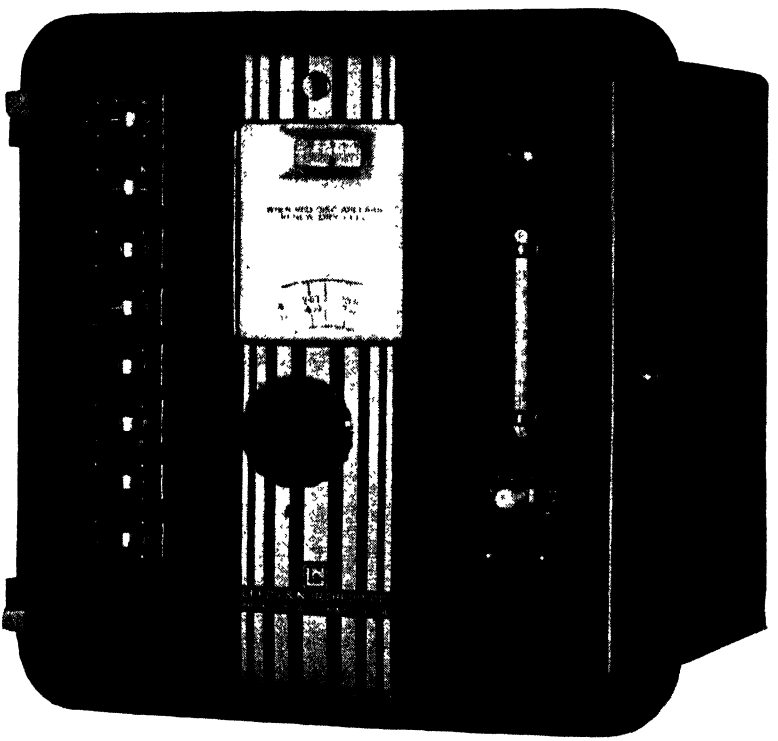

FIG. 129. Thermocouple potentiometer. (Courtesy of Leeds and Northrup Co.)

small, it is necessary that the instrument used be quite free from stray voltages of the same order, which may exist due to thermoelectric effects in the potentiometer itself. Several potentiometers specially designed for this purpose are on the market. One commercial instrument, manufactured by Leeds and Northrup Company, is shown in Fig. 129 and its simplified wiring diagram is given in Fig. 130. Inspection of the diagram will show

that $R_1$, $R_2$ and $R_3$ form the main part of a potentiometer circuit, shunted by $R_s$ and $R_c$. The latter is a coil of high temperature coefficient, designed to change the shunting resistance with changes in room temperature so as to compensate for these changes (Cf. Art. 127). This is necessary because the instrument is designed to be used without a cold junction, and the potentiometer itself acts as a reference junction. The potentiometer current, supplied by the dry cell $B$, contained within the instrument, is adjusted by the series resistor $R$. When the four pole, triple

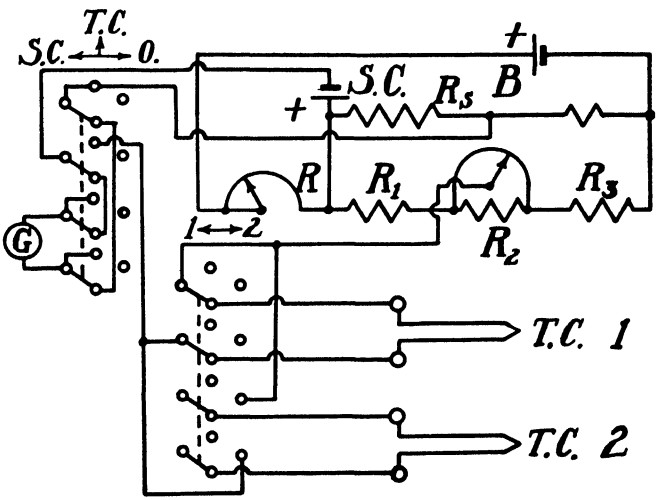


FIG. 130. Self contained thermocouple potentiometer.

throw switch, shown at the top left of both Fig. 129 and Fig. 130, is thrown to the *S.C.* position the galvanometer may be brought to zero by adjustment of $R$, standardizing the instrument. Below this switch are six other switches (only one of which is shown in Fig. 130) which enable any one of twelve thermocouples to be connected to the potentiometer when the top switch is in the *T.C.* position. This balancing is accomplished by adjustment of $R_2$. The knob controlling this resistor also turns a dial which is calibrated directly in degrees for the particular types of couples specified for the instrument.

Included on the indicator of Fig. 129 but not shown on the diagram are two optional switches, one for obtaining a double range and the other for making external connections to a portable potentiometer for checking purposes. The instrument may be supplied calibrated for iron-constantan, chromel-alumel, copper-constantan or platinum-platinum-rhodium couples.

Indicators of the type just described are intended for routine temperature measurements. When high precision is needed either a standard precision potentiometer, or, better, a low resistance instrument free from the effects of thermal E.M.F.'s, such as one of the Wolff "thermokraftfrie" or the Wenner designs should be used.

Another precaution to be observed in the use of thermocouples is the elimination of thermal E.M.F.'s along the leads. Since these leads are subject to variations in temperature along their lengths, in general, any inhomogeneity, either physical or chemical, may cause a potential difference which will seriously affect the readings. The means of eliminating this effect will be pointed out in the next article.

**124. Construction of Thermocouples.**

While almost any combination of metals will generate a thermal E.M.F., certain pairs have been commonly adopted, due to their high thermoelectric powers and their resistance to corrosion. Three of the most common pairs, together with the temperature range for which they find particular use, are:

1. Copper-Constantan (— 270° C. to 500° C.).
2. Chromel-Alumel (0° C. to 1000° C.).
3. Platinum-Platinum with 10 per cent Iridium (1000° C. to 1600° C.).

As a guide in the construction of couples, the approximate thermoelectric powers * of various metals and alloys against

* The thermoelectric power of a metal against platinum is defined as the temperature derivative of the potential difference produced by a thermocouple having one element of the metal and the other of platinum.

platinum, in microvolts per degree difference of temperature, at temperatures of 0° C. to 100° C. (mean values), are given below:

| | Microvolts/Degree |
|---|---|
| Bismuth-tin (Bi 90, Sn 10) | 35 |
| Chromel P (Ni 90, Cr 10) | 30 |
| Nichrome (Ni 58.5, Fe 22.5, Cr 16, Mn 3) | 25 |
| Iron | 18 |
| Platinum-iridium (Pt 90, Ir 10) | 13 |
| Silver | 9 |
| Gold | 6 |
| Platinum-rhodium (Pt 90, Rh 10) | 5 |
| Manganin (Cu 84, Mn 12, Ni 4) | 4 |
| Copper | 3 |
| Chromel (Fe 55, Ni 25, Cr 20) | 3 |
| Nickel | −18 |
| Constantan (Cu 60, Ni 40) | −36 |
| Alumel (Ni 94, Mn 2.5, Al 2, Si 1, Fe 0.5) | −38 |
| Bismuth-antimony (Bi 90, Sb 10) | −87 |

For any of these metals in combination, the thermoelectric power can be found by subtracting their tabulated powers.

Since the junction should possess a low and constant resistance, the two metals are usually welded together. This may be accomplished in any one of several ways:

1. By twisting the cleaned wires together and inserting the tip into an oxygen-gas flame until a bead of the two metals forms at the end.
2. By twisting and spot welding the ends electrically.
3. By striking an electric arc between the two wires, fusing them together, or by inserting the cleaned and twisted wires into a carbon arc.
4. By brazing or soldering the two wires.

When the couple has been formed it should first be tested for constancy of resistance by connecting the leads to the terminals of a sensitive galvanometer. The galvanometer acts as the cold junction as well as a meter. If the couple is heated slightly by dipping into a beaker of water slightly above room temperature, the galvanometer should deflect. Try moving the two wires and see whether the deflection changes. If the joint is properly made no change will occur.

If the joint is satisfactory, the leads should be tested for

inhomogeneities. To do this, leave the junction in the beaker and run a Bunsen flame over the leads. Inhomogeneities will be shown by sudden deflections as the flame passes the points where they occur. If any are observed, they may usually be removed by annealing the leads electrically. Pass a current sufficient to heat the leads to a dull red for about five minutes and test again. If two or three treatments of this type do not remove the trouble, it may be well to make a new couple.

Since thermocouples are generally used in the presence of either heated gases or of molten metals, both of which may be extremely corrosive, they must be protected against deterioration. For this purpose they are enclosed in tubes of glass, quartz, porcelain or other corrosion resisting materials. In addition to

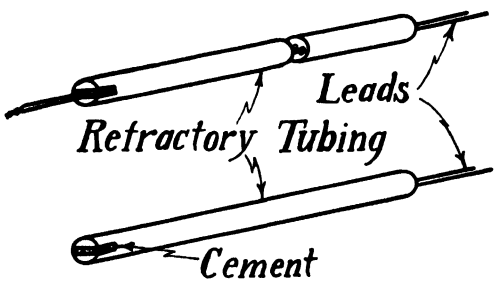


FIG. 131. Thermocouple construction.

ability to resist corrosion the couple jacket must have mechanical strength if it is to be used in metals, since the metal, in freezing about it, may otherwise shatter it. It must also possess a fairly high heat conductivity and a low heat capacity in order to follow changes of temperature rapidly. Two very good materials are Usalite, manufactured by the Stupakoff Laboratories, and Alundum, manufactured by the Norton Company.

A very satisfactory construction for a couple is shown in Fig. 131. Two holed thermocouple tubing, about ¼″ in diameter, is slotted with a hack saw and the couple wires are inserted as shown at the top. The junction is then drawn into the slot and covered with a refractory cement which is baked in place. If greater mechanical strength is needed the entire assembly may be placed in a heavier tube of refractory material or metal, but this increases the heat capacity.

**125. Calibration of Couples.**

The simplest means of calibrating a thermocouple is, of course, to compare it directly with another couple or other thermometer. This method, however, involves the necessity of holding a space of considerable size at a constant temperature while readings are being taken. In addition, it is a secondary calibration, involving all the errors of the standard instrument. Consequently, couples are often calibrated directly against fixed points on the temperature scale. The primary points, of course, are the freezing and boiling points of pure water at 760 mm. pressure. For points below the former, the freezing points of various liquids or the boiling points of liquified gases may be used. For points above 100° C., for which couples are most commonly used, the melting points of certain pure elements and salts are the standards. The following list gives a number of well determined and commonly used points:

MELTING POINTS

| | |
|---|---|
| Cu (Saturated with oxide) | 1063° C. |
| NaCl | 800 |
| Al | 657 |
| Zn | 419 |
| Pb | 327 |
| Sn | 232 |
| Naphthalene | 80 |
| $H_2O$ | 0 |
| Hg | − 39 |
| Normal Ethylaniline | − 64 |
| Diethyl Ether | −116 |
| Ethyl Chloride | −141 |

BOILING POINTS

| | |
|---|---|
| S | 445 |
| Napthalene | 218 |
| $H_2O$ | 100 |
| $O_2$ | −183 |
| $N_2$ | −196 |

These temperatures are, of course, for the pure materials. Addition of impurities will usually make the point more or less indefinite, so that accurate calibrations are very difficult. For the temperatures above room temperature, an electric furnace is probably the most convenient form of apparatus. To prevent oxidation of the metals they should be melted in a graphite crucible and covered with a layer of flake graphite or charcoal dust.

The general procedure is to melt the material, insert the thermocouple (protected by its jacket) and then allow the whole to cool. A sketch of a typical set-up is shown in Fig. 132.

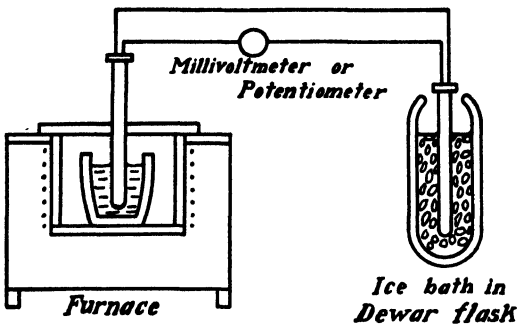


FIG. 132. Calibration of a thermocouple.

If the E.M.F. across the couple is plotted as a function of time as the metal cools, a curve similar to Fig. 133 *A* will, in general, be obtained. As the molten metal cools, the temperature drops along the curve *A–B*, until the freezing point is reached. Then,

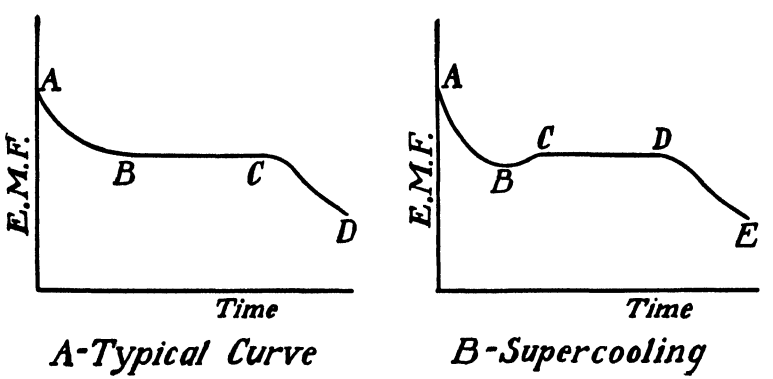


FIG. 133. Cooling curves.

during the transition from the liquid to the solid phase, the temperature, and hence the E.M.F., remains constant, giving the portion *B–C*, after which the temperature again falls. The voltage during the time *B–C* therefore corresponds to the melting point of the metal. In some cases the phenomenon of super-

cooling takes place, giving a curve of the type of Fig. 133 *B*. The temperature, in this case, drops below the freezing point before the liquid begins to solidify. This is an unstable state, however, and the E.M.F. will return to the proper value, giving the flat portion *C–D*.

It is essential, of course, that the horizontal part of the curve be long enough to indicate its exact position, and to separate it from accidental variations in the curve. This necessitates a fairly slow cooling. Extremely slow cooling, however, wastes time, and in any particular case the proper cooling conditions must be determined by experiment. If the loss of heat with the crucible in the furnace is too slow, it may be moved into the outside air; if too rapid, the current in the furnace may be reduced rather than shut off during the cooling process.

When boiling points are used for determination, the liquid may merely be allowed to boil in air, and will maintain a constant temperature. The barometric pressure should, of course, be read and corrected for. When expensive liquids are used, they may be refluxed by means of a long water cooled tube above the boiling liquid.

When three or four determinations of the type described have been made, it will be possible, in general, to fit an empirical calibration curve to the results. For most couples, it will be found that the electromotive force can be expressed quite well by the equation:

$$E = AT + BT^2 \qquad (1)$$

where $T$ is the difference of temperature of the two junctions (the cold junction being maintained at constant temperature) and $A$ and $B$ are constants. Since $E$ is known for certain definite temperatures, pairs of simultaneous equations may be set up and solved for $A$ and $B$. For future use, the resulting curve may be calculated and plotted.

## EXPERIMENT 49

### CALIBRATION OF A THERMOCOUPLE IN THE RANGE 0° C. TO 500° C.

Test the couple as outlined above and insert in the jacket. The wiring diagram will be either that of Fig. 128 or 130, depending on the measuring instrument used.

Keep the cold junction immersed two or three inches in the ice bath, and for the hot junction use any three of the melting points in the proper range. When these three points have been determined, use the highest and lowest of them to determine the constants of Eq. (1), Art. 125. Using these constants calculate the reading at the third temperature used, and check this against the observed value. If the check is within the desired accuracy (usually about one degree) the formula is satisfactory. If the result does not check, the trouble may be either experimental error or an actual variation of one of the freezing points, due to impurities. Consequently, it is usually better to choose a fourth metal for a new determination than to check over the old runs.

Calculate the calibration curve from Eq. (1) and plot temperature in degrees Centigrade (with cold junction at zero) as a function of meter reading.

**126. Low Temperature Calibration.**

At temperatures below the ice point the technique is slightly different than for high temperature couples. It is desirable, for example, that the test for homogeneity be made in the same range of temperatures as that in which the couple is to be used. A satisfactory way of making this test, therefore, is to use liquid air or some other low temperature bath in place of the Bunsen flame. A "U" tube of glass may be inserted in the bath and the wire pulled through this tube for the test. In the actual construction of the couples, pyrex glass or quartz tubes are quite satisfactory for protective jackets. Another difficulty which has been experienced with the copper-constantan couples when used at low temperatures is an aging effect that sometimes takes place when the couple is first used. When this takes place the thermoelectric power of the couple decreases with time, but approaches an asymptotic value after a little use.

For the calibration, any of the freezing or boiling points given above may be used. The liquids may be frozen by immersion in liquid air, or, for a limited range of temperatures, in solid carbon dioxide, then allowed to melt, with a heating curve taken exactly as was done with the metals at high temperatures. It is best to keep the couple in contact with the container. The cooling rates, which will depend on the amount of liquid, its freezing

point and the type of container used, may be controlled by the use of baths. For example, with ethyl chloride, it is sometimes desirable to place the test tube containing the liquid and the couple in a larger tube containing mercury. The whole is then frozen in liquid air and allowed to warm in the room. The large mass of mercury prevents too rapid melting of the chloride. It is well to close the mouth of the test tube during the time it is at low temperature, in order to prevent water vapor from the atmosphere from collecting on and mixing with the liquid being tested, and thus changing its freezing point.

**127. Reference Junction Correction.**

As was explained in Art. 123, both junctions of a thermocouple produce E.M.F.'s which are functions of their absolute temperatures. In precision laboratory work one *reference junction* is always held at a constant temperature, usually, for convenience, at 0° C. In commercial practice, however, this is not always feasible, and the reference junction is usually kept at room temperature, which may vary to some extent. If the voltage were a linear function of temperature this would cause no trouble, as the room temperature could be read with a mercury thermometer and the voltage of the couple would be proportional to the difference of the temperatures of the two junctions. This, unfortunately, is not the case, and a *reference junction correction* * must be applied.

If the full calibration curve of the couple is known, the correction may easily be computed. Suppose that the E.M.F. across the reference junction at the calibration temperature is $V_0$, and that the couple is used with reference junction at a temperature $t'$ so that the voltage across it is $V_{t'}$. Then if the voltage across the measuring junction is $V_t$, the actual voltage measured will be, in accordance with Art. 123,

$$V = V_t - V_{t'}, \tag{1}$$

which leads to

$$V_t - V_0 = V + (V_{t'} - V_0). \tag{2}$$

* Often referred to as the *cold junction correction*, but this term is ambiguous, since couples are used for temperatures both above and below those of the room.

But the calibration curve gives $V_t - V_0$ as a function of temperature. Therefore, if $t'$ is known, $V_{t'} - V_0$ can be read from the curve and thus $V_t - V_0$ can be obtained. But this is exactly the voltage which would be read if the reference junction were at the proper temperature, and the true temperature can be read from the calibration curve, using this voltage in place of $V$.

In commercial instruments this reference junction correction is sometimes approximated automatically by inserting in the circuit a resistance, located near the cold junction, of the proper temperature coefficient nearly to balance the change of E.M.F. at the junction. If the readings are taken with a millivoltmeter, Eq. (1), Art. 123, indicates how this might be accomplished, while with potentiometer measurements various schemes are used, depending on the particular circuit.

**128. The Resistance Thermometer.**

Another electrical property which may be used for the measurement of temperature is the change of resistance with change of temperature. In its simplest form the *resistance thermometer* consists of a Wheatstone bridge with one arm containing a resistance whose variation with temperature is known and which is inserted in the region whose temperature is desired, while the other three arms are composed of resistances of negligible temperature coefficient, and are at room temperature. The bridge, of course, must be of sufficient sensitivity so that the heating effect of the currents will be negligible. The choice of the metal depends on the conditions of use, copper or nickel being suitable for low temperatures while platinum is widely used for temperatures up to 1500°.

The only serious difficulty with the arrangement outlined is that not only the resistance element is at a temperature above that of the room, but also its leads are at temperatures which are variable along their length and which are not necessarily functions of the measured temperature alone. It becomes necessary, therefore, to compensate for these variations. In the commercial instrument manufactured by Leeds and Northrup Co. the compensation is accomplished in the manner shown in Fig. 134. Three leads of identical material and resistance are carried to the thermometer element. This removes the junction of the bridge

to a point at the higher temperature. If an equal arm bridge is used (as is desirable for maximum sensitivity) the effect of temperature on leads $A$ and $B$ is exactly self compensating, since it produces identical changes of resistance in two adjacent arms of the bridge. The contacts $S$ and $S_1$ are moved by a dial

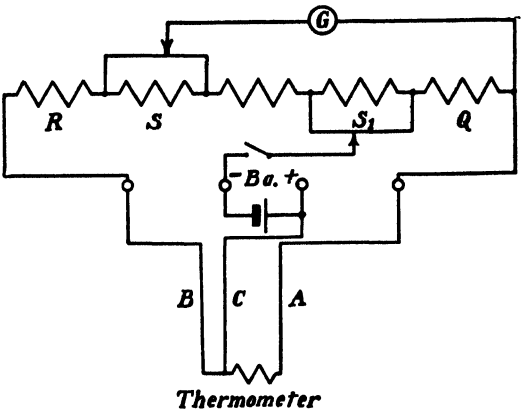


FIG. 134. Circuit of resistance thermometer.

on the face of the instrument, which is calibrated to read directly in degrees. The thermometer element itself is a small spool of wire mounted on a mica bobbin and enclosed in a protective jacket similar to those used for thermocouples. The apparatus is shown in Fig. 135.

## EXPERIMENT 50

### CALIBRATION OF A RESISTANCE THERMOMETER AGAINST A STANDARDIZED THERMOCOUPLE

This experiment will be indicative of the methods used for secondary standardization. Mount the thermometer and a thermocouple, whose calibration is known, close together in the electric furnace. Heat the furnace to the highest temperature which you desire to use. When both instruments have come to a steady reading decrease the current through the furnace, so as to cool it slowly. Slow cooling is necessary in order that the time lag of the instruments may not be appreciable. At regular in-

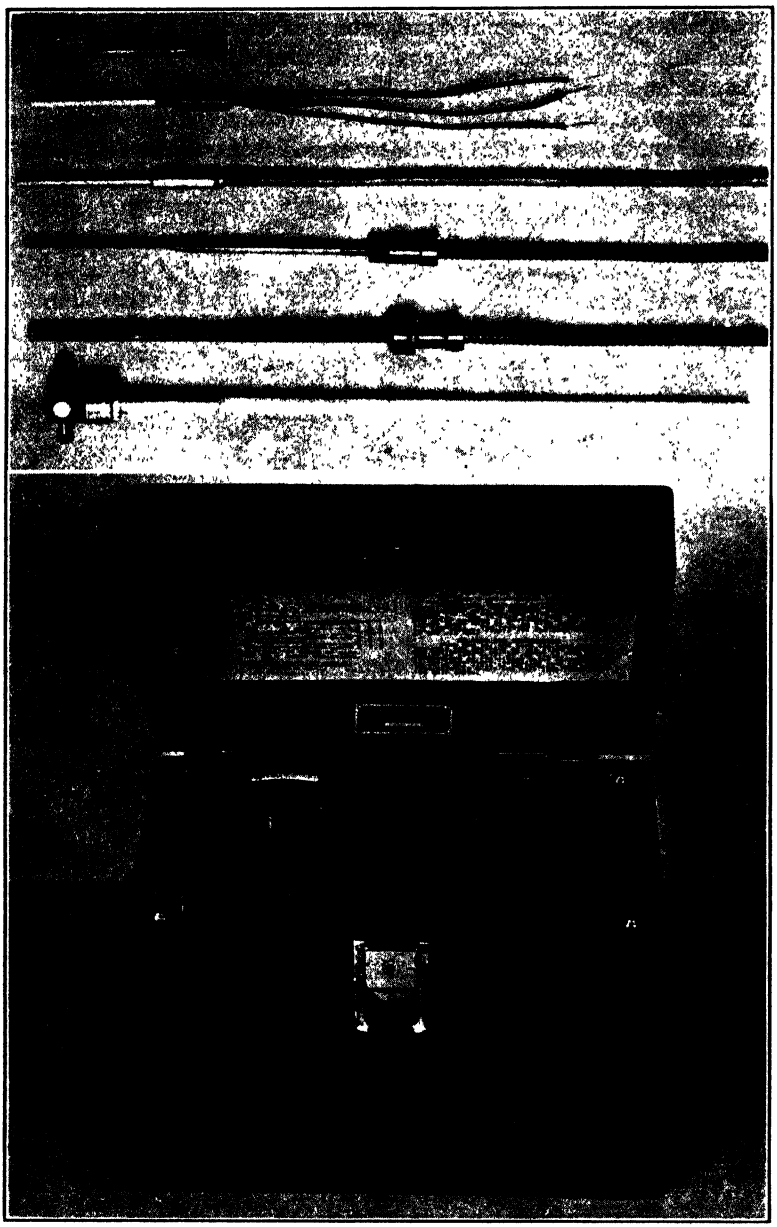

FIG. 135. Resistance thermometer. (Courtesy of Leeds and Northrup Co.)

tervals read the two indicators and plot their readings against time on a single sheet. Then, corresponding to any particular time, both the reading of the resistance thermometer and the temperature can be obtained, so that a new curve of readings of the thermometer as a function of temperature can be plotted.

**129. Thermistors.**

When small changes of temperature are to be measured the available thermoelectric powers of couples or the temperature coefficients of resistance of the metals may not be sufficient to give the required accuracy. In this case it may be advantageous to use a resistance thermometer in which the primary element is of metallic oxide instead of a pure metal. Such elements are known as *thermistors*, since their resistances depend so greatly on temperature. Many of the metallic oxides are *semi-conductors*, and these have the property that the resistance decreases exponentially with increase of temperature. A thermistor element may be used in a bridge circuit where it has the advantage of being of high resistance, so that contact and lead resistances become negligible. This high resistance introduces one difficulty, since appreciable currents in the element will cause it to heat to a temperature above its surroundings, consequently good voltage or current control on the bridge is essential. Thermistor elements have not yet been made with the stability of the metals, but with calibration in place they offer temperature measuring devices of high sensitivity and low thermal capacity. Commercial thermistors are manufactured by the Western Electric Company.

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# CHAPTER 12

## RADIATION MEASUREMENTS AND THEIR APPLICATIONS

### 130. Electromagnetic Waves.

So far we have dealt chiefly with the flow of electricity in conductors or with the storing of electrical energy in electric and magnetic fields. In the present chapter we shall consider the measurement of electrical energy as it is transferred through space in the form of *electromagnetic waves*.

The methods used to measure the *intensity* of such radiation, which is the energy transferred by it across unit area in unit time, vary widely according to its *wave length*. The technique for the long *radio waves* is somewhat specialized and, since the basic ideas of the work have been developed in Chapter VIII, the student who wishes more detailed information is referred to texts on radio communication. For measuring the intensity of the *heat* or *infra-red* radiation, having wave lengths intermediate between the radio waves and *visible light*, the *radiometer*, *bolometer* and *vacuum thermocouple* are available. Only the last two will be considered here. For visual radiation there are available, in addition to these instruments, the *visual photometer*, the *photo-electric* cell and *photographic* methods. The last two may be used for wave lengths extending into the *ultra-violet* region. As we pass to still shorter wave lengths—to the x-rays and the gamma rays—we find the *ionization chamber* and the *Geiger counter* being used.

### 131. Laws of Heat Radiation.

In 1801 Young proved light to be a wave motion and sixty years later Maxwell developed the electromagnetic theory. The next step was to determine the mechanism and laws of radiation from heated bodies. The first important success in this line was the establishment of the *Stefan-Boltzmann law*, which states that every body is radiating heat at a rate proportional to the fourth power of its absolute temperature. It is, of course, also absorbing

heat at a rate proportional to the fourth power of the absolute temperature of its surroundings. The degree on the absolute temperature scale may be taken the same as that on the Centigrade scale but the zero is at approximately −273° C. We will designate absolute temperature hereafter as *degrees Kelvin* (° K.).

For most purposes the total energy emission from a body is not as important as the emission in a certain range of wave lengths. This problem of the distribution of energy throughout the *spectrum* as a function of wave length was investigated by Rayleigh, Jeans, Planck, Wien and others. The final expression, obtained first empirically and then theoretically, by Planck, is not as convenient for computation as an earlier expression given by Wien. In the ordinary temperature range the difference in the two expressions is negligible. The Wien distribution law states that the energy emitted per unit time in an interval of wave length $d\lambda$ from a black body is:

$$E_\lambda d\lambda = C_1 \lambda^{-5} e^{-(c_2/\lambda T)} d\lambda, \qquad (1)$$

where $C_1$ and $c_2$ are constants, $\lambda$ is the wave length and $T$ the absolute temperature of the body. The *black body* referred to is defined as a body which reflects none and absorbs all of the radiation falling upon it.

**132. The Experimental Black Body.**

No materials which are "black," according to the above definition, exist in nature; but, as first pointed out by Lummer and Kurlbaum, it is possible to construct an almost perfect substitute in the laboratory. If a large cavity is pierced with a small hole for observation, any light entering this hole will in general suffer a large number of internal reflections before it can re-emerge from the hole. Moreover, since a loss is sustained at each reflection, almost 100 per cent of the light is absorbed within the cavity, which therefore acts as an almost perfect black body.

The black body furnace shown in Fig. 136 is built according to this principle and follows quite closely the design of Forsythe and Worthing.* The furnace is constructed about a cylindrical tube of refractory material, surrounded by a thick layer of heat insulat-

* Forsythe and Worthing, *Astrophysical Journal*, Vol. 61, p. 196 (1925).

ing material. The heating coils, wound with chromel or nichrome wire, capable of reaching temperatures of about 1100° C., are usually wound for convenience in three sections as shown, with somewhat closer spacing on the ends than in the middle, since the heat losses from the former regions are greatest. For laboratory

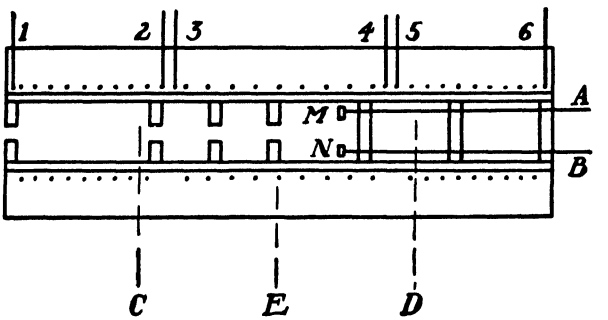


FIG. 136. The black body furnace.

purposes, it is well to have the partitions, which are molded of some refractory material such as alundum, removable. The partitions at the rear of the furnace are cast with two small holes to carry the wires *A–B*, while those at the front have concentric holes, about 1/4 to 3/8 inches in diameter, for observation.

## EXPERIMENT 51

### CALIBRATION OF A BLACK BODY FURNACE

In addition to the condition for blackness, it is necessary that the furnace be at a constant temperature over a great part of its length, in order that the $T$ of Eq. (1), Art. 131, may be defined. This necessitates a determination of the proper currents for the three windings.

Set up the apparatus as shown in Fig. 137. The partitions of Fig. 136 are replaced by partitions of asbestos board in which holes have been bored to insert the thermocouples. These are placed so that their elements will be located approximately at the positions denoted by *C*, *D* and *E* in Fig. 136. The heating coils are arranged for separate control and so that the current in each

one may be read separately, and a single indicating instrument and cold junction are made to serve for all three couples.

As a first approximation, take a current about twenty per cent greater in the end coils than in the center. The order of magnitude of the currents to be used for the particular furnace will be given by the instructor. With the current on, take readings of each couple at five or ten minute intervals. Plot three curves of temperature against time. As soon as these curves begin to bend, approximate extrapolations may be made, and the currents in

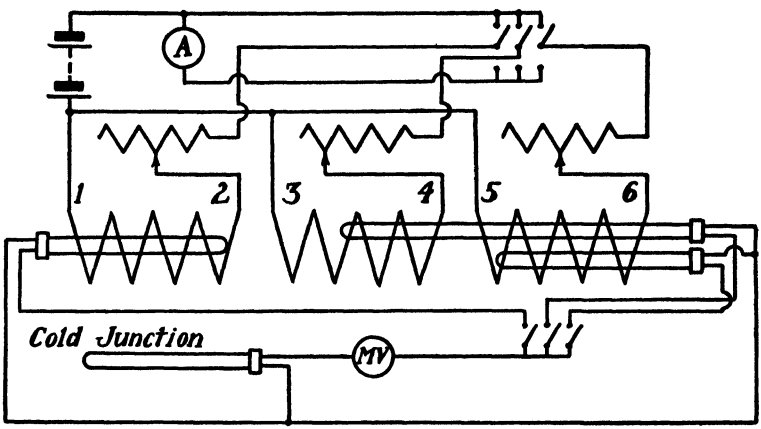


FIG. 137. Calibration of a black body furnace.

the three windings adjusted to bring the three portions of the furnace to the same equilibrium temperature. Continue this procedure until each couple gives a constant reading and all three couples give the same temperature within five degrees.

Record the equilibrium temperature and the three currents necessary to produce it, then increase or decrease the currents, keeping their ratios constant, and determine another point, in the same manner. Repeat until four points are obtained, distributed throughout the range 900 to 1100° C. Plot curves of the three currents against temperature, on a single sheet.

### 133. The Optical Pyrometer.

The simplest means of measuring intensities in the visible region of the spectrum is by visual comparison of the unknown

with a source of known intensity, as in the simple photometer. This principle is made use of in the *optical pyrometer*, which is undoubtedly the most elegant instrument for measurement of temperatures from about 1000° or 1100° upward. The most common form of this instrument, the disappearing filament type, is shown schematically in Fig. 138. It consists of a telescope, which may be focused on the unknown source, with a small calibrated lamp, mounted with its filament in the focal plane of the telescope. Between the lamp and the eyepiece is an absorb-

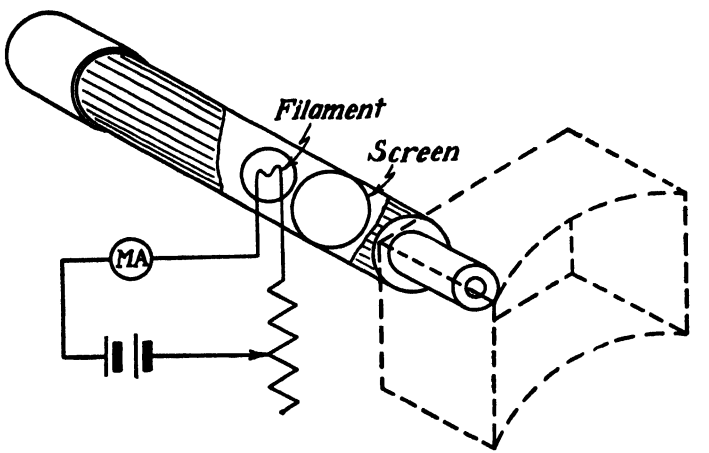


FIG. 138. The disappearing filament optical pyrometer.

ing screen, which transmits a very narrow band of wave lengths (usually in the red region, at 6650 Angstrom units, or 0.665 micron). From Art. 131, Eq. (1), it may be seen that for a given wave length, $\lambda$, and a given width of band, $\Delta\lambda$, the intensity is a function of the temperature only:

$$I = k \int_{\lambda}^{\lambda+\Delta\lambda} \lambda^{-5} e^{-(c_2/\lambda T)} d\lambda.$$

But the brightness of the lamp filament is a function of the current only, and consequently, if the image of the source and the filament are brought to equal brightness, by varying the current through the lamp, a direct relationship exists between the temperature of the source and the current as read by the filament

milliammeter. The comparison is brought about by changing the brightness of the filament, which appears against the background formed by the source image, until the filament just disappears. If the current is too low the filament will appear dark against a bright background; if too high, it will be bright against a dark background. After a little practice the exact point of disappearance can be judged with considerable nicety.

In practice the simple instrument just described would have several serious disadvantages. The optical system, without appropriate aperture limiting stops or adequate corrections, would not give a sufficiently defined image to allow complete matching, and the usual deflection milliammeter cannot be read to the accuracy with which a match can be made. Consequently a

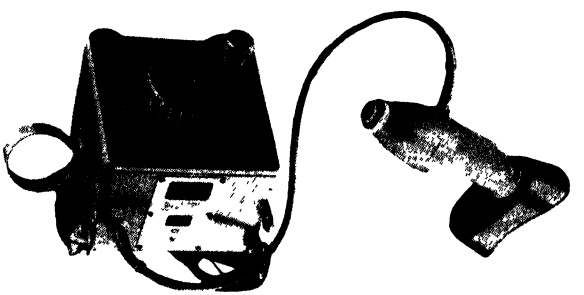

FIG. 139. The 8621 optical pyrometer. (Courtesy of Leeds and Northrup Co.)

satisfactory optical pyrometer is considerably more involved than that shown in Fig. 138, but is identical with it in principle. For precision work the best instrument thus far designed is probably that of Fairchild and Hoover, to whose original paper a reference will be found at the end of the chapter. Their instrument, however, is rather expensive and not conveniently portable, and most commercial instruments represent a compromise between it and the simplified design.

A convenient portable instrument, as manufactured by the Leeds and Northrup Co., is shown complete in Fig. 139. The optical system of the telescope is compound, and the bulb has flat windows inclined to the axis so as to separate any reflections from the image. By careful optical design diffraction difficulties are minimized. Screw adjustments are provided for focusing

the eyepiece on the filament and the objective lens on the surface being measured. In the No. 8621 instrument shown there are two ranges, of 1400–2250° F. and 1950–3200° F. (760–1230 and 1065–1560° C.) black body temperature, obtained by snapping an absorbing screen in and out behind the objective lens. In the similar No. 8622 pyrometer a third range of 2700–5200° F. (1480–2815° C.) is added. A potentiometer is used to measure

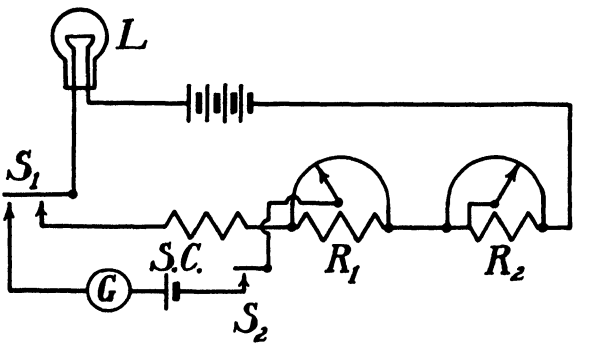


FIG. 140. Circuit of the 8621 pyrometer.

the filament current and the scale is calibrated directly in Fahrenheit, Centigrade, or Kelvin degrees. The wiring diagram is shown in Fig. 140. Four dry cells, the standard cell *S.C.* and the galvanometer *G* are included in the control box. The switch $S_1$ is closed by a hand grip on the telescope. The large knob on the side of the control box regulates the filament current by adjusting the resistance $R_2$ and at the same time, by means of a friction drive, changes $R_1$ so that the potential drop due to the lamp current will approximately balance the E.M.F. of the standard cell when the switch $S_2$ is closed. This switch is closed by depressing the small knob on the side of the box, after which the galvanometer may be brought to zero deflection by turning the knob, which controls $R_1$.

## EXPERIMENT 52

### CALIBRATION OF AN OPTICAL PYROMETER AT THE GOLD POINT

Set up the black body furnace with the refractory partitions in place and focus the pyrometer on the hole in the innermost

partition. This may be accomplished easily by removing the first partitions and shining a bright light on the final one. Between the iron clamps *M–N*, shown in Fig. 136, a strip of thin

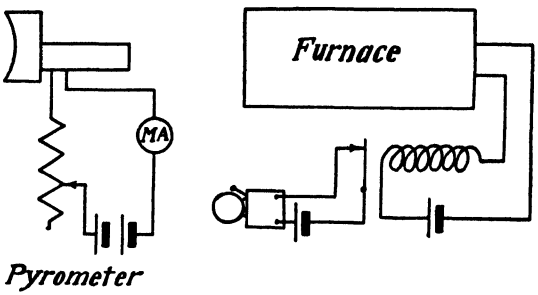


FIG. 141. Calibration of an optical pyrometer at the gold point.

gold sheet or a piece of gold wire is mounted and these clamps are connected to a relay and alarm bell system as shown in Fig. 141.

From the calibration curve of the furnace determine the currents necessary to reach a temperature slightly above the melting point of gold (1063° C.), say 1080° C. As the furnace heats, watch the opening through the pyrometer and as soon as the hole appears bright against its background, start taking pyrometer readings. Take readings at regular intervals of, say, two or three minutes, recording the time, in hours, minutes and seconds, for each reading. It is well, in taking these, to approach the balance point alternately from a too bright and a too dull filament, in order that personal errors may be eliminated. With most observers, the points obtained by one approach will be consistently low, while by the other they will be consistently high, so that a mean current-time curve will approximate quite closely the true curve. As the furnace reaches the gold point the strip will melt, opening the relay circuit and ringing the bell. Record the time at which the bell rings and take six or eight readings after this. Plot the reading-time curve for the pyrometer and obtain from this curve the pyrometer reading at the gold point. This is then the reading corresponding to a black body temperature of 1063° C. or 1336° K.

**134. Gray Bodies—Extension of Pyrometer Calibration.**

The calibration of the pyrometer could, of course, be extended by using metals other than gold and repeating the procedure of Experiment 52, but such a method meets with two serious objections:

1. The black body furnace is a bulky and inconvenient form of apparatus for laboratory work and its use at temperatures much above 1100° necessitates platinum or tungsten windings, and operation in a vacuum or hydrogen if the latter are used.
2. Materials with well determined melting points above that of gold are rare.

While the laws of emission from materials other than black bodies cannot be calculated theoretically, it is very fortunate that certain materials have been found empirically to follow the black body law except for the introduction of an essentially constant factor, the *emissivity*. Such materials are known as *gray bodies*. Chief among these, because of the high temperatures at which it can be used, is tungsten. The law of emission for such bodies is:

$$E_\lambda = C_1 \epsilon \lambda^{-5} e^{-(c_2/\lambda T)} \qquad (1)$$

where $\epsilon$ is the emissivity. The values of this quantity for tungsten have been determined at $0.665\mu$, the optical pyrometer wave length, by Forsythe and Worthing (loc. cit.) and are given as a function of temperature:

| Temperature (° C.) | Emissivity |
|---|---|
| 1000 | 0.456 |
| 1200 | 0.452 |
| 1400 | 0.448 |
| 1600 | 0.443 |
| 1800 | 0.439 |
| 2000 | 0.435 |
| 2200 | 0.431 |
| 2400 | 0.427 |
| 2600 | 0.423 |
| 2800 | 0.419 |
| 3000 | 0.415 |

The dependence of the emissivity on wave length is small, and a very good approximation may be obtained by increasing the

emissivity by 0.4 per cent for every thousand Angstroms decrease in wave length.

We may use these data on tungsten to find the temperature of a heated tungsten filament by the use of the optical pyrometer. For, if the black body temperature, as read by the pyrometer, is $S$, we have, from Wien's distribution law:

$$I = C_1\lambda^{-5}e^{-(c_2/\lambda S)}, \tag{2}$$

while the true temperature, $T$, is defined by the equation:

$$I = C_1\epsilon\lambda^{-5}e^{-c_2/\lambda T}. \tag{3}$$

Equating the right hand sides of these equations and passing to natural logarithms:

$$\log_e \epsilon - \frac{c_2}{\lambda T} + \frac{c_2}{\lambda S} = 0. \tag{3A}$$

If fractions are cleared, the equation may be solved for $T-S$ by substituting $(T - S) + S$ for $T$, giving:

$$T - S = -\frac{\dfrac{\lambda S^2 \log_e \epsilon}{c_2}}{1 + \dfrac{\lambda S \log_e \epsilon}{c_2}}, \tag{4}$$

where $c_2 = 14330$ micron degrees. This is the most convenient form for computing the correction to be applied when the source is not a black body.

From the above discussion, it is clear that the effect of an emissivity less than one is to decrease the apparent temperature, as observed by the optical pyrometer. Exactly the same effect can be produced by cutting off part of the light reaching the photometer, either by a rotating sectored disc or by light absorbing screens. The latter scheme is used in most pyrometers to increase their range to high temperatures, while the former is commonly used for calibration purposes. Suppose, for example, that a disc with $t$ per cent of its area open is rotated between the source and the pyrometer. Then, if the speed of rotation is sufficiently high, so that the time between successive transits of the open portion is less than the time of persistence of vision, the

apparent effect will be merely a reduction of the brightness of the image in the telescope, in the ratio $100 : t$. Consequently, the difference between the true temperature and the observed temperature may be computed by merely substituting $t/100$ for $\epsilon$ in Eq. (4). The use of this device for calibration purposes will be pointed out in the next experiment.

## EXPERIMENT 53

### CALIBRATION OF AN OPTICAL PYROMETER FROM 1063° C. TO 2000° C.

A tungsten lamp, preferably one with a flat ribbon filament for easy focussing, will be used as the source. Connect this lamp in series with an ammeter and a finely adjustable control rheostat. Focus the pyrometer on the filament, then insert the sectored disc between the lamp and telescope. Either an adjustable disc, with opening variable from 180° down to a few degrees, or a series of fixed sectors may be used. Compute the opening which must be used to give $T$–$S$ equal to about 100° when $S$ is 1336° K. This is most easily done by solving for $t(\epsilon)$ in Eq. (3A). Either set the variable disc at this opening or use the fixed disc having an opening nearest to it.

Now set the current in the pyrometer lamp to that given for the gold point by your previous calibration. Start the sectored disc rotating and achieve a balance by changing the current in the tungsten lamp, holding the pyrometer current constant. As before, approach balance first from too dull a field, then from too bright. Take about three or four pairs of readings of the lamp current in this way. If these are averaged, the current which is necessary to run the lamp at an apparent temperature of 1336° K., through the disc, will be obtained. Stop the disc and again obtain a balance, this time by varying the pyrometer current, holding the lamp current constant at the average value determined above. This will give the true temperature, $T$, of the lamp (uncorrected for emissivity of course). But $T$–1336, and therefore $T$, can be computed from Eq. (4), and the known sector opening, consequently the current corresponding to a temperature $T$, a new point on your calibration curve is determined. Notice

that no correction need be made for the emissivity of tungsten, since this would merely introduce an additive term into both temperatures and would cancel out. The point obtained is for *black body temperature*, since the initial standard point was determined for a black body.

Now compute the opening necessary for $T-S = 200°$ approximately. If this opening is available, repeat the procedure, again using the gold point as your standard. If not, use your newly determined point as a standard and determine a new point. Continue until 2000° C. or the upper limit of the pyrometer has been reached. Plot a curve of pyrometer reading against black body temperature.

Since you took several pairs of readings on each point, you can compute the deviation measure of your observations. Do this and also determine the accuracy with which the disc opening is known for each observation. Combine these properly to find the precision measure for each point, and take the greatest of these precision measures as the residual error of your calibration. Specify this on the curve.

## EXPERIMENT 54

### CALIBRATION OF AN OPTICAL PYROMETER ABSORBING SCREEN

As mentioned above, most pyrometers are equipped with an absorbing screen, used to extend their range. This screen is usually arranged to be flipped in and out of place by a small external lever. Again the effect is merely a reduction of intensity and of apparent temperature, so if $t$ is the transmission of the screen, this quantity takes the place of $\epsilon$ in the above discussion.

To calibrate the screen, choose two or three temperatures near the upper end of the pyrometer range without the screen in place. Focus the pyrometer on the lamp filament and adjust the current through the lamp to give, approximately, one of these temperatures. Take a series of readings to determine the temperature, then insert the screen and again take a series of readings. From the difference of apparent temperatures, compute the transmission, $t$, of the screen by Eq. (3A). Repeat at the other points.

Having determined $t$, compute a new calibration curve for the

pyrometer with the screen in place and plot. Again compute the residual error of your calibration from the previous residual and the deviation measure of the readings.

**135. The Vacuum Thermocouple.**

As explained above, the optical pyrometer is used to measure intensities at a particular wave length. From the temperature obtained in this way, and from the known properties of the emitter (blackness, emissivity, etc.), intensities at other wave lengths may be calculated. This procedure is applicable, however, only when the properties of the emitter are known. For a great many light sources, this is not true and some means of measuring intensities at any wave length is desirable. One of the most useful instruments for this purpose is the vacuum thermocouple.

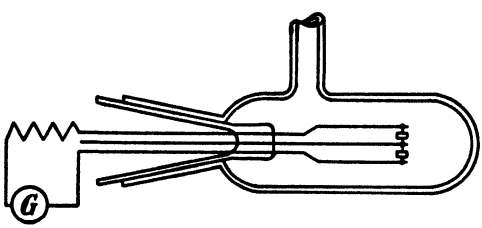


FIG. 142. Vacuum thermocouple construction.

While the construction of vacuum thermocouples varies largely, a somewhat typical arrangement is shown in Fig. 142. Joined to the heavy lead in wires are very fine (of the order of 0.001 mm. diameter) wires, *A–B–A*, the central one of which has a high thermoelectric power against the outer two. At the joints between the wires, *receivers*, of exceedingly thin foil, blackened on the front surface, are mounted. The whole apparatus is then mounted in glass and evacuated to sufficiently low pressure so that the mean free path of the gas molecules is long compared with the radius of the tube. As was explained in the section on resistance manometers, this reduces to very small proportions the loss of heat by conduction in the gas. In the construction shown, the element is inserted by means of a ground glass joint, sealed with wax or stopcock grease, and the couple must be

permanently connected to the vacuum pumps; but in other designs, the element is sealed in and the couple is sealed off from the pumps. The figure also shows three lead in wires, so that either junction may be used independently, although the two junctions are usually used together, so that one acts as a reference junction for the other. When used in this way, fluctuations in room temperature do not affect the readings.

Now suppose that a beam of light is focused on one of the receivers. If the intensity of the image is $I$, the area of the receiver $A$, and the reflectivity of the front surface is $r_1$, the amount of energy absorbed by the receiver per second will be:

$$Q = IA(1 - r_1), \tag{1}$$

and this energy will increase very slightly the temperature of the receiver. When equilibrium has been reached all this energy must be lost by conduction and radiation. The radiation loss is proportional to the difference of the fourth powers of the temperature of the receiver and its surroundings, so that it is given by:

$$Q_r = \sigma A(1 - r_1)\Delta(T^4) + \sigma A(1 - r_2)\Delta(T^4), \tag{2}$$

where $r_2$ is the reflectivity of the rear surface and $\sigma$ is the Steffan-Boltzman constant. The loss due to conduction through the leads is proportional to the first power of the difference of temperature, so

$$Q_c = k\Delta T. \tag{3}$$

Equating the energy loss and gain, we have:

$$IA(1 - r_1) = \{4T^3\sigma(2 - r_1 - r_2) + k\}\Delta T,$$

and if this is solved for the difference in temperature, $\Delta T$, we find:

$$\Delta T = \frac{A(1 - r_1)}{4T^3\sigma A(2 - r_1 - r_2) + k} I = K_1 I. \tag{4}$$

But, over a limited range, the voltage of the couple is proportional to the difference of temperature of the two junctions, so the voltage recorded by the galvanometer is

$$V = P\Delta T = K_1 PI = K_2 I. \tag{5}$$

The galvanometer deflection obtained with a vacuum thermocouple is therefore directly proportional to the intensity of the light falling upon it. Moreover, the sensitivity depends on wave length only in that $r_1$ may be a function of wave length. But if the front surfaces of the receivers are blackened, $r_1$ is a very small quantity and is almost independent of wave length, so that the couple is equally sensitive not only for all colors of the visible spectrum, but also for the ultra-violet or infra-red regions.

A remark may still be made about the electrical connections for the couple. For extreme sensitivity, it is desirable to have as large a portion of the total power as possible dissipated in the galvanometer, and this demands that the galvanometer and leads resistance be approximately equal to that of the couple. Since the critical damping resistance is usually greater than the coil resistance for such low resistance galvanometers, this may necessitate a resistance in series with the couple as shown in Fig. 142. This, it is true, somewhat reduces the sensitivity, but this is usually more than compensated for by the increase in convenience and speed obtained by using the galvanometer critically damped. If the damping resistance is lower than that of the couple a shunt resistance of high value may of course be used.

## EXPERIMENT 55

### DETERMINATION OF INTENSITIES OF THE EMISSION LINES OF A QUARTZ MERCURY ARC

As this experiment will involve the use of some optical apparatus, the student is referred to any good text on optics for an understanding of the principles and use of the instruments involved in the measurements.

The quartz monochromator shown in Fig. 143 is merely a convenient form of spectrometer for isolating the various mercury lines to be studied. Open the entrance slit of the monochromator to a width of about one or one and a half millimeters and open the exit slit to about three-fourths of this width. The instructor will furnish a list of the lines to be used together with the settings of the prism table and the lenses of the instrument. The mercury arc must be of quartz in order to transmit the ultra-violet lines and the quartz condensing lens should be mounted in such a way

that it can be moved along the optical axis without disturbing the alignment. This is necessary because the focal length of the lens is a function of wave length and it must be re-focussed for each line studied. The other quartz lens, between the exit slit and the thermocouple, must be similarly arranged. The thermocouple used will have a quartz window, through which light is focussed on the receiver.

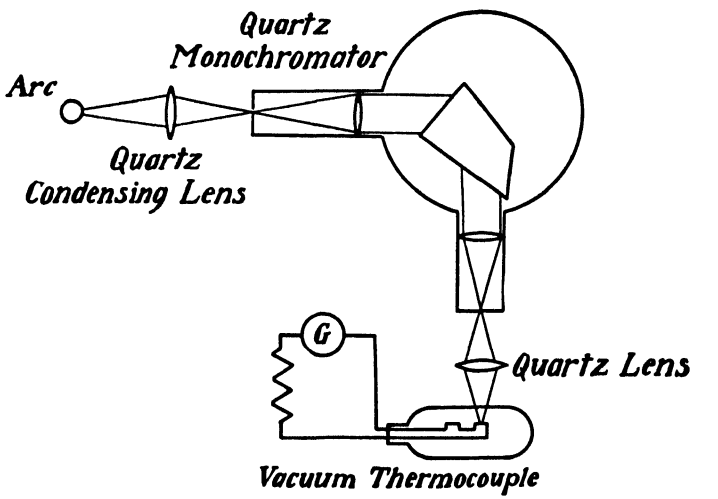


FIG. 143. Measurement of light intensities.

Set the various adjustments of the monochromator for the mercury green line at 5461 A. Slide the condensing lens in and out until a sharp image of the arc is obtained on the slit of the instrument, then focus the second slit on one element of the thermocouple in the same way. Then, for final adjustment, move the condensing lens very slowly until the galvanometer deflection reaches a maximum. When this has been obtained leave the condensing lens in this position and focus the final lens in the same way. Take about three pairs of readings by inserting a shutter in the beam, reading the zero, removing the shutter and reading the deflection, replacing the shutter, etc. If conditions are unsteady, a good result may be obtained by reading at fixed

intervals, say 30 seconds. Record the mean deflection as a measure of the intensity of the line.

Repeat this procedure for each of the lines assigned. Remember that the best focus for the ultra-violet lines is not necessarily that at which the images appear most distinct.

### 136. The Bolometer.

A resistance thermometer can be substituted for the thermocouple in radiation measurements just as in ordinary temperature measurements, and when used in this way is known as the *bolometer*. Two identical blackened resistance strips are mounted in the same way as the two thermal junctions mentioned in the last article, and are connected into a Wheatstone bridge so that at balance the same current flows through each. The additional resistance of the strip upon which the radiation falls is a measure of the intensity of the radiation. Similar considerations as to heat losses enter in both cases.

### 137. The Photo-Electric Cell.

During his researches on long wave length ether waves, Hertz observed that the difficulty of causing a spark to jump between two metal electrodes was somewhat reduced when the electrodes were illuminated with ultra-violet light. Subsequent researches show that this was due to the *photo-electric* effect, or the emission of free electrons from a metal surface by the action of light. It was also shown that, in general, these electrons were removed more easily by short waves than by long, and that every metal possessed a definite *long wave length limit*. Light of wave length greater than this limit is entirely non-effective in producing the effect, while at wave lengths shorter the efficiency of the effect increases until a certain maximum is reached, then decreases slowly.

The results of these researches have been applied to the construction of the *photo-electric cell*. As shown in Fig. 144, such a cell consists merely of a bulb of quartz or glass, over the inside surface of which some metal with a fairly high long wave length limit has been deposited, with the exception of a window, *A*. Light coming through this window falls on the metal film, *B*, and

frees electrons from the surface. A sufficiently high potential is applied between the coating and a *collector*, *C*, to prevent the building up of a large space charge and to provide a means of measurement. When no light shorter than the long wave length limit falls on the cell, the circuit is open and no current flows; but as soon as electrons are freed from the surface, they are accelerated toward the collector and cause a current. Since these currents are, in general, extremely feeble, the scheme shown of using an electrometer or an electrometer tube connected across a high resistance, *R*, is usually used for their measurement. For a limited range of intensities, the current is proportional to the intensity; but the constant of proportionality is a function of the wave length. As was the case with the vacuum tubes considered in Chapter VI, the voltage across the tube affects the current only if it is lower than the *saturation voltage*.

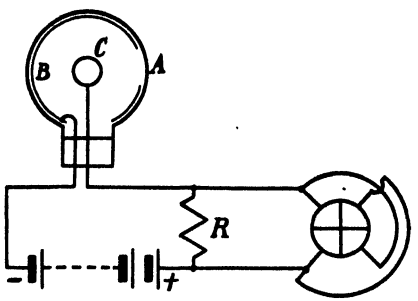

FIG. 144. The photo-electric cell.

## EXPERIMENT 56

### DETERMINATION OF THE LONG WAVE LENGTH LIMIT AND CALIBRATION OF A PHOTO-ELECTRIC CELL

The optical set-up for this experiment will be identical with that of Experiment 55, while the electrical connections will be those of Fig. 144. For *R* use a resistance of from $10^8$ to $10^{13}$ ohms, depending on the cell and electrometer used. This cell should have a window either of quartz or of some glass capable of transmitting the near ultra-violet.

The effect for the lines of the mercury arc studied in Experiment 55 will be determined. Set the monochromator for each of the lines, and focus the condensing lens, as was done in that experiment, for a maximum effect. Record the current produced by each line.

If the current produced by the line is divided by the known relative intensity of the line, the sensitivity of the cell for that wave length will be obtained. Plot the sensitivity as a function of wave length and extrapolate the curve to zero sensitivity, to obtain the long wave length limit of the cell.

**138. The Geiger-Müller Counter.**

One of the most useful devices for the measurement not only of radiation but also of fast moving particles, such as $\alpha$-rays, $\beta$-rays, neutrons, protons, etc., is the Geiger-Müller counter.

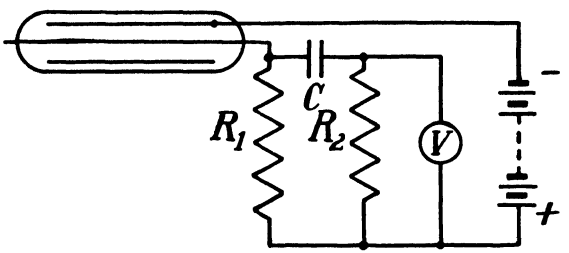


FIG. 145. The Geiger-Müller counter.

The general scheme of such a counter is shown in Fig. 145. The counter itself consists of a fine wire surrounded by a much larger coaxial cylinder of metal, which is maintained at a large negative potential with respect to the wire. If ionization occurs anywhere within the counter the ion is accelerated toward the outer cylinder and the electron toward the wire. The apparatus contains gas or vapor at a pressure of a few centimeters of mercury and the field is sufficiently high so that both the heavy ion and the electron, but particularly the latter, may acquire sufficient energy to ionize another molecule on their first collisions. The resulting particles again are accelerated, and thus a cumulative process begins, leading to a rush of positive charge to the cylinder and of negative charge to the wire. If this process were allowed to continue a continuous discharge would be maintained, but this is prevented by the external circuit. The resistance $R_1$ is made sufficiently high so that the potential drop across it during the sudden rush of charge becomes comparable with the supply

voltage, and most of the current which flows goes to building up a charge on $C$. The resulting drop of potential across the counter stops the cumulative discharge and extinguishes the counter. As the charge leaks off of the condenser it causes a drop of potential across $R_2$, actuating $V$, which is some detecting device such as a vacuum tube voltmeter or a string electrometer. During this process the potential across the counter again increases, restoring it to its original condition, ready for another count. The time taken for this process, and consequently the maximum counting rate, depends on the nature of the materials used in the counter and, of course, on the time constant of the external circuit.

The actual process of discharge in a counter is far more complicated than we have outlined. Heavy negative ions are probably formed by attachment of electrons to gas molecules, and

FIG. 146. The Locher Geiger-Müller counter. (Courtesy of Herbach and Rademan Radio Co.)

photo-electric ionization, due to the light emitted in the recombination of electrons and ions, undoubtedly contributes to the ionization. If *ionizing particles*, such as $\alpha$- or $\beta$-rays or protons are being counted the counter must be provided with a window sufficiently thin to admit the particles and the initial ionization takes place by direct collision in the gas. For $\gamma$-rays or x-rays, which ionize a gas molecule in their path only occasionally, the initial ionization takes place due to secondary electrons ejected from the cylinder by the radiation. For neutrons, part of the counter wall must be made of a material from which ionizing protons or other particles will be ejected by the neutrons.

A counter suitable for the measurement of $\gamma$-rays, as manufactured by Herbach and Rademan, is shown in Fig. 146.

It is usually convenient to have the G-M counter operate some sort of a mechanical counting device rather than for the observer to count the pulses visually on $V$. Since the counter pulses are not sufficient to do this directly, some sort of vacuum

tube coupling is necessary. A satisfactory circuit is shown in Fig. 147. The constants of the circuit are as recommended by Neher and are suitable for the counter of Fig. 146. A pulse in the counter swings the grid of the 57 tube negative, decreases the current through the tube and applies a positive pulse to the grid of the 2A5 tube. The surge of current in the plate circuit of this tube operates the mechanical counter. Many other types of counting circuits, including *scale of two devices*, to divide the count by powers of two when it is too fast for the mechanical counter to record, will be found in the literature.

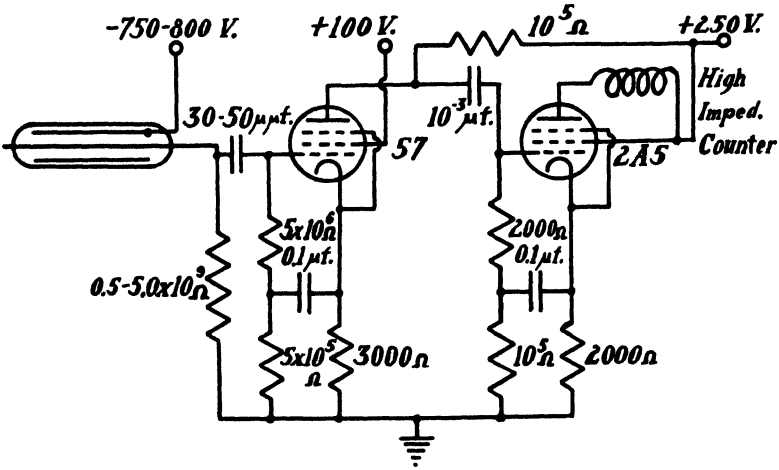


FIG. 147. Recording circuit for a G-M counter.

If a G-M counter has a constant number of ionizing particles entering it while the voltage is changed, the number of counts recorded per unit time will vary somewhat in the manner shown in Fig. 148. No counts will be recorded until the *threshold voltage*, $V_0$, is passed, after which the counting rate will increase rapidly at first, then will remain very nearly constant over the "plateau" from $V_1$ to $V_2$. Above this the counting rate will again increase rapidly with increasing voltage. The proper voltage to use with the counter lies between $V_1$ and $V_2$, as in this range any variations in supply potential will not affect the counting rate appreciably.

## EXPERIMENT 57

## DETERMINATION OF THE CHARACTERISTIC CURVE OF A GEIGER-MÜLLER COUNTER

The electrical circuit for this experiment will be that of Fig. 147. The arrangement of the source for the counter will vary with the type of counter used. If a $\gamma$-ray counter is being tested any $\gamma$-radioactive material may be mounted alongside the counter, and if the counter is for $\alpha$- or $\beta$-rays the appropriate material will be mounted in line with the thin window, which is usually in the end of the tube. In any case the radioactive

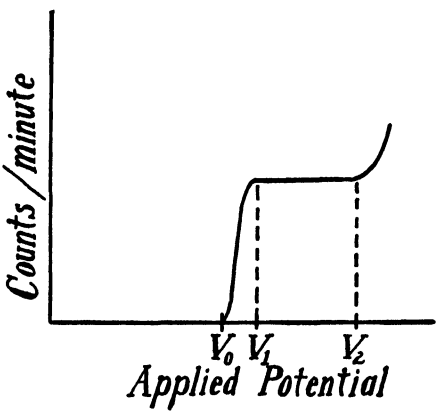


FIG. 148. Characteristic curve of a Geiger-Muller counter.

material should be surrounded by lead of several centimeters thickness in all directions except toward the counter, to protect the observer from the radiations.

For a voltage source use either batteries or a filtered full wave rectifier, with a high resistance voltmeter to measure the potential. With the radioactive material in place increase the potential by steps of approximately 100 volts until two successive voltages give approximately the same counting rate. Leave the counter operating for several minutes at this rate, observing the time and the reading of the mechanical counter at the beginning and end. Calculate the average rate and compare it with the manufac-

turer's statement of the maximum counting rate of the mechanical counter. If it is more than half of the latter the radioactive source is too strong or too close, and should be changed until the rate decreases to below this value. With this adjustment made, decrease the voltage to just below the threshold. Increase the voltage by steps of about 25 volts, determining the time rate of counting at each voltage. Since the counts occur at random, it will be necessary to count for a long enough period at each voltage to average out statistical fluctuations, and this will necessitate a long enough counting time to give at least 100 counts. Having determined the approximate period necessary for this, open the counter circuit. Record the counter reading, take the time when the circuit is again closed and when it is opened at the end of the period. Repeat at each voltage until two or three successive readings show appreciable increases in counting rate.

Plot a curve similar to Fig. 148. Determine from it the maximum and minimum values of voltage at which the counter should be operated.

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# CHAPTER 13

## ELECTROCHEMICAL MEASUREMENTS

### 139. Electrolytic Dissociation.

Thus far we have considered metallic conduction, in which electrons carry the current, and gaseous conduction, in which both electrons and ions contribute. In another class, electrolytic conduction, the charge is transferred entirely by positive and negative ions.

Despite subsequent developments in the theory of electrolytic conductivity, the original electrolytic dissociation theory of Arrhenius still holds qualitatively and the laboratory worker in electrochemistry will also find it quantitatively adequate in most cases. It has long been known that aqueous solutions of certain substances, particularly the acids, bases and salts, are quite good conductors of electricity, while solutions of other substances, such as most of the organic compounds, are no better conductors than the water used as a solvent. It has also been known that solutions differ in other ways from the pure solvent, in that their freezing points are lower, their boiling points higher, and, if they are separated from the solvent by a semipermeable membrane, an osmotic pressure is developed and causes the solvent to diffuse through the membrane. In 1885, Van't Hoff showed that the magnitude of this osmotic pressure could be derived, for dilute solutions, by assuming that the solute, or dissolved substance, acted as a perfect gas contained in the volume of the solution, and followed the general gas law:

$$PV = nRT, \tag{1}$$

where $P$ is the osmotic pressure, $V$ is the volume of the solution, $n$ is the number of mols of the solute, $T$ is the temperature in degrees Kelvin and $R$ is the general gas constant, equal to 0.0821 liter atmospheres per degree, 1.985 calories per degree, or 8.315 joules per degree.

Thermodynamic reasoning leads to the fact that the elevation of the boiling point or the lowering of the freezing point of a

solution is directly proportional to the osmotic pressure, and, consequently, can be easily calculated. But it was soon shown that the solutions which conducted an electric current readily gave abnormal values of the freezing and boiling point changes, so that Eq. (1) had to be modified to:

$$PV = inRT, \tag{2}$$

where $i$ is always greater than one and varies, for a given solution, with the concentration. In 1887 Arrhenius proposed a theory of this abnormal behavior, assuming that the so-called electrolytes dissociate, in solution, into positive and negative ions, each ion then acting, in regard to osmotic pressure, exactly as a new molecule. If the molecules were to dissociate completely, this would lead to integral values of $i$, but if it be assumed that the ratio of dissociated molecules to those undissociated is dependent on the concentration, temperature, etc., the variations of $i$ may also be explained. For example, if at a given temperature and concentration $100x$ per cent of the molecules are dissociated, and each molecule forms $N$ ions (e.g. $N = 3$ for $H_2SO_4$; $H^+$, $H^+$ and $SO_4^{--}$), the total number of equivalent molecules present is

$$(1 - x)n + Nxn,$$

and since the ratio of this number to $n$, the number originally present, is $i$, we have:

$$1 - x + Nx = i \tag{3}$$

or

$$x = \frac{i - 1}{N - 1}. \tag{4}$$

It has been found that, in general, $i$ increases at low concentrations and at high temperatures, so we may infer from (4) that the percentage dissociation also increases under these conditions.

**140. Electrolytic Conduction.**

From Arrhenius' theory the nature of electrolytic conduction is immediately clear. For it accounts for the presence in the solution of positively and negatively charged ions which, as soon as an electric field is established, move toward regions of lower

and higher potential, respectively. Suppose that we have various ions in solution, and that the number of any type per unit volume is $N$, the charge per ion is $ne$ and the velocity of migration is $v$; then the current (by Eq. (1), Art. 60) is:

$$I = \sum Nnev,$$

where the summation is taken over all types of ion in the solution. $N$, of course, is dependent on the concentration and temperature, $n$ on the nature of the solute and $v$ on the field strength in the solution. It might be thought that the current could easily be calculated from this equation, since $v$ might be found from the field strength and the known law of resistance in the liquid. It has been found, however, that the actual velocities are usually much lower than this calculated value, and the discrepancy has been explained by the idea that the ions associate with themselves molecules of the solvent and thereby increase their mass and decrease their velocities.

**141. Measurement of Resistance—Polarization.**

Since the resistance of an electrolytic solution is a function of the concentration, the measurement of specific resistance (or its reciprocal, *specific conductance*) immediately suggests itself as a means of measuring concentration. At first glance it might seem that the simple Wheatstone bridge would be suitable for this measurement, but certain difficulties present themselves when this is tried.

To understand the nature of the first of these difficulties, it is necessary to realize the difference in behavior between ions and atoms in a solution. Consider a weak solution of sodium chloride in water. A certain percentage of the NaCl molecules will have dissociated into $Na^+$ and $Cl^-$. It is well known that neutral sodium reacts violently with water to form the hydroxide and hydrogen, but the ion retains its identity. Now suppose that electrodes of some resistant metal, such as platinum, are inserted in the solution, and a difference of potential is maintained between them by an external source of electricity. The $Na^+$ ions migrate toward the *cathode*, or negative terminal, while the $Cl^-$ ions migrate to the *anode*, or positive terminal. The former, on reaching the electrode, give up their charge and are then normal

sodium atoms, capable of reacting with the water about them. This reaction proceeds:

$$2Na + 2H_2O \rightarrow 2NaOH + H_2.$$

The hydroxide, being soluble, diffuses into the solution and dissociates into $Na^+$ and $OH^-$, leaving the mean concentration of sodium ions unchanged throughout the solution, but causing a high concentration of both these and $OH^-$ ions near the cathode. At the anode, moreover, the $Cl^-$ ions are similarly neutralized and react with the water:

$$2Cl_2 + 2H_2O \rightarrow 4HCl + O_2,$$

and the acid also dissociates, leaving the mean concentration of $Cl^-$ unchanged. Instead of dealing with two types of ion, therefore, we must consider four, and the relative concentrations vary near the electrodes. This causes a change of resistance with time and with current density. Any such change of real or apparent resistance,* caused by changes of concentration due to the flow of current or by the fact that reactions proceed at a finite rate, is known as *polarization.*

The effect of polarization may be overcome by the use of a high frequency alternating current in place of direct current. With alternating current in the simple case considered above, at either electrode NaOH will be formed during one-half cycle and HCl during the next, so that the relative concentration remains unchanged.

But the introduction of alternating current brings with it a second difficulty. The two electrodes in any type of conductance cell act as a condenser and allow the flow of a capacitance current

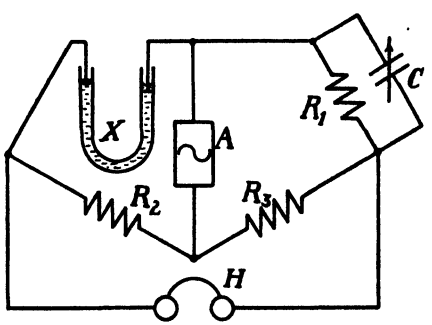


FIG. 149. Measurement of electrolytic conductance.

* The term *polarization* is used to include both actual changes of the resistance of the electrolyte and apparent changes produced by extraneous electromotive forces in the conductivity cell.

which cannot be balanced by the ordinary Wheatstone bridge. The actual arrangement used for measurement of electrolytic resistance is that shown in Fig. 149. Except for the arrangement of the capacitance and resistance in parallel instead of series, it is identical with the Wien capacitance bridge of Art. 102, and the same equations hold:

$$R_x = \frac{R_1 R_2}{R_3}, \tag{1}$$

$$C_x = C\frac{R_3}{R_2} \tag{2}$$

where $R_x$ and $C_x$ are the resistance and capacitance, respectively, of the conductivity cell, $X$.

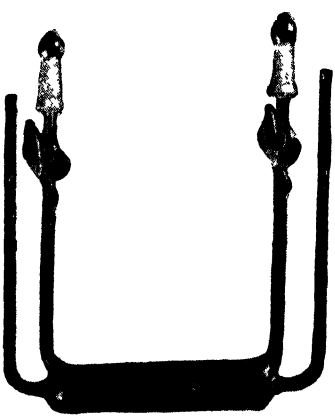

FIG. 150. The Jones Conductivity cell. (Courtesy of Leeds and Northrup Co.)

The cells used for the resistance measurement are generally similar to that shown in Fig. 150. When the resistance has been measured, the specific conductivity might be calculated from the dimensions of the cell, but such a course would require a very simple shape of electrical path and would be subject to errors due to non-uniform diameters, non-parallel electrodes, etc. Consequently, the usual procedure is to calibrate the cell with an electrolyte of known conductivity, and to make future measurements

relative to this. Since very careful work has been done on potassium chloride solutions, they are usually used as the standards. The following table * gives the specific conductance of various concentrations of potassium chloride in water, in mhos (reciprocal ohms) per centimeter.

| Normality | Specific Conductance ($\kappa$) | |
|---|---|---|
| | At 18° C. | At 25° C. |
| 1.0 | 0.09822 | 0.11180 |
| 0.1 | 0.01119 | 0.01288 |
| 0.02 | 0.002397 | 0.002765 |
| 0.01 | 0.001225 | 0.001413 |

It is to be remembered that a normal solution contains one equivalent weight of the solute per liter *of solution*. An *equivalent weight* is the gram-molecular weight divided by the valence of the most highly valent ion formed.

## EXPERIMENT 58

### CALIBRATION OF A CONDUCTIVITY CELL

Weigh out the proper weight of chemically pure potassium chloride for a normal solution and dissolve in distilled water in a volumetric flask, having the flask about half full. If the flask is not calibrated for room temperature heat or cool it to the proper temperature in a water bath and then fill exactly to the mark with distilled water. Carefully wash the flask and repeat this procedure for the 0.1, 0.02 and 0.01 normal solutions.

Set up the apparatus as shown in Fig. 149, using a thousand cycle source at *A*. Carefully wash the conductivity cell with distilled water and dry thoroughly before inserting the normal electrolyte. By successive adjustments of the resistances and the variable capacitance, *C*, eliminate the response in the tuned detector, and calculate the conductance of the solution. From this determine the *cell constant*, which is the ratio of the specific conductance of the solution to the conductance of the cell. If the temperature is between those used in the above table interpolate linearly for the specific conductance. Estimate the error

* Kohlrausch, Holburn and Diesselhorst, *Ann. der Phys.*, Vol. 64, p. 417 (1898).

of the weighing and of the resistance measurement, and calculate from these the deviation measure of the constant.

Again wash and dry the cell and repeat with the other solutions. Combine the four observations as suggested in Art. 9 and calculate the precision measure of the final value of the cell constant.

**142. Electrochemical Definition of Current—Faraday's Law.**

It will be remembered that, in Art. 21, the International ampere was defined as the current which would deposit a definite mass of silver per second from a solution of silver nitrate in water, in accordance with certain specifications. If a current be passed through such a solution, the $Ag^+$ ions migrate to the cathode and there give up their charge to be deposited as metallic silver. If the anode is of pure silver the $NO_3^-$ ions, on reaching it, attack the metal and thus keep the concentration of the solution, as a whole, constant. The basis for the electrochemical definition of the standard of current lies in two experimental laws established by Faraday in 1834, stating that:

1. *When different quantities of electricity flow across the junction between a purely metallic and a given purely electrolytic conductor, the amount of chemical change produced is exactly proportional to the quantity of electricity which flows, regardless of all other factors.*

2. *When the same quantity of electricity flows across such junctions with different electrolytes the amount of chemical change produced, measured in equivalent weights, is the same.*

These laws may be easily understood if we remember that each ion carries a charge $ne$ where $n$ is the valence of the ion. If a total of $M$ grams be deposited, and the ionic weight is $m$, we have, since one gram ionic weight contains $N$ ions (Avogadro's number), for the total number of ions involved:

$$\frac{M}{m}N,$$

and for the total quantity of electricity passed:

$$\frac{M}{m}Nne.$$

But, since one equivalent weight contains $m/n$ grams, this gives the quantity of electricity

$$\int I dt = M_e N e,$$

or

$$M_e = \frac{1}{Ne} \int I dt,$$

where $M_e$ is measured in equivalents. By measuring $M_e$ in this way we remove from the constant of proportionality all reference to the particular ion considered.

**143. The Coulometer or Voltameter.**

The term *coulometer* or *voltameter* is applied to any device which may be used to measure, by electrochemical means, the quantity of electricity passing through a circuit. Chief among coulometers, of course, is the silver coulometer, on the basis of which the International coulomb and ampere are defined and which, with the proper precautions, can be made to yield an accuracy of about 0.001 per cent. Full directions for its construction and use have been given by Rosa and Vinal.* The manipulation, however, is difficult, and the cost is high, so, for most work not requiring extreme accuracy, the copper coulometer is better adapted. The best form of this coulometer is probably that using two electrolytically refined copper plates for anodes with a thin copper cathode between them. The composition of the solution by weight is: water, 100 parts; crystallized copper sulphate, 15 parts; concentrated sulphuric acid, 5 parts; and alcohol, 5 parts. The water, of course, must be distilled.

## EXPERIMENT 59

### CALIBRATION OF AN AMMETER WITH THE COPPER COULOMETER

Set up the apparatus as shown in Fig. 151. Two coulometers are used in series in order to increase the accuracy of the result and to indicate roughly the precision obtainable. Use a current

* Rosa and Vinal, *Bull. of Bur. of Stds.*, Vol. 13, p. 479 (1916).

density of about 10 milliamperes per square centimeter of cathode area. Wash the cathodes thoroughly with distilled water and dry them at a temperature of 120° C. to 150° C. Then weigh to the nearest milligram and insert them in the solution.

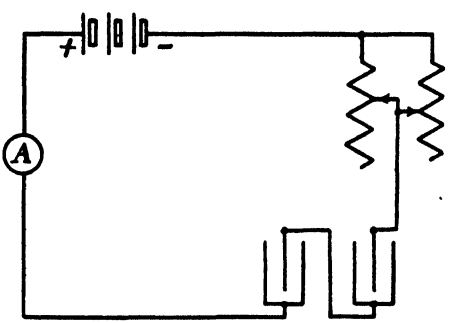


FIG. 151. Calibration of ammeter with the copper coulometer.

Hold the current as nearly steady as possible and plot a curve of current vs. time, taking readings as frequently as necessary to keep the variations from the mean, as read by the meter, under one per cent. After one or two hours remove the cathodes, again wash, dry and weigh. Determine the gain of weight in each case.

Since the equivalent weight of copper is 63.57/2 and that of silver is 107.88, the mass of copper deposited per coulomb is:

$$0.001118 \times 63.57/2 \times 1/107.88 = 0.000330 \text{ gram.}$$

From this and the amount deposited calculate the number of coulombs which flowed. Calculate also the coulombs indicated by the meter, that is, the area under the current-time curve. From these calculate the percentage correction for the meter, at the mean current.

**144. Electromotive Force.**

In addition to being conductors of electricity, electrolytes have the property of producing and maintaining a difference of potential between electrodes of dissimilar metals which are immersed in them. From such *electrolytic cells*, current can be drawn and, consequently, they may be used as sources of energy. Whenever this is done a chemical change takes place in the electrolyte or the electrodes (or both) and the actual rôle of the cell is the transformation of chemical into electrical energy.

A rigorous derivation, along modern lines, of the equation for the electromotive force produced by a cell is beyond the scope of this book. However, it is possible to derive this equation, with sufficient rigor for our purpose, by following the reasoning of Nernst. We have already introduced the idea of osmotic pressure in solutions, and we shall now consider another pressure, that which forces a substance, in contact with a solvent, to go into solution. Let this pressure, for a metal electrode, dipping into a solution of one of its own salts, be known as the *electrolytic solution pressure*, and have the value $P$, while osmotic pressure of the solution is $p$. If $P$ is greater than $p$, the metal will go into solution in the form of positive ions, as indicated in the discussion of electrolytic dissociation. At first thought it would seem that this action would go on until the metal were entirely dissolved, but it is experimentally known that such is not the case. The reason that the action eventually stops is that the metal, by losing positive ions, becomes negative with respect to the solution, and therefore exerts an attractive force on the ions which attempt to leave its surface. Eventually this attraction builds up to such a value that it plus the osmotic pressure just balances the solution pressure, and no more metal dissolves. If the negative charge on the electrode is removed by external electrical connections, the solution will again take place, and will continue at a rate just sufficient to maintain the potential of the electrode.

Now suppose that, during such a process, one equivalent weight of the metal is dissolved. This corresponds to the removal of that amount of metal (considered as obeying the general gas law as in Art. 139) from a region of pressure $P$ to one of lower pressure $p$. The work done in the expansion of a gas is, from elementary mechanics:

$$W = \int p dv, \tag{1}$$

where $v$ is the volume.

But, from Art. 139, Eq. (2), since $n$ was taken as $1/m$, where $m$ is the valence of the metal, and since $i$ is equal to one, all the dissolved metal having gone into solution as ions, we have:

$$pv = \frac{RT}{m}$$

or

$$dv = -\frac{RT}{mp^2}dp. \tag{2}$$

Substituting this value in Eq. (1) and introducing the limits of pressure:

$$W = -\frac{RT}{m}\int_P^p \frac{dp}{p} = \frac{RT}{m}\int_p^P \frac{dp}{p}$$

$$= \frac{RT}{m}\log\frac{P}{p}. \tag{3}$$

But this work has been transformed to electrical energy. The total quantity of electricity involved has been $Ne$ and, if this has been transferred across a difference of potential $V$, then the work done is $NeV$, so that:

$$NeV = \frac{RT}{m}\log\frac{P}{p}$$

or

$$V = \frac{RT}{Nem}\log\frac{P}{p}.$$

Putting in the numerical values,

$$R = 8.315 \text{ joules per degree}$$
$$N = 6.03 \times 10^{23}.$$
$$e = 4.80 \times 10^{-19} \text{ coulombs}$$

and transforming logarithms to the base ten:

$$V = \frac{19.8(10)^{-5}T}{m}\log_{10}\frac{P}{p} \text{ volts.} \tag{4}$$

**145. Normal Electrodes.**

The equation derived in the preceding article is not adapted to direct application because of the fact that $P$ is not a calculable quantity and $p$, while it may be calculated from the general gas law, involves a knowledge of the degree of dissociation in the solution. Moreover the potential of the electrode relative to the solution cannot be measured without inserting another electrode

in the solution, and, in the general case, this electrode will also assume a potential differing from that of the solution.

A number of schemes have been devised to measure directly the difference of potential between an electrode and its solution, but, since they do not agree among themselves, they must be considered as unsatisfactory. Consequently an arbitrary standard is employed in defining this potential. This standard is the *normal hydrogen electrode*, which consists of a strip of platinized platinum, partly immersed in pure hydrogen at atmospheric pressure and partly in a solution of hydrochloric acid of such concentration that it contains one gram-molecule of hydrogen ions per liter. The potential of this electrode with respect to the

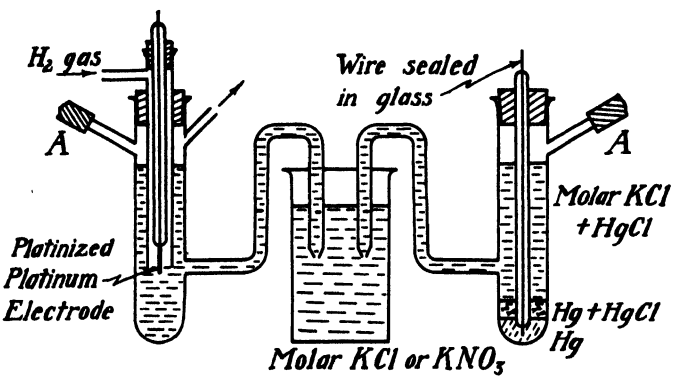


FIG. 152. Normal electrodes.

solution is arbitrarily taken as zero, this being possible because of the arbitrary nature of the zero of potential. As the normal hydrogen electrode is inconvenient to set up and use, a secondary standard, the *normal calomel electrode*, is commonly used in its place. This electrode is quite similar to the left side of the Weston standard cell as shown in Fig. 17, and consists of a mercury electrode, over which is spread a layer of paste made by grinding mercurous chloride (calomel) with mercury and moistening with a molar solution of potassium chloride. The solution, above this, consists of molar potassium chloride saturated with the almost insoluble mercurous chloride. The potential of the

normal calomel electrode with respect to its solution on the hydrogen electrode scale is 0.283 volt.

A typical laboratory set up of a normal calomel electrode and a hydrogen electrode arranged for measurement against each other is shown in Fig. 152. The middle vessel is used to bring the two solutions into contact. In the type of hydrogen electrode shown, the hydrogen is bubbled over the platinized platinum electrode. The side arms connecting to the middle vessel are filled by sucking on the rubber tubes at $A$–$A$ and closing these off with a pinch cock as soon as sufficient solution has been drawn over to fill the tube. Various other types of electrodes are manufactured by electrical instrument makers.

**146. Ionic Concentration Measurements.**

One of the principal applications of the results of the last few articles is the measurement of ionic concentrations in solutions. We shall consider the measurement of hydrogen ion concentrations in acid solutions. We may apply Eq. (4) of Art. 144 to this case, remembering that the valence of the molecular hydrogen gas is two, while that of the ions is one, and that the osmotic pressure of a solution is $kC$, where $C$ is the concentration of the solute, to obtain

$$V = 19.8(10)^{-5}T(\tfrac{1}{2}\log_{10} P - \log_{10} k - \log_{10} C_H) \text{ volts} \quad (1)$$

and, since, by definition, $V$ equals zero when $P$ equals one atmosphere and $C_H$ equals one gram-molecule per liter, we have

$$\log_{10} k = 0$$

or

$$V = 19.8(10)^{-5}T(\tfrac{1}{2}\log_{10} P - \log_{10} C_H) \text{ volts},$$

and

$$-\log_{10} C_H = \frac{V}{19.8(10)^{-5}T} - \frac{\log_{10} P}{2}. \quad (2)$$

For convenience the hydrogen ion concentration is usually expressed as

$$p_H = -\log_{10} C_H.$$

## EXPERIMENT 60

## MEASUREMENT OF HYDROGEN ION CONCENTRATION

Set up the electrode combination as shown in Fig. 152, or as in the manufacturer's directions, using for the acid solution an unknown concentration of $H_2SO_4$ supplied by the instructor. Measure the voltage across the combination with a potentiometer, making certain that the hydrogen has bubbled for sufficient time (usually a few minutes) to bring the potential to a steady value. Also read the barometric pressure and the temperature.

Suppose that the voltage thus measured is $V'$. The potential of the calomel electrode with respect to the solution is $+0.283$ and the $V$ of Eq. (2) is that of the solution with respect to the electrode, so

$$V = V' - 0.283 \text{ volt.}$$

Substitute this value in Eq. (2). Correct the barometric pressure for the vapor pressure of water at room temperature to find the partial pressure of the hydrogen, and express this pressure in terms of atmospheres (760 mm.). $T$, of course, is absolute temperature. Calculate the $p_H$ of the solution.

Rinse the electrode vessel and the hydrogen electrode with distilled water and then with the new unknown before filling the cell with the second sample.

### 147. The Glass Electrode.

While the hydrogen electrode is the primary reference standard for the measurement of $p_H$, it has several serious disadvantages, since the electrode may be "poisoned" by some solutions, particularly those occurring biologically, and may be made inaccurate by the presence of strong oxidizing or reducing agents. As a result it is replaced in many cases by a glass electrode which has the advantage of being extremely inert. It has been shown by a number of workers that an electrode which is composed of a thin soft glass bulb, containing an acid solution to which contact is made by a platinum wire, follows quite accurately changes in $p_H$. For the range from $p_H$ one to nine the glass electrode is consistently 0.352 volts positive with respect to a hydrogen electrode in the same solution. Outside of this range the potential difference varies and the glass electrode is not so satisfactory,

since the potential depends on the nature of the other ions in the solution, and is particularly sensitive to sodium. Some glasses are more suitable than others, and the best thus far available seems to be a sodium, calcium, silicate glass prepared by the Corning Glass Works as 015 glass. A commercial electrode of this glass is shown in Fig. 153.

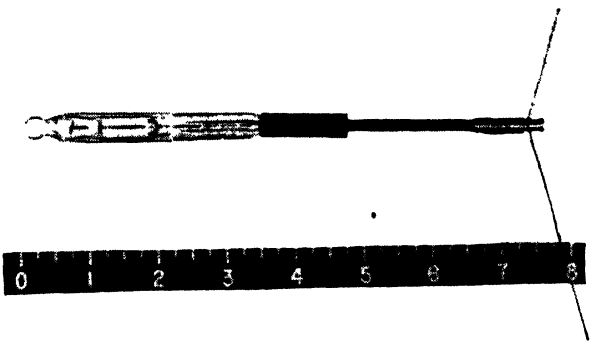


FIG. 153. Glass electrode. (Courtesy of Leeds and Northrup Co.)

While the glass electrode has important advantages from the viewpoint of chemical inertness it has the disadvantage that it has very high electrical resistance. As a result even very small currents drawn by the measuring device may cause appreciable errors and trouble is experienced with pick-up unless the measuring circuit is carefully shielded against electrostatic disturbances. A great deal of the early work with the glass electrode was done with electrometers but these have the disadvantages of requiring critical adjustment and of possessing only low sensitivity. The best present practice is to use vacuum tube amplifiers in conjunction with potentiometers. An amplifier designed for this purpose by the Leeds and Northrup Co. is shown schematically in Fig. 154. This outfit uses a RH-507 Westinghouse tube as an inverted triode, with the plate used as a control element and the grid as an anode. The anode is operated at 4.5 to 9.0 volts, and the control electrode at $-1.0$ to $-3.0$ volts with respect to the negative end of the cathode. Under these conditions the control current drawn from the cell never exceeds $2 \times 10^{-12}$ amperes. The filament current is controlled by the resistors $R_1$ and $R_2$ and the plate bias by $R_5$. A galvanometer

is used to measure the anode current, and its deflection may be controlled by $R_3$ and $R_4$. The scheme used for measurement is similar to that shown in Fig. 56, the unknown E.M.F. being balanced by an adjustable potential difference supplied, between the terminals marked "Pot," by the use of a precision poten-

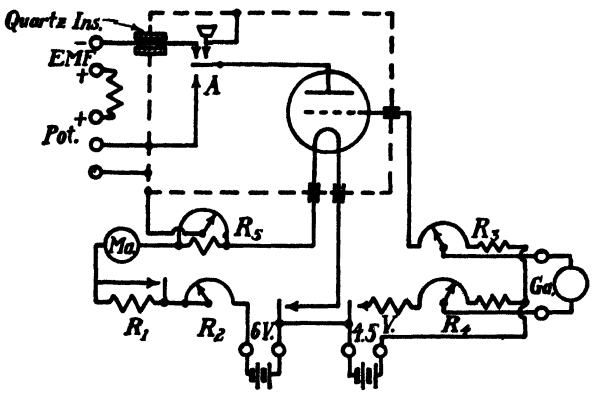


FIG. 154. Amplifier for use with glass electrode.

tiometer. In the use of the instrument the unknown potential is applied to the "EMF" terminals and the galvanometer is adjusted to any convenient deflection by the use of $R_4$ and $R_5$. The compensating potential is then adjusted until the reversal of switch $A$ makes no change in the deflection. A series resistor of five megohms prevents the flow of any appreciable polarizing current during the adjustment and at the final balance the current is practically zero. The instrument is useful not only for hydrogen ion concentration measurements but also for the determination of potential differences in any high resistance circuit in which the galvanometer currents with the conventional potentiometer might affect the measurements. As an example, photoelectric currents are conveniently measured with the outfit. The assembled apparatus for $p_H$ measurements is shown in Fig. 155.

In using the glass electrode the experimenter is limited by the fact that no adequate theory, such as that which exists for the hydrogen electrode, has been developed for this instrument. Consequently the glass electrode must be calibrated. This is done by the use of *buffer solutions* which are aqueous solutions of a weak acid and a salt of the same acid and which have the

property of maintaining their $p_H$ values in spite of the addition of small amounts of acid or base. Such solutions are supplied at known $p_H$ values by manufacturers of measuring equipment. After the electrode has been calibrated with such buffers so that its potential with respect to the reference electrode is known at one or two values of ion concentration, the difference between this potential and that computed for the hydrogen electrode may be found and applied to future readings. The calculations then become those used in Exp. 60.

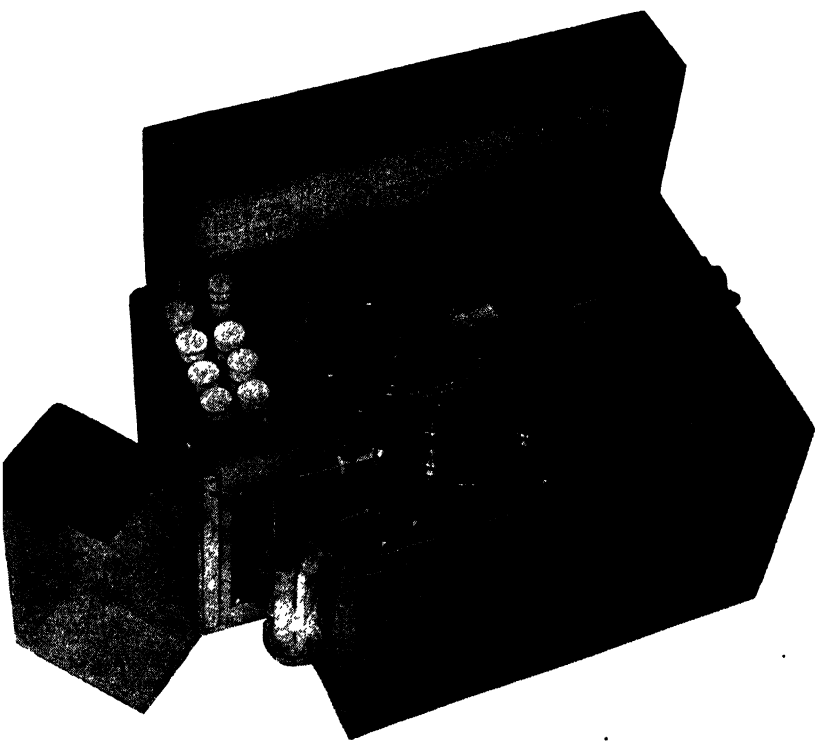

FIG. 155. Hydrogen ion concentration assembly. (Courtesy of Leeds and Northrup Co.)

## REFERENCES

MAC INNES, *The Principles of Electrochemistry*, Reinhold, 1939.
DOLE, *Experimental and Theoretical Electrochemistry*, McGraw-Hill, 1935.

# APPENDIX A

## UNITS

The MKSC system of units used in this text is one of a number of sets of units which has gained acceptance. It has been chosen both because it results in a simplification of many of the formulae involved in electrical measurements and because it essentially coincides with the *practical system* which is used almost universally in engineering practice and in terms of which nearly all laboratory instruments are calibrated. The most commonly used systems in addition to this are the *cgs-electrostatic* and the *cgs-electromagnetic* sets of units, frequently referred to as e.s.u. and e.m.u., respectively. Since many texts use these units, and since it may occasionally be necessary to transform to or from them, the table shown below, giving the relations between the three systems, may be found useful. As stated in Art. 12, it is necessary to fix arbitrarily the dimensions of one quantity, in addition to length, mass and time, in order to establish a set of electrical and magnetic units. In the MKSC system this fourth unit is taken as the coulomb. In the electrostatic system the condition is imposed by making the dielectric constant dimensionless, in the electromagnetic a similar choice is made for the permeability. Because of the value of dimensional reasoning, particularly for checking equations, the dimensions of the various units are given in the table for each of the three systems. The relations between the various units are given for the proposed new legal standards, and are in error by as much as several parts in 100,000 for the present International units. The velocity of light has been taken as $2.998 \times 10^8$ meters per second.

| MKSC units | | Equivalent Quantities | | | |
|---|---|---|---|---|---|
| | | e.s.u. | | e.m.u. | |
| Quantity | Dimensions | Quantity | Dimensions | Quantity | Dimensions |
| 1 meter | $M$ | 100 centimeters | $c$ | 100 centimeters | $c$ |
| 1 kilogram | $K$ | $10^3$ grams | $g$ | $10^3$ grams | $g$ |
| 1 second | $S$ | 1 second | $s$ | 1 second | $s$ |
| 1 newton | $MKS^{-2}$ | $10^5$ dynes | $cgs^{-2}$ | $10^5$ dynes | $cgs^{-2}$ |
| 1 joule | $M^2KS^{-2}$ | $10^7$ ergs | $c^2gs^{-2}$ | $10^7$ ergs | $c^2gs^{-2}$ |
| 1 coulomb | $C$ | $2.998\times10^9$ e.s.u. | $c^{3/2}g^{1/2}s^{-1}$ | $10^{-1}$ e.m.u. | $c^{1/2}g^{1/2}$ |
| 1 ampere | $S^{-1}C$ | $2.998\times10^9$ e.s.u. | $c^{3/2}g^{1/2}s^{-2}$ | $10^{-1}$ e.m.u. | $c^{1/2}g^{1/2}s^{-1}$ |
| 1 volt | $M^2KS^{-2}C^{-1}$ | 1/299.8 e.s.u. | $c^{1/2}g^{1/2}s^{-1}$ | $10^8$ e.m.u. | $c^{3/2}g^{1/2}s^{-2}$ |
| 1 ohm | $M^2KS^{-1}C^{-2}$ | $10^{-11}/8.988$ e.s.u. | $c^{-1}s$ | $10^9$ e.m.u. | $cs^{-1}$ |
| 1 henry | $M^2KC^{-2}$ | $10^{-11}/8.988$ e.s.u. | $c^{-1}s^2$ | $10^9$ e.m.u. | $c$ |
| 1 farad | $M^{-2}K^{-1}S^2C^2$ | $8.988\times10^{11}$ e.s.u. | $c$ | $10^{-9}$ e.m.u. | $c^{-1}s^2$ |
| 1 weber | $M^2KS^{-1}C^{-1}$ | 1/299.8 e.s.u. | $c^{1/2}g^{1/2}$ | $10^8$ maxwells | $c^{3/2}g^{1/2}s^{-1}$ |
| 1 amp. turn | $S^{-1}C$ | $3.767\times10^{10}$ e.s.u. | $c^{3/2}g^{1/2}s^{-2}$ | $0.4\pi$ gilberts | $c^{1/2}g^{1/2}s^{-1}$ |
| 1 amp. turn/meter | $M^{-1}S^{-1}C$ | $3.767\times10^8$ e.s.u. | $c^{1/2}g^{1/2}s^{-2}$ | $1.257\times10^{-2}$ oersteds | $c^{-1/2}g^{1/2}s^{-1}$ |

# SUBJECT INDEX

Note: Roman numerals refer to chapters, **bold face** to experiments and plain type to pages.


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